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Offering Strategies and Simulation of Multi Item Dynamic Auctions of Energy Contracts

Luiz A. Barroso, *Senior Member, IEEE*, Alexandre Street, *Member, IEEE*, Sergio Granville and Mario V. Pereira, *Fellow, IEEE*

***Abstract*— the objective of this work is threefold. We firstly present an optimization model for a price-taker hydrothermal generation company (Genco) to devise bidding strategies in multi-item dynamic auctions of long-term contracts. The bidding model calculates a willingness-to-supply curve (WSC), which takes into account the key issues on the auctioned contracts, such as its time horizon, the risk factors that affect the future contract outcomes, interdependence between auctioned products, and the agents' risk profile. Then, the risk profile of the Gencos are represented as piecewise linear utility functions and a practical specification approach is proposed. Finally, we present a simulator of a dynamic multi-item contract auction, where the set of auction rules for multiple products is implemented. The auction convergence price can be estimated from the successive application of the bidding model to each individual player at each round in the auction simulator. A real multi-product descending clock auction is simulated for the Brazilian power system under the proposed bidding scheme.**

***Index Terms* -- Power system economics, forward contracts, contract auctions, portfolio optimization, utility function.**

I. Introduction

Procurement of energy has always been an important function for electric utilities and consumers in order to serve their electricity supply needs. Objectives in energy procurement span from very short-term needs (for example, procurement of energy for day-ahead, real-time delivery or reserves) to long-term needs (energy purchase for supply adequacy purposes). In the case of emerging economies, the fast load growth makes the primary objective of energy procurement be the entrance of *sufficient* capacity in the most *efficient* way. Efficiency, in this case, basically means the lowest possible cost to the consumer.

The need to procure sufficiency with efficiency for the electricity supply has motivated the introduction of auctions of energy contracts as mechanisms to complement trading at the spot market. Energy contract auctions can be carried out to procure energy from the existing generation – contracts serve as a hedging mechanism for buyers and sellers – and/or they can be organized to facilitate capacity expansion. In this case, the contracts can establish delivery in a few years, which is highly desirable for new entrants to compete with existing suppliers. In addition, a forward contract provides revenue stability to the new entrant, which facilitates generation financing.

Energy contract auctions have been extensively used all around the world, such as in the US (New Jersey [1], Illinois [2] and New England [3]), Latin America (Brazil and Chile [4], Peru [5], and Colombia [6]) and in Europe (Spain [7]). They have been implemented as centralized or non-centralized processes with products ranging from forward contracts to call options. Reference [8] presents a good survey. The design of the auction mechanism has ranged from simple sealed-envelope pay-as-bid auctions (Chilean and Peruvian approaches) to more sophisticated approaches, such as the dynamic descending price-clock auctions involving in some cases different products (adopted in US, Brazil, Colombia and Spain). Reference [9] provides a survey for South America.

The existence of dynamic and iterative auctions creates relevant challenges under a bidder perspective and under a market designer standpoint. For each bidder, the question is how to develop bidding strategies that maximize revenue taking into account the long-term uncertainties, its risk profile and interdependence between auction products. Bidders are also interested in simulating the auction rules in order to estimate the auction convergence price and to test a bidding strategy. Conversely, regulators and market designers will be interested in analyzing and simulating bidding schemes in order to test a proposed auction mechanism. These tests aim to define the auction key parameters (duration and maximum number of rounds, price decrements, impact of reserve prices, etc) and to prevent abusive actions so that achieving efficiency in the contracting process.

A. Objective

The objective of this work is threefold:

- We firstly present an optimization model for a price-taker hydro generation company (Genco) to devise bidding strategies in multi-item dynamic auctions of mid-term contracts. The model calculates a willingness-to-supply curve (WSC) – introduced in [10] – which takes into account the key issues on the auctioned contracts, such as its time horizon; the risk factors that affect the future contract outcomes; interdependence between auctioned products and the agents' risk-profile. The WSC is the generator's *best-response function* during the auction.
- The risk profile of the Genco is represented as a

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piecewise linear utility function. A practical specification approach for the utility function is proposed.

- Once a methodology of calculating the risk-averse WSC is presented and exemplified, we finally present a simulator of a dynamic multi-item auction. Then, the set of auction rules for many interdependent products is implemented and the auction final price can be estimated from the successive (iterative) application of the WSC model to each individual player at each round. The proposed methodology is tested by means of a case study with real data from the Brazilian power system, in which a multi-product descending clock auction is simulated under the proposed bidding scheme for Gencos.

This work is based upon the energy auctions carried out in Brazil since 2004. Overall, amongst auctions for contract renewal and auctions for new capacity, the country has carried out 30 auctions, involving about 4,000 TWh in mid-long term energy contracts, settled for around 30 years, and whose transactions total about 500 billion USD. Each auction demand is based on the distribution company's projection and the product auctioned is a standardized forward energy contract[1]. The auction mechanism follows a hybrid design, which mixes a descending price clock scheme with a final pay-as-bid round. This work studies the very first auction carried out in 2004, where three energy contracts for future delivery were simultaneously auctioned in a dynamic process. The organization and success of this auction was of great importance and required the development of a framework to simulate the proposed auction designs and model agent behavior. In other words, a simulation tool to investigate the robustness of the rules as a whole was needed by the organizing entities. The auction organization and design would become a benchmark for future auctions to be carried out. The models presented here are the result of active research carried out at the time of the auction, which produced decision support tools used by the bidders during the auction itself and by auction designers to test the auction rules.

B. Contributions and paper organization

While strategic bidding for day-ahead spot markets has been extensively studied for the past 10 years, bidding simulation of dynamic energy contract auctions is a recent research area. In [11] a static competitive game model with risk-averse bidding is constructed to simulate the outcome of contract auctions in the Chilean market. Using portfolio concepts authors have observed the preferences of Gencos to sub-products offered in the Chilean auctions. In [12], the bidding strategy of a wind power producer in auctions for 3-6 month energy contracts is presented. The auction follows a descending clock format and the bidding strategy is developed by maximizing a risk measure for each possible auction price. In [13] auctions mechanisms are studied through auction theory by using Bayesian equilibrium concepts. Two sealed (static) bid auction formats are reviewed: a single object first-price auction and single object second price auction. These formats are analyzed under a pseudo common value and symmetric equilibrium framework. The developed models are applied to assess diverse elements (reserve price, winners curse and bidding strategy) in the Chilean energy contract auctions. Bayesian equilibrium is also used in [14] to analyze bilateral contract auctions carried out in Brazil in 2003 where Nash equilibrium was found through the expected payoff matrix of each agent in a static game.

With respect to the existing literature, the contributions of this work are the following: our bidding model represents fully each player's characteristics and portfolio and all uncertainties are fully represented by means of scenarios in a dynamic way (thus overcoming the static representation of some previous references). In this sense, our model shares some similarities to the one presented in [12] for wind producers. We, however, aim at using an alternative risk measure (utility functions) and we discuss in details practical aspects that emerge when applying such measure to define a risk preference in the mid-term[2]. We also combine our bidding model with the development of a dynamic auction simulation model, not presented in all the previous references. Such simulator enables auctioneers to assess the design of complex iterative descending price auctions and paves the way for a wide range market design analysis such as: testing of different auction mechanisms, rules, and parameters as well as their robustness to different risk-attitudes. The use of the simulator to test bidding strategies and assess the auction convergence price[3] will be of interest to Gencos. Our model is a practical one that has been applied by several Gencos to develop and assess their bidding strategies. We will then discuss some practical issues on the modeling of system components and risk profiles.

The remainder of the paper is organized as follows. Section II provides an overview of the Brazilian physical and regulatory framework in order to contextualize the environment in which Gencos are embedded. Section III describes the modeling of the bidding process of Gencos in the energy auction. Section IV introduces a simulation framework used to model a dynamic auction process. Section V presents computational results and Section VI concludes.

II. Auctions for Competitive Energy Procurement

A. Energy auctions in the Brazilian power sector

The Brazilian power system is the largest in Latin America with 106,000 MW of installed capacity (2009) and a consumption of 400 TWh/year. Hydropower is the predominant resource in the country and accounts for 71% of the country's installed capacity, but more than 90% in terms of energy production. As described in [15] the country relies upon energy contract auctions as the mechanism for energy procurement for regulated (or captive) users. Although

[1] Actually two types of financial agreements are negotiated: forward contracts (where one single price is traded) and options (where two payments are negotiated: strike and premium prices).

[2] As will be discussed, the proposed bidding tool allows the use of any other risk measure such as VaR or CVaR.

[3] Observe that the simulator does not calculate an "equilibrium" price because the auction price will be the result of a convergence following a specific set of rules.

contracts are financial instruments, they should be "anchored" by a physical generation capacity. This is measured in terms of a firm energy certificate (FEC) [16], which measures the energy contribution of a project in a dry year. FEC's are measured in MWh/year, or in average MW (avgMW), and are calculated and assigned by the Ministry of Energy to every project. These auctions are divided in specific auctions for *new energy* (carried out three and five years in advance of delivery; the contracts are driven for the construction of greenfield capacity only that will cover the forecasted load increase) and auctions for *existing energy*, which are intended to cover the existing load (contract renewal) and unexpected demand growth. Both auctions are centrally organized and carried out jointly for the set of Discos and follow a dynamic auction design (descending price clock auction). The contracting level in the system is close to 100%.

While the system relies on competition for contracts to achieve the market efficiency, the system dispatch remains cost-based and carried out in a centralized way by an independent system operator (ISO). Hydro plants are dispatched based on their expected opportunity costs ("water values"), which are computed by a multi-stage stochastic optimization model that takes into account a detailed representation of hydro plant operation and inflow uncertainties (see [16]). The ISO dispatch model needs to simulate the future system behavior under many possible resource conditions (hydrological scenarios), in order to obtain today's water value and optimal system schedule. A byproduct of such simulation is a set of simulated scenarios of the future generation of each generator in the system and the related optimal system water values, which in Brazil play the role of energy "spot" (short-term) prices. These spot prices are then used to settle energy imbalances (differences between produced and contracted volumes) and are very volatile, thus turning energy contracts essential mechanisms for the commercial feasibility of generators (see [15]).

B. *The Auction Mechanism*

The first procurement auction carried out under the revised regulatory model in Brazil was done in 2004 and offered three types of long-term energy contracts, referred to as Product 1 to Product 3. Each product was an eight-year financial energy supply contract with start dates in 2005, 2006 and 2007, as shown in Fig. 1.

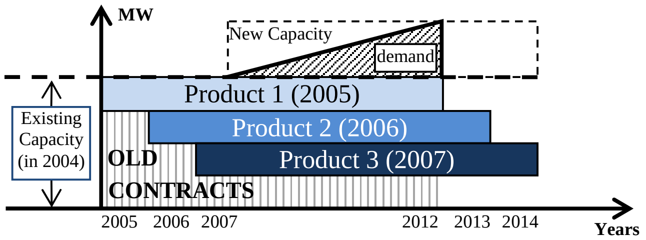

Fig. 1 – Products offered in the 2004 energy auction

Demand for the contracts came from Brazilian electricity distribution companies (with no government interference) and was over 17,000 average MW in total.

The auction mechanism followed a multi-product dynamic descending clock auction similar in overall design to those described in the recent literature [18] and which have recently been used to auction 'virtual' electricity generation capacity in France and elsewhere [18]. In the descending auctions, the auctioneer starts at a very high price, which is successively lowered over a set of rounds (in a reverse auction) while the total supply remains greater than total demand. When the total offered (by Gencos) energy meets total demand, the remaining bidders win by selling their items at their final prices (uniform pricing). In a nutshell, the process works as follows: each round, the auctioneer announces the items' price. There is a price 'clock' for the item and each bidder then indicates the total quantity of each product he desires to sell at the current prices. In a multiple-item auction the total quantity offered is the sum of each bidder's quantity at that price for each product. In subsequent rounds, the price decreases if there is excess supply for each item, and the bidders again express the total quantity at the new price. As long as there is excess supply, the price decreases. The price decrement is determined following the best practice methods, essentially in relation to the extent of the excess supply. This process is repeated until there is no excess supply - or that the excess supply is minimal - and that the incumbent prices are smaller or equal to the reserve prices. The tentative prices and assignments of quantities to each bidder then become final in a uniform pricing approach.

The full auction mechanism has several details (see [20] and [21]). We highlight the activity rule, which is intended to enhance price discovery by motivating each bidder to bid throughout the auction in a manner that is consistent with the bidder's true interests. The most common activity rule is that a bidder cannot bid in a subsequent round if he has failed to bid in the previous one. This translates into a rule where agents are allowed to switch products to respond to changes in the relative prices throughout the auction; the only constraint is that the total offered quantity cannot exceed the total quantity offered in the previous round.

The Brazilian auction mechanism has also implemented a second phase (after the dynamic auction is concluded), formed by a multi-product sealed-bid discriminatory final round. This work discusses only the bidding strategies and design of the uniform-price dynamic auction ("Phase I"). This was the phase that invoked most fear to auction participants in this first auction given their inexperience on the topic.

III. A MODEL FOR BIDDING IN ENERGY AUCTIONS

The problem faced by the participant is to compute the quantity to be offered to each product for each possible price vector that could appear during the auction. It is a best-response function and will be presented next.

A price-taker Genco is assumed. This assumption is justified by the fact that the presence of complex auction rules based on a dynamic process with a multiproduct environment turns any price-maker strategy quite difficult to be developed. In addition, every auction is quite different from the other, the frequency of such auctions is not as much as of the day-ahead spot auctions and the amount of information released by the auctioneer is very limited in each auction (auctioned demand is not public, for example). This creates difficulties to use information of past auctions to devise a strategic bidding and the iterative computation of price-taker best-response bidding functions turns out to be the immediate option. Finally, in the Brazilian auctions, competition has been very high and market power has not been a concern.

A. Bidder revenue stream

The products auctioned were fixed-price financial forward contracts for differences. The energy delivery risk belongs to seller, who is not obliged to physically produce the contracted amount, but must clear the differences between energy production and total contracted amount in the spot market. The revenue stream of a Genco is composed of three terms: the fixed price contract revenue (deterministic) and two stochastic components due to the positive or negative market clearing and generation expenses (variable costs). Assuming the uncertainties are represented by scenarios, the revenue stream for each contracting period *t* (assumed to be monthly based) and simulated scenario *s* of a Genco owing n_u generation units and with n_c forward contracts is given by:

$$R_{t,s} = \sum_i P_i \cdot E_{i,t} \cdot h_t + (\sum_i G_{i,t,s} - \sum_i E_{i,t} \cdot h_t) \cdot \pi_{t,s} - \sum_i G_{i,t,s} \cdot c_{i,t}. \quad (1)$$

Where *i* indexes the contracts (set of contracts $\{1,\ldots, n_c\}$), *j* indexes the generation units (set of units $\{1,\ldots,n_u\}$), and *s* indexes the scenarios ($\{1,\ldots,S\}$). $G_{j,t,s}$ is the generation amount of unit *j* in each period *t* and simulated scenario *s* (in MWh); $E_{i,t}$ is the energy amount (in avgMW) of contract *i* at period *t* (includes existing and candidate auction contracts); P_i is the price of contract *i* (in $/MWh), h_t is the number of hours in period *t* and $c_{j,t}$is operating cost (in $/MWh) of unit *j* in period *t*.

In expression (1) each contract can have different starting dates, duration and monthly profiles. Such differences can be addressed by means of the $E_{i,t}$ quantity variable, which should cover the time horizon for each contract, assuming zero value at periods in which contract *i* is not available. We assume the contract supply profile is fixed and flat ($E_{i,t} = E_i$ for all *t* in which the contract holds and zero otherwise) and this is known in advance. The energy profile $E_{i,t}$ can be addressed by means of an indicator coefficient $q_{i,t}$, which values one during the whole contract *i* time horizon and zero in the remaining periods, multiplied by the average nominal contract energy amount e_i (in avgMW). Hence, Gencos decisions consist in defining how much of each contract should be signed in terms of nominal average energy (vector $\mathbf{e}=[e_1,\ldots,e_{n_c}]^T$).

B. Risks and uncertainties for generators in the contracts

Energy contracts provide an adequate hedge against spot price volatility in the case of thermal plants, since its price-quantity risk can be limited to its own variable operating cost and available capacity. In the case of hydro-based Gencos, however, bilateral contracts might not be sufficient to provide a complete hedge. Because the Brazilian system is hydro-dominated, spot prices are higher in drought situations, which are exactly when hydro plants have lower production capability and need to purchase energy to meet their contract obligations. In other words, there is a negative correlation between spot price and hydro-plant production. This creates a contracting dilemma for hydro: if lightly contracted, the hydro will be exposed in low price periods which, occur frequently and may last for years; if heavily contracted (e.g. in 100% of its FEC amount), it will be exposed to extremely high prices in the dry periods, when it may not produce enough to meet its contract (hydrological risk, as discussed in [22]). This situation is made even worse by the variability of individual hydro production in different basins, which have different hydrological regimes. This means that hydro plants may be often forced to transact substantial amounts of energy in the spot market, which might disrupt their revenue streams depending on the spot prices.

The uncertain parameters in our model are the production of the generators and the spot prices. They were represented by means of scenarios through a “fundamental” (or “structural") forecasting, where spot prices and generation scenarios are calculated from the solution of a generation stochastic dispatch model. Such model simulates the Brazilian hydrothermal scheduling for a set of hydrological conditions (Monte Carlo simulation) during the study horizon. We use the same stochastic hydro-thermal dispatch model that is used to calculate the optimal system operation policy of the Brazilian power system (as described in Section II) for a defined supply and demand scenario, taking into account inflows uncertainties. The result of the simulation is a set of *S* scenarios for each stochastic variable ($G_{j,t,s}$ and $\pi_{t,s}$) for each time step *t*.

C. Risk-aversion modeling

Several risk measures can be used to represent a risk profile. One example is the Conditional Value at Risk (CVaR) [23], which has recently become very popular as a risk measure. In this work, however, we decided not to use the CVaR approach and the risk preference of the decision maker is represented through a von-Neumann Morgenstern Expected Utility (EU) functional instead[4]. The EU functional takes into account the whole range of scenarios by “translating” monetary outcomes into “utility units” and expresses agents’ preferences through the expectation of the resultant “risk-adjusted” utility scenarios [15]. The objective of a rational agent is to maximize the expected utility.

The agent risk profile is characterized by the *form* of the utility function. A risk-neutral Genco would have an affine UF. This means that a revenue increase has the same impact, in absolute terms, as a revenue decrease of the same magnitude. Instead, a risk-averse agent would have a concave UF, as shown in Fig. 2. In this case, the loss of a “bad” outcome is not “compensated” by the gain of a “good” one with the same magnitude, which means that the marginal increasing rate decreases as long as the revenue outcome increases. In this figure a decrease (-d) from a reference

[4] CVaR could be introduced in our model without additional complexity.

wealth outcome (R_0) results in a utility loss of DU^{dw}, which is greater than the utility gain (DU^{up}) due to a wealth increase of the same magnitude (+d) on R_0. This is the main characteristic of a risk-averse agent, which is the case of this work. Risk-taking agents, on the other hand, would have a convex UF.

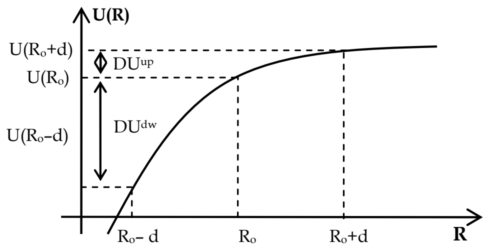

Fig. 2 – Concave UF representing a risk-averse profile.

D. Piecewise linear representation of utility functions

Instead of nonlinear smooth UF's, such as the logarithmic and negative exponential functions, this work adopts a piecewise linear representation of the UF (PLUF) as shown in Fig. 3.

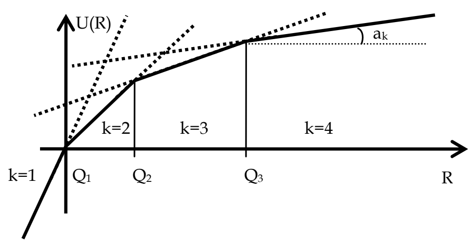

Fig. 3 – Risk aversion profile through the PLUF.

Two reasons have motivated the use of a PLUF. The first is related to the flexibility needed in real life implementation: in the long-term, decision makers are, generally, not interested in specifying the risk aversion for the entire revenue domain but only for specific points in which Gencos' financial situation (*status quo*) may change. This is supported by the fact that Gencos' investors are generally locally risk-neutral, but globally averse. Such change occurs when a given decision makes the total wealth cross a critical revenue point, e.g., the operational breakeven, resulting in a worse annual financial demonstrative. Thus, for small revenue variations, between such critical points, Gencos' decision makers should behave as risk-neutral agents (linear UF). The adjustments in the breakpoints and slopes of the PLUF define different risk-averse behaviors. The second reason is to avoid the representation of nonlinear smooth curves, which would require the use of nonlinear algorithms to solve the best-response problem. It is worth mentioning that PLUF's can approximate any concave smooth UF with a demanded precision level.

The EU of a discrete random revenue (R) defined by a set of S outcomes and associated probabilities $\{R_s, p_s\}_{s=1,\dots,S}$ can be represented through a linear programming (LP) problem:

$$E[U(R)] = \mathrm{Max}_{(u)} \textstyle\sum_s u_s \cdot p_s \quad (2)$$

Subject to:

$$u_s \le a_k \cdot R_s + b_k \quad \forall\, k, s \quad (2.1)$$
$$u_s \in \Re \quad \forall\, s. \quad (2.2)$$

Where, k represents the segments index ($\{1,\dots,K\}$), a_k represents the slope of the k-th segment, b_k represents the linear coefficient of the k-th segment and u_s is a decision variable that represents the UF of each revenue scenario s. Fig. 3 shows a PLUF with four segments (Q_k represents a break point, where the marginal utility changes from a_k to a_{k+1}).

E. Specification of the PLUF

Many different approaches can be used to estimate an agent UF [15]. Our proposed approach is to construct a PLUF of a Genco in order to express the marginal benefit of being in different revenue ranges and to represent it based on financial or accounting performance indexes. To do that, a Genco needs to collect a set of critical revenue points together with the marginal utility slope coefficients, which represent the marginal satisfaction of the Genco when changing from one point to another. Some approaches seemed to be of interest to Gencos when applying this methodology in real-life case studies.

In the first one, Gencos calculate reference revenues – the breakpoints of Fig. 3 – and define the marginal utilities (slopes) according to a given criterion such as the impact of reaching lower breakpoints. For example, some Gencos associate the slopes with the possibility of not reaching financial viability of the company. In this case, whenever the cash-flow approaches lower reference revenues (breakpoints) the slope increases. In the second approach, still within the reference revenue approach, Gencos calculate the slope among different breakpoints of the PLUF according to the "spread" a bank would charge in loan to a company with such revenue as a guarantee. In general, the piecewise linear representation gives a lot of flexibility and degrees of freedom for Gencos to express their risk aversion according to concrete financial indicators. One can adapt this idea in many different ways in accordance to any sort of relevant index that can be mapped into Genco's revenue dimension. Examples of alternative accounting indexes that can be used are debt-to-equity targets, expected spot revenue without contracts, expected revenue of risk-neutral decisions, etc. The only requirement is to keep marginal utilities coefficients non-increasing in order to keep the UF concave.

Observe that the calibration of the PLUF is a key issue to define a plausible risk profile. This is not an easy task. Although it is well-recognized in the literature that utility specification is a personal and subjective issue, the authors experience in real problems has shown that the risk profile can be easier expressed if viewed as marginal penalties for specified revenue blocks as a PLUF.

F. Best-response model: the Willingness-to-Supply Curve

The strategy for participating in a multi-product descending dynamic clock auction can be summarized as follows: for each price-vector released from the auctioneer (price of each product), a given Genco should choose the quantity-vector (quantity to be offered in each product) that maximizes the resultant revenue stream. This is similar to a best-response portfolio optimization problem under the absence of market power, but applied to an iterative auction. This supply function can be represented by a willingness-to-supply (WSC) curve, which is an optimal bidding curve to be used during the contract auction rounds.

The main idea of the curve is to generate a mapping between candidate contract prices and optimal quantity bids (see Fig. 4) for a price-taker Genco. Multi-product auctions require a multi-dimensional WSC.

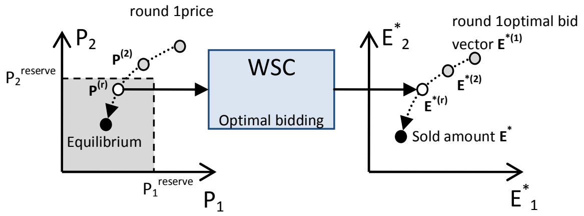

Fig. 4 – Bi-dimensional WSC. The gray area is the feasible region for prices (the item's prices are lower than the reserve prices). The right side illustrates a possible bidding trajectory due to a given WSC map.

The WSC, which is a function of the price vector $\mathbf{P}=[P_1, \ldots, P_{nc}]^T$ of the products, depends on the uncertainties (spot prices and future generation), on the risk profile, and timing characteristics: initial dates and durations. The optimal quantity bids for a Genco should be limited to its maximum amount of annual FEC's at the first round and to the total offered quantity in the previous round. Such constraint allows agents to switch their quantities among products to respond to changes in the relative prices throughout the auction, but guarantees that the total auction offer be monotonically decreasing (important to ensure auction's convergence [18]).

The following parametric (on prices vector per round) stochastic LP model provides the mathematical formulation for the portfolio-based optimal bidding strategy under uncertainty for a Genco at any auction round *r*:

$$\mathbf{e_r}^*(\mathbf{P}) = \text{argmax}_{(u,e,R)} \textstyle\sum_t \sum_s u_{t,s} \cdot p_s \cdot (1+J)^{-t} \quad (3)$$

Subjected to:

$$u_{t,s} \leq a_k \cdot R_{t,s} + b_k \quad \forall\, k, t, s \quad (3.1)$$

$$R_{t,s} = \textstyle\sum_i P_i \cdot e_i \cdot q_{i,t} \cdot h_t + [\sum_j G_{j,t,s} - \sum_i e_i \cdot q_{i,t} \cdot h_t] \cdot \pi_{t,s} - \sum_j G_{j,t,s} \cdot c_{j,t} \quad \forall t,s \quad (3.2)$$

$$\textstyle\sum_i e_i \leq Q_{r-1} \quad (3.3)$$

$$u_{t,s} \in \Re \;\forall\, t,s \;\text{ and }\; e_i \in \Re_+ \;\forall\, i \quad (3.4)$$

In model (3)-(3.4), expression (3) assesses the expected utility functional for a separable per-period PLUF maximizer agent whose timing preference (inter-period) is accounted through the risk-free discount factor $(1+J)^{-t}$. Expression (3.1) was introduced before in (2.1); (3.2) matches expression (1) by substituting the per-period contract energy amount ($E_{i,t}$) by its average nominal amount (e_i – *decision variable*) multiplied by its indicator coefficient ($q_{i,t}$), which values one during the time horizon of contract *i* and zero otherwise. Expression (3.3) handles the non-increasing amounts per-round and the total FEC constraint. We assume Q_0 = FEC and that Q_r is updated with the sum of the optimal bids ($\sum_i e_i^*$) of the previous round. The Genco's existing portfolio can be accommodated by extending the number of contracts and fixing the associated e_i variables to the known amounts. For the sake of simplicity we will omit them in the formulation; however, we remark that they should be considered because a bidding strategy is affected by the synergies between existing and candidate contracts. Moreover, as will be further detailed at section IV.C, the existing contract portfolio of the Genco will play an important role during the auction process in order to incorporate in the bidding model the information of past bids.

Finally, observe that committing resources too early (for example, 1 year ahead) comes at the expense of an opportunity cost (sales at the spot market). This tradeoff between a fixed contract committed years ahead and sales at the (volatile) spot market is explicitly taken into account in the willingness-to-supply curve (eq. (3.2) of model (3)).

IV. A Simulator of Dynamic Contract Auctions

Once a methodology to calculate a WSC is presented, the next step is to incorporate it into an iterative process that simulates the descending-price clock auction mechanism and rules. As mentioned earlier, dynamic auctions are sophisticated mechanisms and dynamic multi-unit auctions are quite complex themselves. For this reason, the ability to simulate such complex auction rules can be a very powerful mechanism for market designers – that can test rules and parameters among the many design options before implementing them – and for generators, who can estimate the auction final price.

A. *Implementing auction rules in a simulator*

The design of an auction simulator is nothing more than implementing the set of auction rules under evaluation. Several different rules can be implemented. For example, Brazil, Colombia and New England have been recently using descending price clock auctions to allocate forward contracts (Brazil) and reliability options (Colombia and NEPPOOL) ([3][6][28]). The auction conceptual design is the same for them all, but there are specific rules on reserve prices elasticity, stopping criteria, and intra-round bidding in the Colombian and New England's auction rules, which are not observed in Brazil. These rules could be implemented in our proposed framework as well.

B. *The auction rules implemented*

In this work we have developed an auction simulator that implements the Brazilian auction rules described in Section II.B (see the Appendix and [21] for a detailed description of these rules). The rules have, in essence, seven main steps:

1. Auctioneer defines auctioned demand for each product.
2. Auctioneer defines the opening auction price and the reserve price (i.e., maximum price) for each product.
3. Auctioneer defines a price-decrement rule (for example, higher decrements if more oversupply is observed);
4. Auctioneer releases a vector of prices for each product.
5. Each generator optimizes its quantity bidding strategy using the portfolio based WSC model of Section III. The WSC model is loaded with scenarios and data of each individual Genco. Alternative bidding models can be used

in this step, such as “wizards” (pre-programmed strategies following a specific rationale) or even human beings bidding on behalf of Gencos.

6. Auctioneer evaluates if the total supply matches the total demand (the summation among all product[5]). If not, prices of products in which supply exceeds demand are reduced following the rule defined in 3 and returns to step 4. If yes, go to next step.
7. Auctioneer verifies if the product’s price is smaller than its reserve prices. If yes, the auction finishes for this product and supply and demand are cleared at the given prices. If not, then demand is reduced so as to ensure a little oversupply and the process returns to step 4.

Fig. 5 depicts the main scheme of the auction simulator process that implements these rules (see Appendix for details). For a given price vector at each round (provided by the “auctioneer”), each agent returns quantities to be sold at the actual round prices. The optimization module described in Section III can be run to return the optimal quantities to be sold at the actual round prices taking into account the risk-profile of each agent. The auctioneer processes the results according to the auction rules and another round is started. The process goes on until the auction convergence criteria are met.

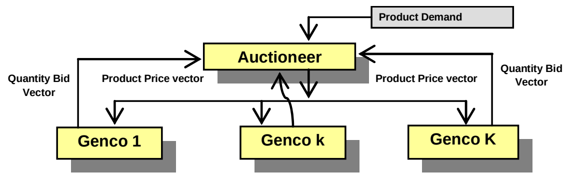

Fig. 5 – Bidding scheme of each auction’s round.

C. Design and platform of the auction simulator

The auction simulator was implemented in a distributed way, requiring a network of computers using the TCP/IP protocol. Each remote computer (station) could represent one or more bidders during the auction process. The platform is composed of two main modules: the auctioneer “module”, which is responsible for setting the key auction parameters (number of products, demands, reserve prices, decrements, etc), to release price vectors and receive quantity bids, and to stop the auction process; and a module for the bidders, which basically receive prices and return quantity bids according to some preset rule. Such rules can vary from simple user-defined pre-programmed bidding curves (“wizards”) until complex optimization problems such as the WSC portfolio optimization model presented in Section III.

V. Case Studies

This section presents some applications of the WSC and the auction simulator proposed in this work. We will focus our case studies on the same 2004 energy auction that has originated the research presented in this paper.

A. Modeling of generation companies

We have modeled the 11 most representative Gencos present in the 2004 auction. The portfolio of existing selling contracts for each of these companies was represented in greater details by means of an exhaustive research on public databases. The use of representative scenarios for the uncertain parameters is a key point in the presented methodology. Hence, we have relied on the official technical data available from the Independent System Operator website and on its same dispatch model to simulate the scheduling of the system to produce 200 scenarios of energy spot prices and production for each generator of each company represented. The study horizon goes from 2005 (starting date of the first contract) until 2014 (ending date of the last contract being auctioned). Most of the energy supplied was hydro energy. Approximately 59% of the available hydro supply is accounted for by three federal government-owned companies, 26% by state government-owned companies, and 15% by private players.

The utility functions used in this case study to model Gencos’ risk attitude were constructed to reflect the regret of not achieving the existing contracting portfolio price[6]. An annual revenue target was established for each Genco as being the average price of each existing portfolio times each available FEC for bidding. Intermediary revenue break points were set to represent percentages of each Genco’s target revenue. Together with such break points, a set of marginal utility slopes were defined according to Table I.

Table I - PLUF segments

Segment	Revenue segment upper limit (% of annual target revenue)	Segment slope (utility / Millions of R$)
1	50	2.0
2	70	1.5
3	100	1.2
4	+ ∝	1.0

B. Willingness to supply curve

We will first illustrate the results of the WSC model presented in Section III for a given Genco. Due to the multi-dimensional nature of the WSC, 4-dimensional even in the case of two-products, we show here the single-product WSC of a hydro-thermal Genco having 100 avgMW of FEC. Two cases are provided: risk-averse, using the PLUF (Table 1), and risk-neutral.

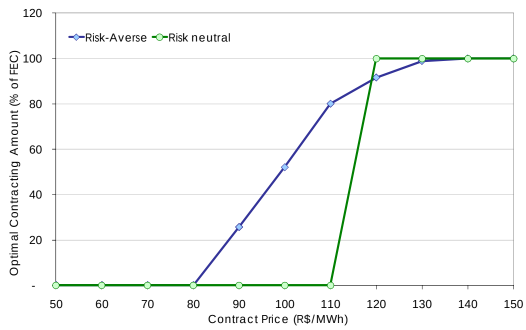

Fig. 6 – WSC for both risk-averse and risk-neutral offers (1 USD = 1,7 R$).

[5] There are specific rules to deal with undersupply for all products, in this case a demand reduction is applied. For more details see [25].

[6] Authors do not intend to suggest a risk-attitude for each Genco. The purpose is to exemplify the usage of the proposed methodology in a realistic manner. Nevertheless, the presented way in which the PLUF is specified is not “far from” the way most of the Gencos adopted for this auction.

The risk-neutral curve is stepwise and shifts from 0 to 100% of the total FEC (100 avgMW) at an indifference contract price (ICP) of 120 R$/MWh. This is the contract price for which the total Expected Net Present Value (ENPV) of the candidate contract is greater than the ENPV of the spot sales. From this point on, the risk-neutral agent will be willing-to-supply as much energy as possible in such contract. On the other hand, risk-aversive agents will search for a mix between spot and contract sales in order to mitigate the portfolio risk according to each risk preference, even if the contract price is lower than the ICP. This behavior can be verified in Fig. 6. Furthermore, the risk-averse WSC captures the hedging effect due to the risk of not producing the total contract amount, the well-known "quantity risk", by not contracting 100% of its total FEC, at prices even higher than the ICP. Thus, a risk-averse Genco maintains a safe margin not contracted, to be cleared at the spot market, to hedge against scenarios with deficit of production and high spot prices.

The WSC can be calculated to every Genco modeled. These curves provide the Genco's best-responses to each price-vector of the products. If a single product was being auctioned, the price that clears the aggregated WSC of all companies and the demand could be a proxy for the auction outcome. In a multiple product auction, however, this does not strictly hold because of the complexity of the auction rules, interdependence between different products and the preferences of each Genco over them all. For this reason, in order to calculate the auction final (convergence) price, the complete auction simulation is carried out next.

C. Auction simulation

The auction simulator described in Section IV was executed having the WSC optimization model to calculate the bidding at each round. Each product demand and reserve prices were set according to Table II (prices are shown in Brazilian currency – "*Reais (R$)*").

TABLE II - PRODUCT INFORMATION

Products (start of each contract)	Total Discos Demand (avgMW)	Starting Price (R$/MWh)	Reserve Price (R$/MWh)
2005	14,658.0	106.00	80.00
2006	6,879.0	106.00	90.00
2007	1,586.0	106.00	96.00

Fig. 7 illustrates the evolution of the prices of each product per auction round.

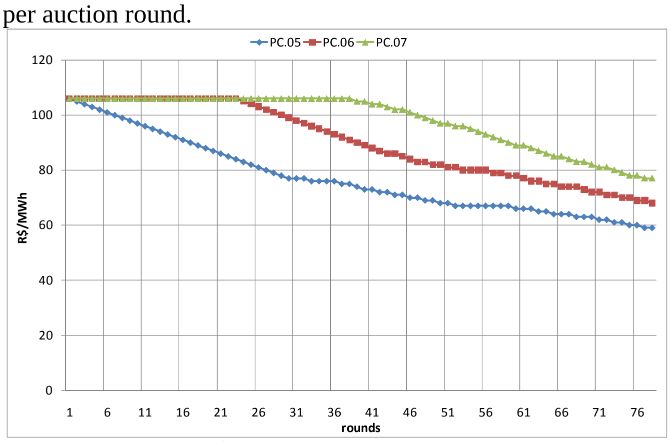

Fig. 7 –Prices per round of auction simulation

We firstly observed the auction convergence after achieving its stopping criteria in 78 rounds. This is an interesting market design result: in the Brazilian auction each round was supposed to last (fixed) 15 minutes. The large number of rounds until convergence coupled with this fixed duration could have been used by authorities to avoid a long and exhausting auction (15 x 78 = 19,5 hours, in this simulation). Instead, authorities decided to stick to the fixed round duration, ignoring the model results, and the outcome was an auction which lasted 6 hours and a subsequent (similar) one carried out in 2005 that lasted 18 hours. From 2006 the fixed round duration was abolished.The evolution of the product prices per round reflects the preference of Gencos between contracts: for the same price, earlier contracts are preferred. Depending on the relative price difference between products, Gencos start to bid in other products, whose prices then start to fall because of an excess of supply.

Table III provides the final prices of each product in the last round. Table IV provides the final contracted amounts of each Genco per product.

TABLE III
FINAL AUCTION PRICES (1 USD = 1,7 R$)

(R$/MWh)	2005	2006	2007
Final Prices	59,0	68,0	77,0

TABLE IV
ENERGY ALLOCATED TO EACH GENCO (AVGMW AND % OF FEC).

Gencos	2005	2006	2007	Total	% Contracted
furnas	5.379	369	257	6.005	86%
chesf	3.123	1.074	362	4.559	99%
cemig	2.275	156	31	2.462	73%
cesp	2.416	0	0	2.416	70%
eletronorte	1.991	197	0	2.188	97%
tractebel	1.145	479	54	1.678	96%
copel	900	215	0	1.115	92%
duke	580	0	0	580	68%
cien	196	0	0	196	51%
ceee	250	44	0	294	78%
cdsa	154	0	0	154	66%
outros	225	0	0	225	100%
Total	**18.634**	**2.534**	**704**	**21.872**	**85%**

The above results illustrate a few of the several outputs that can be produced by this type of simulation tool. Finally, a key challenge for the successful application of this type of model is a correct calibration. As discussed in Section III.C, eliciting risk-averse utility functions is challenging and the calibration of the simulation model is even more. A model badly calibrated can produce weird and unreasonable results. In the author's experience, the calibration of the model is a time-consuming task that involves a lot of knowledge and experience in the market being simulated. This, however, does not prevent or limit the model application from producing useful bidding and policy insights and results.

VI. CONCLUSIONS

This work has provided a mathematical formulation for long-term WSC of risk-averse and price-takers Gencos. Such model has shown to be useful for Gencos when bidding in dynamic contract auctions and for regulators in order to estimate long-term prices. Furthermore, a vast range of applications and market monitoring studies can rise from this methodology. By exploring the sensitivities between the auction final prices and the inputted parameters, such as, Gencos' risk-profiles, reserve prices and total demand, one can assess the impact of each isolated or combined changes in the forward prices.

We envision several applications for the methodology and simulator presented in this work to carry out several analyses for the "checks and balances" in the hard task of auction design. Some of them are the definition of critical parameters of the auction (number and duration of rounds, definition of reserve prices, definition of number of products to be simultaneously auctioned) and understanding the decision space of bidders.

For future work, we suggest the modeling of the Phase 2 of the auction, which includes a final pay-as-bid round once the dynamic phase is concluded. A very first approach is to use the auction simulator presented here to estimate the auction convergence price, which will be the marginal bid in the next phase, and thus devise a bidding strategy for Phase 2 that undercuts this value while at the same time maximize profits (which leads to the well-known strategy of bidding the expected marginal clearing price). More sophisticated approaches include sampling demand realizations for Phase 2 and modeling of strategic behavior.

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APPENDIX – DETAILED AUCTION RULES

This Appendix presents in detail the rules of the auction selected to be implemented in the simulator described in Section IV.

At the beginning of the process the auctioneer sets the initial prices for each product as well as their volumes being auctioned. Each bidder is allowed to offer standard lots of 1 average MW each. Transferring quantity lots between products is allowed, but it is required that the total amount of energy over all products should be monotonically decreasing[7].

At each round Gencos bid the quantity they are willing to supply to each product at their incumbent prices. The auctioneer sums up the total bids for each product and informs bidders about the status of their energy lots: "free lots" or "restricted lots". Free lots are those allocated to products where supply exceeds demand, namely "open products". These product prices will be reduced in the subsequent round. Restricted lots are those allocated to products where demand exceeds supply, namely "closed products". The price of a closed product will not decrease from one round to another. While free lots should be reoffered at every round – they are allowed to be reduced in the current product or resubmitted to other closed or open products – "restricted lots" cannot move while they are allocated in a closed product.

After each round, Gencos adjust their WSC models by fixing (constraining) their restricted lots in each product as if they were part of their existing portfolios, since such amounts are in a closed product and will be sold at the incumbent prices if the stopping criterion is achieved.

The auction stopping criterion is based on two factors: total supply and demand balance and the reserve prices, which are externally defined and reflects the consumers' willingness-to-pay. The first condition of the stopping criterion requires that total supply (lots) over all products meets total demand whereas the second one requires that each product price be lower than its respective reserve price. If total demand is not exceeded by total supply, but the reserve price is not achieved for at least one product, there is a demand reduction for the product whose incumbent price is higher than the reserve price.

The joint requirement of decreasing quantity offers and demand reduction ensures a finite-step convergence for the auction under a reasonable assumption that no Genco would bid more than zero in the case of prices being zero. This can be easily verified in the case where reserve price condition is already achieved: if the supply and demand condition is not met, at least one product will face a price decrement, and since such decrement is positive and bounded below, despite any possible lots movement among products, prices will still decrease to zero unless quantities are decreased and the stopping criterion is reached. In the case where the supply and demand condition is already met and there is at least one product price greater than its reserve price, a demand curtailment will take place in this product, implying in a price decrement that will inevitably lead prices to the stopping criterion. In the general case, where none of the conditions are attended, prices and quantities will decrease until one of the aforementioned cases is verified.

Finally, once achieved the stopping criterion, the auction finishes. All "restricted lots" of each Genco at the last round become committed and are then negotiated with Discos by means of a cross-bilateral contracting scheme where contracts quantities that each Genco sells to each Disco is a proportion of the Genco's final offer. The proportion is taken with respect to each Disco's declared demand.

[7] This condition is ensured by constraining Gencos to provide decreasing quantity bids. It is incorporated in the bidding model by constraint (3.3).

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VII. BIOGRAPHIES

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