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Introduction

Flows with particles in suspension, i.e. the analysis of small rigid particles suspended in a viscous fluid is one of the main focuses of engineering research and it is still a challenging and important task in many applications such as sedimentation, rheology and fluidized suspensions. The coupling between the suspending liquid flow and the particles' motion is the central point in the complete understanding of these flows.

Moreover, the study of the evolution of the configuration of particles at an interface between two immiscible fluid phases is also a very important research area, since it occurs in many engineering and industrial processes like slurries transport and drying processes of micro and nano suspension coating. In this processes it is important to consider the effect of the capillarity force that contributes to the formation of clusters of particles.

Different numerical techniques have been developed over the years to model and simulate flows with suspended particles. In the first type of methods, the velocity and pressure fields of the fluid flow around each individual particle is determined by solving the Navier–Stokes system by some discretization method, as finite element method in the works by Hu et al (1992, 1996) (21, 22). The hydrodynamic forces acting on each particle is calculated from the fluid flow solution and is used in the equation that describes the particle motion. The finite element mesh has nodes over the fluid–solid boundaries, that move with the particle. Therefore, a new mesh needs to be computed at each time step, making this class of methods extremely expensive.

A second class of methods is based on what is called the fictitious domain method. The first work was proposed by Glowinski et al (1999) (19) and later improved by other researchers (8, 9, 32, 40, 41). The entire domain, including fluid and particles, is discretized by the same mesh and the Navier–Stokes equations are solved on the entire domain, but the velocity in the portion of the domain that contains a particle is constrained to have rigid body motion. This approach eliminates the need for remeshing at each time step. Typically, the transient response is solved explicitly and an iterative procedure is used to solve the velocity and the pressure field separately.

Previous works on floating particles' simulation may be divided in two classes. Works on the first class of floating particles methods (38) aim to develop numerical simulations for particle motions in fluid interfaces assuming that the particles are initially at rest along the interface. The particles are moved in a direct simulation respecting the fundamental equations of motion of fluids and solid particles. The work of Singh et al (2005) (38) uses Level Set together with the fictitious domain method to simulate the equilibrium position of particles at the interface of two fluids and also to approximate the interface's geometry until the flow reaches its steady configuration.

The second class of works follows the approach used by Fujita et al (2006) (15) to develop structure formation simulators for colloidal nano particles during drying. In this works, instead of performing the full discretization of the Navier–Stokes equations, and compute the interaction forces between the particles and the fluids explicitly, it describes the motion of the particles using other models like the Langevin equations, in which forces exerted on each particle consist of contact force, capillarity force, Brownian force, van der Walls force, electrostatic force and fluid drag force, evaluated in the limit of creeping flow ($Re = 0$).

To the best of our knowledge, there is no previous work that solves the Navier–Stokes and rigid body equations to perform the simulation of the flow and of the flotation of particles embedded on one or more immiscible fluid phases. The aim of this work is to fill this gap, using a fictitious domain formulation based on Lagrange multipliers that we numerically solve using the finite element method.

1.1
Contributions

The main contribution of this thesis is to propose a new formulation for the numerical simulation of incompressible flows with suspended rigid particles using a fully implicit and coupled approach. Moreover, we also propose an extension of this method to the simulation of floating particles at the interface between two fluid phases. Our approach is based on the fictitious domain method and uses a Lagrange multipliers formulation, which avoids remeshing around the particles at each time step. The formulation avoids explicit projection methods and time–split integrators, solving at each time step the complete weak formulation through a single non–linear system of fully coupled equations. Finally, we also propose a new scalable topological data–structure for triangle and quadrangle meshes, and a suitable set of classes for a modularized finite element implementation of our method.

1.2
Thesis outline

This work uses the following chapter structure. In chapter 2 we define some notations and review the governing equations for Newtonian incompressible fluids and rigid body particles. In chapter 3 we derive the variational formulation for the simulation of Newtonian incompressible flows and also validate our finite elements code using the lid driven cavity problem. In chapter 4 we propose the new method to solve flows with suspended particles, which is based on the fictitious domain and Lagrange multipliers methods. In chapter 5 we extend our method to handle the simulation of particles floating at the interface between two fluid phases. In chapter 6 we propose a new topological data structure for mixed triangle and quadrangle meshes that is very efficient and useful to develop finite element codes, and describe our fictitious domain finite elements implementation. In chapter 7, we present our results and, finally, in chapter 8 we make some conclusions and suggestions for future works.