3
The Optimal Mechanism

We can use the decomposition implied by Abreu et al. [1] to write the Bellman equation that characterizes the frontier of equilibrium values that can be attained in this environment.

Define $\underline{v}$ as the expected value for a player when the *other* one always chooses the allocation; that is, $\underline{v} = E_\theta \left[u\left(\theta_i, \theta_{-i}\right) \right]$. Analogously, define $\overline{v}$ as the players' payoff when his preferred action is always taken, $\overline{v} = E_\theta \left[u\left(\theta_i, \theta_i\right) \right]$. We assume that there exists a $\kappa > 0$ so that $\underline{w}_i - \kappa > \underline{v}$, $i = 1, 2$. In words, the payoff a player collects in case he exercises his outside option is bounded away from the payoff he gets when his opponent takes all decisions.

Let $D = \left\{ \left\{ a\left(\theta\right), w\left(\theta\right) \right\}_\theta \right\}$ be the set of all deterministic mechanisms. In order to induce enough convexity in our problem, we allow for arbitrary convex combinations of elements in D. We do so by allowing the mechanism to condition play on the realization of a public random device x which has a uniform distribution on $[0, 1]$. The set we consider is:

$$R = \left\{ \left\{ a\left(\theta, x\right), w\left(\theta, x\right) \right\}_{\theta, x} \mid \text{for every } x,\ \left\{ a\left(\theta, x\right), w\left(\theta, x\right) \right\}_\theta \in D \right\}$$

Then, the program of interest is:

$$V\left(v\right) = \sup_{\left\{ \left\{ a(\theta,x), w(\theta,x) \right\}_{\theta,x} \right\} \in R} E_{\theta,x} \left[\left(1-\delta\right) u\left(a\left(\theta, x\right), \theta_n\right) + \delta V\left(w\left(\theta, x\right)\right) \right] \quad \text{(P1)}$$

subject to

$$E_{\theta,x} \left[\left(1-\delta\right) u\left(a\left(\theta, x\right), \theta_2\right) + \delta w\left(\theta, x\right) \right] = v \quad \text{(PK)}$$

$$\begin{aligned} &E_{\theta_2,x} \left[\left(1-\delta\right) u\left(a\left(\theta, x\right), \theta_1\right) + \delta V\left(w\left(\theta, x\right)\right) \right] \geq \\ &\quad E_{\theta_2,x} \left[\left(1-\delta\right) u\left(a\left(\hat{\theta}_1, \theta_2, x\right), \theta_1\right) + \delta V\left(w\left(\hat{\theta}_1, \theta_2, x\right)\right) \right] \forall \theta_1, \hat{\theta}_1 \in \Theta \quad \text{(IC}_1\text{)} \end{aligned}$$

$$E_{\theta_1,x}\left[(1-\delta)\,u\left(a\left(\theta,x\right),\theta_2\right)+\delta w\left(\theta,x\right)\right]\geq$$
$$E_{\theta_1,x}\left[(1-\delta)\,u\left(a\left(\theta_1,\hat{\theta}_2,x\right),\theta_2\right)+\delta w\left(\theta_1,\hat{\theta}_2,x\right)\right]\forall\theta_2,\hat{\theta}_2\in\Theta \quad (\mathrm{IC}_2)$$

$$V\left(w\left(\theta,x\right)\right)\geq \underline{w}_1 \forall\theta\in\Theta^2 \text{ and } \forall\; x\in[0,1] \quad (\mathrm{IR}_1)$$

$$w\left(\theta,x\right)\geq \underline{w}_2 \forall\theta\in\Theta^2 \text{ and } \forall\; x\in[0,1] \quad (\mathrm{IR}_2)$$

The constraints are standard. The promise keeping (PK) constraint requires that, if agent two is promised discounted expected utility of v, the mechanism must choose an action $a(\cdot,\cdot)$ and continuation values $w(\cdot,\cdot)$ that deliver such promise. (IC_1) and (IC_2) state that, given a truthful report of the other agent, it must be optimal for agent i to report truthfully his preference shock.[1] Finally, the last two constraints are the participation constraints for agents 1 and 2, respectively.

Defining $\overline{w}_2 = V^{-1}\left(\underline{w}_1\right)$,[2] we can write the Participation Constraints as

$$w\left(\theta,x\right)\in\left[\underline{w}_2,\overline{w}_2\right]\forall\theta\in\Theta^2, x\in[0,1] \quad (\mathrm{IR'})$$

Early work (e.g. Casella [4], Jackson and Sonnenschein [8]) has shown that the repeated taking of joint actions allows for significant improvements over a one shot framework. The efficiency gains are attained by allowing the actions to be linked over time. A player who reports to have a more extreme preference shock – as measured by its distance from $\frac{1}{2}$ – is granted, relatively to a one shot case, more weight on the current action, relinquishing future decision power.

Define $a^*(\theta) = \operatorname{argmax}_a E\left[u\left(a,\theta_1\right)+u\left(a,\theta_2\right)\right]$, to be the (ex-ante) Pareto efficient allocation, and let $v^{FB} = E\left[u\left(a^*\left(\theta,x\right),\theta_1\right)+u\left(a^*\left(\theta,x\right),\theta_2\right)\right]$ be the total surplus when action $a^*(\theta)$ is taken in all periods.

Under repeated decision taking, if either the Participation Constraint is ex-ante or players are forced to participate, v^{FB} can be arbitrarily approximated – but not attained – by equilibrium payoffs when players become patient. Carrasco and Fuchs [3], however, show that this can only be accomplished through the continuing variation in decision power. This variation, in turn, will necessarily lead to one of the players becoming the dictator: in the long run, one of the players will be promised $\overline{v}$.

In the current setting, this is not feasible because, whenever a player is promised sufficiently low continuation values, he will exercise his outside

[1] We make use of the Revelation Principle (Myerson [11]).
[2] Note that from the envelope theorem $V(\cdot)$ is strictly decreasing, thus it has an inverse.

option. As the mechanism cannot grant unbounded power to a player ex-post, it will not approximate efficiency ex-ante.

Theorem 1 (Inefficiency) *There exists $\epsilon > 0$ such that, for all $\delta \in [0, 1)$, the sum of the agents' payoffs is no larger than $v^{FB} - \epsilon$.*

Therefore, irrespective of how patient agents are, any feasible mechanism that satisfies the ex-post participation constraints will deliver outcomes that are bounded away from the efficient ones. Indeed, efficiency calls for intertemporal decisions to be linked: an agent who is given relatively more weight on a current decision has to relinquish future bargain power. The way through which the mechanism grants an agent a lower future bargain power is by promising him lower continuation values. The outside options place a lower bound on what a mechanism can promise to any single agent, impeding the mechanism to implement the efficient intertemporal trade of decision power.