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The Model

Two agents, $i = 1, 2$, have to take, repeatedly, a joint action, a, over time. At each period $t \in \{0, 1, ...\}$, they receive privately preference shocks $\theta_i \in [0, 1]$. The preference shocks are i.i.d. over time and across players, and are drawn from a distribution $F(\cdot)$, with density $f(\theta_i) > 0$, which is symmetric around $\frac{1}{2}$.

Player i's instantaneous (Bernoulli) utility function is

$$u(a, \theta_i),$$

with

$$u(\theta_i, \theta_i) \geq u(a, \theta_i) \ \ for \ all \ a,$$

and

$$\frac{\partial^2 u(a, \theta_i)}{\partial a \partial \theta_i} > 0 > \frac{\partial^2 u(a, \theta_i)}{\partial a^2}.$$

Put in words, their preferences are single peaked, and θ_i represents agent i's favorite action.

We also impose that their preferences are symmetric around $\frac{1}{2}$: [1] for all $a, \theta_i \in [0, 1] := \Theta$.

$$u(a, \theta_i) = u(1 - a, 1 - \theta_i)$$

As preferences and the density of types are symmetric around $\frac{1}{2}$, the problem itself is symmetric around $\frac{1}{2}$. Hence, one can measure how extreme a preference shock θ_i is in terms of its distance from $\frac{1}{2}$.

After the players observe their preference shocks, they make reports $\tilde{\theta}_i$, $i = 1, 2$. A public history at time t, h^t, is a sequence of (i) past announcements of all players, and (ii) past realized actions:

$$h^t = \left\{ \varnothing, \left(\tilde{\theta}_1^1, \tilde{\theta}_2^1, a^1 \right), ..., \left(\tilde{\theta}_1^{t-1}, \tilde{\theta}_2^{t-1}, a^{t-1} \right) \right\}$$

[1] Note that, in particular, this holds whenever an agent with type θ_i is indifferent between any two actions a and b that are equidistant from θ_i.

Given the reports and the history of the game, a history dependent allocation is determined according to a contract, which is a sequence of functions of the form

$$\left\{ a_t \left(\tilde{\theta}_i, \tilde{\theta}_{-i}, h^{t-1} \right) : [0,1]^2 \times [0,1]^{3(t-1)} \to [0,1] \right\}_{t=1}^{\infty}$$

This contract is chosen a priori before the agents learn their preference shocks.

Let H^t be the set of all public histories of length t. A public strategy for player i is a sequence of functions $\{\tilde{\theta}_i^t(\cdot,\cdot)\}_t$, where

$$\tilde{\theta}_i^t : H^t \times [0,1] \to [0,1]$$

Each profile of strategies $\tilde{\theta} = \left(\left\{ \tilde{\theta}_1^t(\cdot) \right\}_t, \left\{ \tilde{\theta}_2^t(\cdot) \right\}_t \right)$ defines a probability distribution over public histories. Let $\delta \in [0,1)$ denote the common discount factor. Player i's discounted expected payoff is given by

$$E\left[(1-\delta) \sum_{t=0}^{\infty} \delta^t u^t(a(\tilde{\theta}^t); \theta_i^t) \right],$$

where the expectation is taken with respect to the probability distribution over public histories induced by the strategy profile.

We analyze this game using the recursive methods developed by Abreu et al. [1]. More specifically, letting $W \subset \mathbb{R}$ be the set of Public Pure Strategy Equilibria (PPSE) payoffs for the agents, we can decompose the payoffs into a current utility $u(a, \theta_i)$ and a continuation value $w_i(\tilde{\theta}) \in W$:

$$E_\theta[(1-\delta)u(a\left(\tilde{\theta}\right), \theta_i) + \delta w_i(\tilde{\theta}_i, \tilde{\theta}_{-i})].$$

In other words, any PPSE can be summarized by the actions to be taken in the current period and equilibrium continuation values as a function of the announcements.

Player i has an outside option which grants him, for any given contingency, and any period of time, life-time utility of $\underline{w}_i$. Hence, for each of the agents it must be the case that

$$w_i(\theta) \geq \underline{w}_i \text{ for all } \theta.$$

We interpret $(\underline{w}_1, \underline{w}_2)$ as defining the initial distribution of power within the partnership, and seek to derive its implications for the joint actions chosen and the long-run distribution of power.