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On the Asset and Liability Management for pension funds: a multistage stochastic programming model and a equilibrium risk measuring method

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Abstract

This paper proposes an Asset and Liability Management (ALM) for pension funds via multistage stochastic programming and an equilibrium risk measuring method.

The ALM of a pension fund consists in finding the optimal investment policy given the stochastic nature of the asset returns and the liability cash flows. Since it refers to a dynamic portfolio, the most suitable approach would be a multistage stochastic programming model. However, computational restrictions don't allow covering the entire pension fund's planning horizon. Thus, several articles in literature have proposed an arbitrary fixed capital requirement obtained independently on the investment policy adopted to approximate the effects of the non-considered periods.

Whereas the fund's actual opportunity cost, we propose a method for measuring and controlling the equilibrium risk which bootstraps the portfolio return scenarios embedded in the optimal solution in order to approximate the liability discount rate distribution for the periods beyond the considered planning horizon.

Key words: Asset Liability Management(ALM), Stochastic Programming, Pension Funds, Solvency Risk, Equilibrium Risk, Bootstrap

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1. Introduction

The term Asset and Liability Management (ALM) designates the practice of managing a business coordinating decisions and actions taken with respect to assets and liabilities. The ALM is crucial pursuit for any organization which receives and invests money in order to fulfill capital requirements and future cash demands. Moreover, ALM can be defined by the Society of Actuaries "as an ongoing financial management process of formulating, implementing, monitoring, and revising strategies related to assets and liabilities in an attempt to achieve organization's financial objectives given its risk tolerances and other constraints". According to each context, ALM can have substantially different aspects. For instance, derivative traders understand assets and liabilities as similar entities traded in the financial market while ALM of pension funds is focused on deciding an optimal investment policy while liabilities cannot be changed. The main financial objective of the latter financial institution is to ensure the payment of lifelong benefits by investing contributions. Hence, the investment policy must assure two conditions: equilibrium and liquidity - long and short term solvency, respectively.

The first condition states that the value of the assets should always be large enough to pay all benefits until the plan extinction. In other words, the solvency capital (the difference between the total asset value and the net present liability value) should be positive. The second condition states that the investment program should provide enough cash to pay current liabilities, which means that cash level must always be positive. Solvency capital and cash level are affected both by investment policy (decision variables) and asset returns and liability payments (risk factors).

In this paper, we propose a multistage stochastic programming model for an ALM and a new method for measuring and controlling the equilibrium risk of a pension fund in Brazil. Several articles in literature which proposed stochastic programming models for optimal allocation (Bradley and Crane, 1972; Kallberg et al., 1982; Zenios, 1995; Carino and Ziemba, 1998; Kouwenberg, 2001; Hilli et al., 2007), could only measure the equilibrium risk at the end of the considered planning horizon by comparing the final wealth in each scenario to a fixed capital requirement obtained independently on the investment policy adopted. However, in an ALM problem, the liabilities discount rate should be taken as the portfolio return (Veiga, 2003), which depend on the stochastic asset returns and the investment policy. In order to solve this problem, we propose an iterative method to measure the equilibrium risk.

This method discards the fixed capital requirement approximation and uses the portfolio return scenarios embedded on the stochastic programming to generate, via bootstrap, the liability discount rate distribution. Then, we can estimate the net present value distribution of benefits beyond the stochastic programming planning horizon and the related insolvency probability.

This paper is organized as follows: After the introduction, we present the stochastic programming model and all processes to produce the necessary input to the optimization problem. These processes include a stochastic model for economic risk factors, a scenario tree generation method and financial models for the assets and liabilities involved. After that, the equilibrium risk measuring method will be developed and then the results and conclusions will be presented.

2. Multistage stochastic programming model

A multistage stochastic programming model is an optimization problem under uncertainty which the discrete form is solved based on an event tree. Each tree node is associated to a possible state of the system and has a unique predecessor (representing a unique history) and several successors (representing several possible outcomes in the future). In this paper, we describe the ALM as a linear maximization problem which, given an initial portfolio allocation, defines the capital movements between the asset classes as the decision variables. The asset classes, indexed as $i = 1, ..., 4$, are, respectively, stocks, properties, bonds and cash. The model also includes loans, to cover possible cash shortages and transaction costs. Cash balance and asset inventory constraints are represented, as well as the regulatory and market liquidity ones. The objective function is the final wealth expected utility with a penalty for the insolvent scenarios.

A stage is denoted by $t \in 0, ..., T$ and a node related to stage$t$ is defined as $n_t \in (N_{t-1} + 1), ..., N_t$, where N_t is the number of nodes at stage t. Consider $N_{-1} = 0$. For illustrative purpose, the event tree has 5 stages and a conditional branching structure given by $1 - 10 - 6 - 6 - 4 - 4$. Indeed, the stages can have different lengths, for instance, the first and the second have one year, the third has three years, the fourth has five years and the last one has ten years. This length structure would lead to a 20-year planning horizon. This structure is represented by Figure 1.

For the optimization model development, first a list of decision variables and parameters is provided. Parameters are divided in deterministic (with

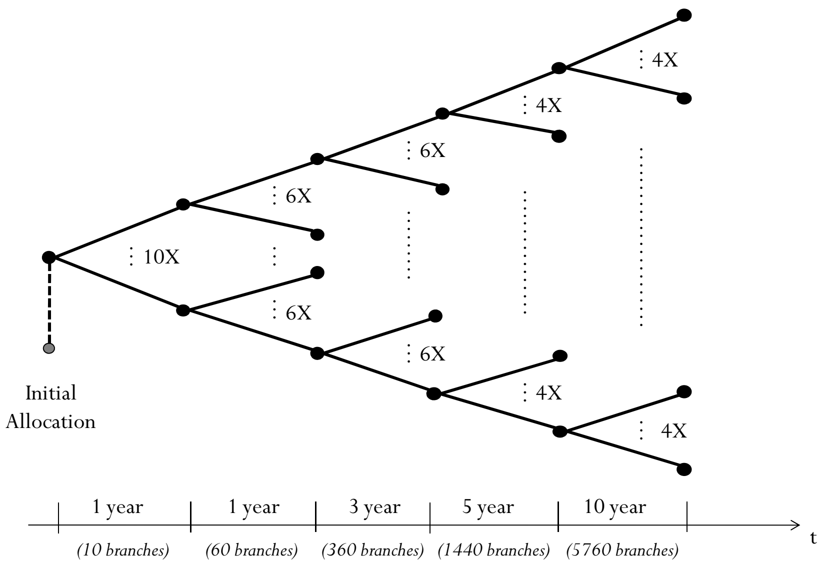

Figure 1: Event tree

no uncertainty) and stochastic (risk factors).

- Decisions variables
  - $c_i(n_t)$: amount bought of asset class i at node n_t
  - $v_i(n_t)$: amount sold of asset class i at node n_t
  - $e(n_t)$: loan obtained at node n_t
  - $a_i(n_t)$: amount invested in asset class i at node n_t
  - $y(n_T)$: positive solvency capital at node n_T
  - $w(n_T)$: negative solvency capital at node n_T
- Deterministic parameters
  - pe: insolvency penalization
  - bo: solvency bonus
  - sp: spread between the borrowing rate and the short term interest rate
  - ma: maximum stock allocation (%)

- ct: transaction cost (%)
- cc_i: market buying capacity of asset class i
- cv_i: market selling capacity of asset class i
- $\overline{a}_i$: initial allocation of asset class i
- L: capital requirement at the end of planning horizon ($t = T$)

- Stochastic parameters
  - $l(n_t)$: nominal liability cash flow at node n_t
  - $r_i(n_t)$: return of asset class i between two linked nodes n_{t-1} e n_t

2.1. Objective function

The objective function is defined by (1).

$$\sum_{n_T=n_{T-1}+1}^{N_T} p(n_T)[bo \cdot y(n_T) - pe \cdot w(n_T)] \tag{1}$$

It represents the pension fund terminal wealth expected utility which penalizes the insolvent scenarios at the end of the planning horizon. Given that $pe > bo$, the wealth utility at a terminal node n_T is described as a piecewise linear concave function representing the risk-averse pension fund's preferences. Then, for the insolvent scenarios, we have $y(n_T) = 0$ and $w(n_T) > 0$. On the other hand, for the solvent scenarios, we have $y(n_T) > 0$ and $w(n_T) = 0$. For the illustrative example, $pe = 2$ and $bo = 1$.

These constraints have a modified version for $t = T - 1$. It defines the variables $y(n_T)$ and $w(n_T)$ used on the objective function. The concave characteristic of the objective function ensures that if $y(n_T) > 0$ then $w(n_T) = 0$, and if $w(n_T) > 0$ then $y(n_T) = 0$. Thus, $y(n_T)$ is how much the final wealth exceeds the capital requirement L. Similarly, $w(n_T)$ is how much the final wealth lacks the capital requirement L. For each pair of linked node n_{T-1} and n_T the constraint we define (2).

$$L + y(n_T) - w(n_T) = \sum_{i=1}^{4} [(1 + r_i(n_T))a_i(n_{T-1})] - l(n_{T-1})$$
$$-(1 + sp + r_3(n_T)).e(n_{T-1}) - ct. \sum_{i=1}^{4} [c_i(n_T) + v_i(n_T)] \tag{2}$$

2.2. Asset inventory constraint

The asset inventory constraint specifies that the future value of an asset class i at node n_t is equal to the present value of the same asset class adjusted for buying and selling at node n_{t+1}, for each pair of linked nodes(n_t, n_{t+1}) . Note that the asset class $i = 4$ (cash) is not included because there is no meaning in “buying” or “selling” cash. For $t \in \{0, 1, ..., T-2\}$, for each pair of linked node (n_t, n_{t+1}) and for $i \in \{1, 2, 3\}$, we define (3) as an asset inventory constraint. Moreover, for $i \in \{1, 2, 3\}$, we define (4) as the initial asset inventory constraint.

$$a_i(n_{t+1}) = (1 + r_i(n_{t+1}))a_i(n_t) + c_i(n_{t+1}) - v_i(n_{t+1}) \tag{3}$$

$$a_i(n_0) = \overline{a}_i + c_i(n_0) - v_i(n_0) \tag{4}$$

2.3. Regulatory constraint for stock allocation

The Brazilian law determines a maximum stock allocation of 70% of the portfolio. Given $ma = 70\%$ the stock allocation at each node is bounded as follows.

$$a_1(n_t) \leq ma \sum_{i=1}^{4} a_i(n_t), \quad \forall t = 0, ..., T-1 \tag{5}$$

2.4. Market liquidity constraint

This constraint represents the fact that large pension funds are cannot transact a great amount of money without affecting the respective market prices. So the transactions are bounded by the market capacity.

$$c_i(n_t) \leq cc_i, \quad \forall t = 0, ..., T-1, \quad \forall i = 1, 2, 3 \tag{6}$$
$$v_i(n_t) \leq cv_i, \quad \forall t = 0, ..., T-1, \quad \forall i = 1, 2, 3 \tag{7}$$

3. Stochastic model for economic risk factors

The optimization model will give a realistic solution if, and only if, the risk factors are appropriately modeled. These risk factors include economic random variables related to the financial market and the economy as a whole. Thus, a stochastic model for the economic risk factors will be developed to forecast these random variables over the planning horizon.

	x_1	x_2	x_3	x_4	x_5
Mean	2.66%	3.46%	9.04%	18.58%	6.26%
Median	2.96%	4.05%	6.14%	17.22%	4.52%
Maximum	13.98%	11.58%	50.18%	31.11%	45.85%
Minimum	-7.26%	-10.36%	-6.10%	11.40%	-30.88%
Std. Dev.	4.57%	4.46%	9.83%	4.71%	17.20%
Skewness	-0.069	-0.841	1.982	1.079	0.11
Kurtosis	3.18	3.89	8.31	3.85	2.99

Table 1: Statistics, time series 1996Q2-2007Q2

The stochastic model for economic risk factors chosen is a Vector Auto-Regressive based on Dert (1998). The variables are chosen to model appropriately the asset returns used as inputs for the optimization problem. The mean vector is specified exogenously and can be interpreted as a sensitivity parameter for the model response to the economic risk factors. This model has quarterly data with a sample representing the Brazilian economy from 1996Q2 to 2007Q2.

$$X_q - \mu = \alpha(X_{q-1} - \mu) + \epsilon_q, \quad \epsilon_q \sim N(0, \Sigma) \tag{8}$$

$$x_{j,q} = log(1 + y_j q), \quad \forall j = 1, \ldots, 5 \tag{9}$$

Where

$$y_{j,q} = \begin{cases} output\ growth\ rate, & j = 1 \\ rental\ growth\ rate, & j = 2 \\ inflation\ rate, & j = 3 \\ interest\ rate, & j = 4 \\ stock\ return, & j = 5 \end{cases}$$

The time series statistics are described in Table 1. As we see, the historical mean of Brazilian interest rate is too high because of some international crisis (Asia-1997 and Russia-1998) and the hyperinflation process remnants at the beginning of the sample. For this reason, the mean of the economic variables must be determined exogenously, for example: $\mu = (4\%, 11\%, 4\%, 10\%, 12\%)'$.

The other coefficients (Σ and α) are estimated using the Ordinary Least Squares method, and are given by Tables 2 and 3.

	x_1	x_2	x_3	x_4	x_5
x_1	0.001886	0.000347	0.000881	-0.000285	0.000084
x_2	0.000347	0.001927	0.001555	-0.000020	0.000123
x_3	0.000881	0.001555	0.008173	-0.000026	0.003296
x_4	-0.000285	-0.000020	-0.000026	0.000683	-0.000531
x_5	0.000084	0.000123	0.003296	-0.000531	0.036031

Table 2: Covariance matrix, quarterly 1996Q2-2007Q2

	$x_{1,q-1} - 0.04$	$x_{2,q-1} - 0.11$	$x_{3,q-1} - 0.04$	$x_{4,q-1} - 0.10$	$x_{5,q-1} - 0.12$
$x_{1,q} - 0.04$	-0.148696	-0.185559	-0.045314	-0.229618	0.086285
	(0.15288)	(0.13395)	(0.07164)	(0.17779)	(0.03676)
$x_{2,q} - 0.11$	-0.422790	0.114880	0.064160	-0.919097	0.030370
	(0.15454)	(0.13541)	(0.07242)	(0.17972)	(0.03716)
$x_{3,q} - 0.04$	0.040368	-0.176425	0.418518	0.176557	-0.174167
	(0.31828)	(0.27888)	(0.14915)	(0.37014)	(0.07654)
$x_{4,q} - 0.10$	-0.008872	-0.078176	0.067767	0.657944	-0.088930
	(0.09198)	(0.08059)	(0.04310)	(0.10697)	(0.02212)
$x_{5,q} - 0.12$	-0.431145	0.612786	-0.139294	0.066837	0.140459
	(0.66828)	(0.58556)	(0.31317)	(0.77717)	(0.16070)

Table 3: α coefficient, standard deviation in ( ), sample: 1996Q2-2007Q2

4. Scenario tree generation method

The scenario tree generation method is based on the "Adjusted Random Sampling" of Kouwenberg (2001). Some modifications were introduced in order to take into account the different time intervals between nodes in our event tree. We modify the notation of the equation (8) to make the correspondence between the stage t of the event tree and the quarter q of the stochastic model. So, X_q^t is the risk factor vector of q quarters ahead stage t. The stochastic model is rewritten as (10).

$$X_q^t - \mu = \alpha(X_{q-1}^t - \mu) + \varepsilon_q^t, \varepsilon_q^t \; N(0, \Sigma) \tag{10}$$

The first step of the method is to generate a deterministic one-quarter forecast from the beginning of stage t for each predecessor node.

$$X_q^t - \mu = \alpha \left(X_{q-1}^t - \mu \right) \tag{11}$$

The first $(N_t - N_{t-1})/2$ values of $\varepsilon_q^t \sim N(0, \Sigma)$ are randomly generated:

$$\varepsilon_q^t (n_t) \, N(0, \Sigma), \quad \forall n_t = N_{t-1} + 1, \ldots, N_t/2 \tag{12}$$

In order to guarantee the mean and the other odd central moments as zero, as stated by the Normal distribution, we take the antithetic values.

$$\varepsilon_q^t\left(n_t+N_t/2\right)=-\varepsilon_q^t\left(n_t\right),\quad \forall n_t=N_{t-1}+1,\ldots,N_t/2 \tag{13}$$

Another adjustment is made in order to fit the variances of the tree structure and the stochastic model. This adjustment is made for each component j of the vector $\varepsilon_q^t\left(n_t\right)=\left(\varepsilon_{1,q}^t\left(n_t\right),\ldots,\varepsilon_{5,q}^t\left(n_t\right)\right)'$.

$$\eta_{j,q}^t\left(n_t\right)=\frac{\sigma_j}{\sqrt{\frac{1}{N_t-1}\sum_{i=1}^{N_t}\left(\varepsilon_{j,q}^t\left(n_t\right)\right)^2}}\varepsilon_{j,q}^t\left(n_t\right),\quad \forall n_t=N_{t-1}+1,\ldots,N_t \tag{14}$$

This process is repeated for all quarters of stage t computing N_t-N_{t-1} independent scenarios. The last observation of each scenario belonging to stage t will initialize a set of conditional branches of stage $t+1$ restarting all over the process. Considering $Q(t)$ the number of quarters that compound stage t, the initialization for a given predecessor node n_t is represented as follows:

$$X_0^{t+1}=X_{Q(t)}^t\left(n_t\right) \tag{15}$$

5. Asset pricing model

The asset pricing is an important part of ALM process. It consists of transforming the economic risk factors into the asset class returns. The stock return (16) is modeled as the return of stock index, the properties return (17) is modeled as the return on the rental activity, the bonds return (18) is the short term interest rate plus a deterministic spread and, finally the cash return is the short term interest rate (19). Consider a pair of linked nodes (n_t,n_{t-1}), the returns are given as follows:

$$r_1\left(n_t\right)=\frac{stock\ index\left(n_t\right)}{stock\ index\left(n_{t-1}\right)}-1 \tag{16}$$

$$r_2\left(n_t\right)=\frac{rental\ activity\left(n_t\right)}{rental\ activity\left(n_{t-1}\right)}-1 \tag{17}$$

$$r_3\left(n_t\right)=interest\ rate\left(n_t\right)+spread \tag{18}$$

$$r_3\left(n_t\right)=interest\ rate\left(n_t\right) \tag{19}$$

Following (16), (17), (18) and (19) the asset returns are calculated as functions of economic risk factors. Since the economic factors are represented as components of the vector $y_q^t(n_t) = \left(y_{1,q}^t(n_t), \ldots, y_{5,q}^t(n_t)\right)'$ and $x_{j,q}(n_t) = ln\left(1 + y_{jq}^t(n_t)\right), \quad \forall j = 1, \ldots, 5$, the returns of each node are computed as follows:

$$r_1(n_t) = exp\left(\frac{1}{Q(t)}\sum_{q=1}^{Q(t)} x_{5,q}^t(n_t)\right) - 1 \tag{20}$$

$$r_2(n_t) = exp\left(\frac{1}{Q(t)}\sum_{q=1}^{Q(t)} x_{2,q}^t(n_t)\right) - 1 \tag{21}$$

$$r_3(n_t) = \left[exp\left(\frac{1}{Q(t)}\sum_{q=1}^{Q(t)} x_{4,q}^t(n_t)\right) - 1\right] + spread \tag{22}$$

$$r_4(n_t) = exp\left(\frac{1}{Q(t)}\sum_{q=1}^{Q(t)} x_{4,q}^t(n_t)\right) - 1 \tag{23}$$

6. Liability model

We created an artificial data base of a pension fund with 110200 participants distributed as 45000 active, 60000 retired 5200 pensioners. All the participants have a defined benefit plan to which they contribute, along with the plan sponsor, 16% of their salary to receive a benefit of 90% of the last salary after retirement. Let I_{death} be a dummy variable assuming 1 when the participant is dead and 0 otherwise. Similarly I_{ret} assumes 1 when the participant is retired and 0 otherwise. Thus, the contribution of participant p at year k is defined in (24) while the benefit of participant p at year k is defined in (25).

$$contribution(p, k) = 0.16 salary(p, k)(1 - I_{death})(1 - I_{ret}) \tag{24}$$

$$benefit(p, k) = 0.9 last\_salary(p, k)(1 - I_{death}) I_{ret} \tag{25}$$

Considering a deterministic age of retirement, the expected contribution and benefit of participant p at year k are defined, respectively, in (26) and(27). The expected values are used due to the large number of participants and the independence assumption over their time of death.

$$E\left[contribution\left(p,k\right)\right] = 0.16 \cdot salary\left(p,k\right) \cdot \left(1 - q_{age(p)}\right) \cdot \left(1 - I_{ret}\right) \qquad (26)$$

$$E\left[benefit\left(p,k\right)\right] = 0.9 \cdot last\ salary\left(p,k\right) \cdot \left(1 - q_{death}\right) \cdot I_{ret} \qquad (27)$$

The expected values are accumulated for all participants in (28).

$$RF\left(k\right) = \sum_{p=1}^{10200} \left\{E\left[benefit\left(p,k\right)\right] - E\left[contribution\left(p,k\right)\right]\right\} \qquad (28)$$

7. Equilibrium risk measuring method

The equilibrium risk is defined as the insolvency probability, i.e., the probability that the pension fund won't meet all obligations until its extinction. In other words, insolvency is a state at a defined instant of time where the total asset value is smaller than the net present value of the fund's liabilities, namely the technical reserve.

The total asset value at a determined instant of time is easily calculated by the sum of the amount invested in each asset class. On the other hand, the technical reserve calculation is more complicated because the net present value of the fund's liability cash flows needs a discount rate which, following Veiga (2003), should be the fund's portfolio return. In the first years under study, this calculation is implicitly done by the stochastic optimization model. However, the liability horizon is, usually, longer than the period chosen to optimize the investment policy. Thus, the portfolio return for each scenario, given the optimal decisions, is known only for these first years while some assumptions are needed to model the remaining period.

Choosing a fixed discount rate is not necessarily a good approximation for the portfolio returns of this remaining period nonetheless the previous papers in the literature have assumed this approach for mostly based on a regulatory statement of the country under study. Thus, their approximation gives rise to an arbitrary technical reserve and, consequently, a meaningless equilibrium risk measure.

In order to solve this inadequacy, we propose a new method to better estimate the equilibrium risk. First an optimal solution is obtained with

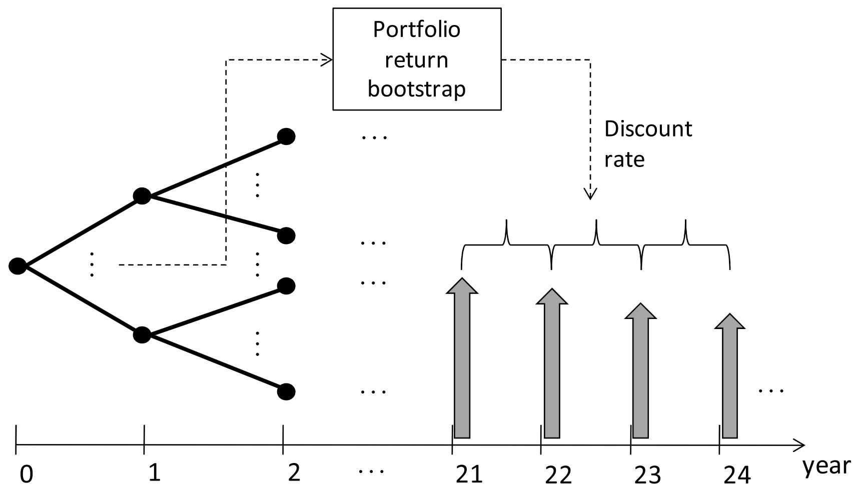

Figure 2: Bootstrapped discount rate

a null capital requirement ($L = 0$). After that the discount rate empirical distribution is obtained bootstrapping the portfolio return embedded on the stochastic programming model. Then, a sequence of the liabilities cash flows is discounted to the end of the stochastic programming horizon using different sequences of bootstrapped portfolio return in order to approximate the technical reserve distribution at the terminal stage.

To form a bootstrapped sequence of the returns we choose the returns $r(n_s)$ according to the following probabilities. Let S and N be random variables that represent, respectively, the stage and the node to be bootstrapped as a future portfolio return. This process is based on the joint distribution of these two variables 29.

$$P\left(S = s, N = n\right) = P\left(N = n|S\right) P\left(S = s\right) \tag{29}$$

The conditional distribution of N given S and the marginal distribution of S are described as the following probability functions:

$$P\left(N = n|S\right) = \frac{1}{N_s - N_{s-1}} \tag{30}$$

$$P\left(S = s\right) = \frac{Q\left(s\right)}{\sum_{t=1}^{T} Q\left(t\right)} \tag{31}$$

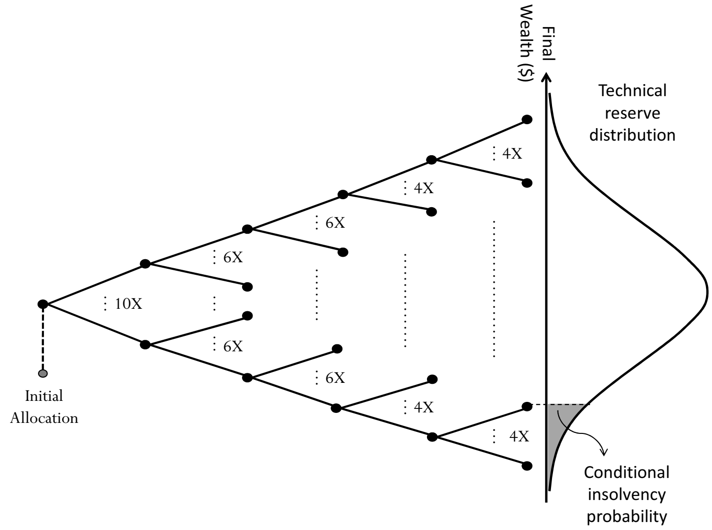

Figure 3: Equilibrium risk measure

Using the stochastic discount rate, the technical reserve distribution can be estimated and consequently a conditional insolvency probability is calculated for each terminal node of the tree structure. Indeed, the conditional insolvency probability is calculated as the maximum p-value which the technical reserve is greater than the wealth of each terminal node. Finally, the insolvency probability at the root node will be the average of all a conditional insolvency probabilities since all scenarios have the same probability.

To sum up, the insolvency probability has to be compared to the fund's risk tolerance to accept the optimal solution. If the equilibrium risk is on an acceptable level then the optimal allocation is defined. But if the equilibrium risk is too high, there are two possibilities to decrease the insolvency probability without changing the initial wealth: raising the insolvency penalization or changing the capital requirement (L) from zero to one quantile of the technical reserve distribution. To test the possible equilibrium risk control we implemented the latter iterative method.

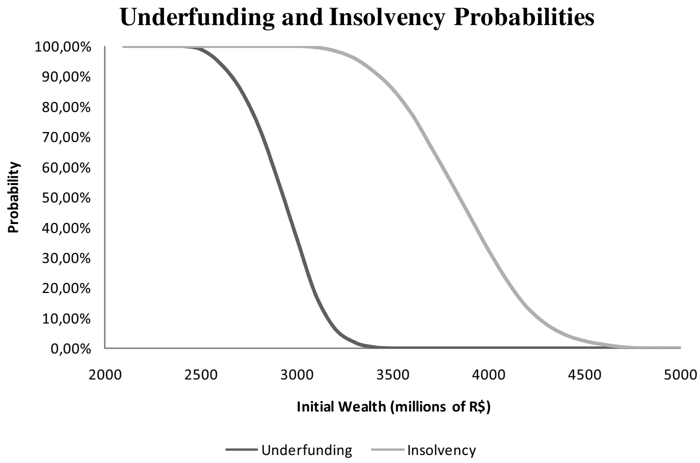

Figure 4: Underfunding and Insolvency probability

8. Illustrative example

The illustrative example has the objective of comparing two non-arbitrary equilibrium risk measures: the underfunding probability and the insolvency probability. An underfunding state is defined as a negative wealth at the end of the stochastic programming horizon while the insolvency state is described by a deficit at the end of the fund existence. Since the stochastic programming horizon is smaller than the fun existence, the underfunding probability is a lower bound for the insolvency probability, underestimating the actual equilibrium risk. The underfunding probability will be calculated as a proportion of the scenarios negative terminal wealth and the insolvency probability will be calculated with the bootstrap method proposed in this paper. Furthermore, the iterative method which increases the capital requirement (L) will also be tested.

First, it is proposed to run the whole process with different initial wealth considering a null capital requirement (Figure 4). This example confirms the theoretical result that the underfunding probability underestimates the actual equilibrium risk, the insolvency probability.

Second, it is proposed, for several different initial wealth, an influential analysis of capital requirement on the insolvency probability. Four cases are considered:

- Case 1: Null capital requirement
- Case 2: Iterative method
  - Step 1: Null Capital requirement
  - Step 2: Capital requirement as the average technical reserve
- Case 3: Iterative method
  - Step 1: Null Capital requirement
  - Step 2: Capital requirement as the reserve with risk correction (1% significance)
- Case 4: Capital requirement as a predetermined value (real discount rate: 6% by Brazilian law)

It is confirmed that when the capital requirement is increased the insolvency probability is decreased. Figure 5 also shows that the differences between each case are small suggesting that the main factor that influences the equilibrium risk is the initial wealth.

9. Conclusions

This paper proposed an ALM multistage stochastic programming model for a Brazilian pension fund and a new methodology of measuring and controlling the equilibrium risk. To do this, the whole process was divided into small parts and each one was described in details. A flowchart (Figure 6) can summarize the whole process.

The process begins with the estimation of the stochastic model coefficients used to generate the risk factor tree structured scenarios. So, financial models are used to do the asset pricing and the liability cash flow calculation. After that, the asset returns and the liability cash flows are used as the stochastic programming inputs to find an optimal investment policy with null capital requirement. Then, the optimal portfolio returns are bootstrapped to obtain the technical reserve distribution. Finally, using this result, the

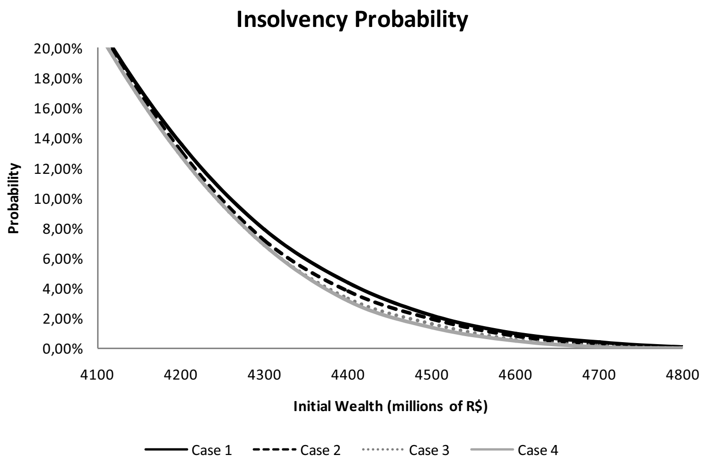

Figure 5: Insolvency probability

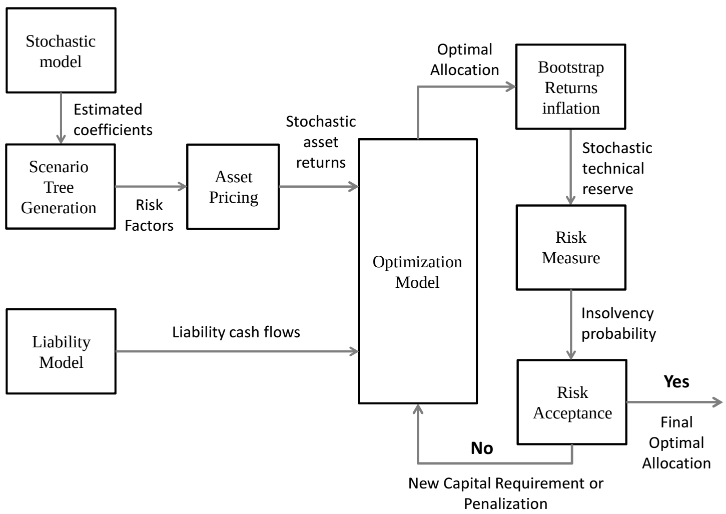

Figure 6: Flowchart

insolvency probability is estimated as the average of the conditional insolvency probabilities calculated for each terminal node. If the risk acceptance criteria is satisfied, then the optimal allocation is defined, else another solution is obtained with a higher capital requirement or a higher insolvency penalization.

In conclusion, this proposed methodology can give a better estimate of the equilibrium risk involved in a pension fund. In fact, the underfunding probability (previous work non-arbitrary risk measure) underestimates the long term risk of a pension fund since it is much smaller than the insolvency probability. Moreover, it was also tested an iterative method, increasing the capital requirement, to control the equilibrium risk. For instance, on the illustrative example, this approach actually decreased the insolvency probability however it shows just small improvements controlling the equilibrium risk.

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