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**Optimal Trajectory Definition and Control  
for a Terrestrial Vehicle in a Closed Track**

**DISSERTAÇÃO DE MESTRADO**

**DEPARTAMENTO DE ENGENHARIA ELÉTRICA**

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DO RIO DE JANEIRO



**Sergio Santiago Ribeiro**

## **Optimal Trajectory Definition and Control for a Terrestrial Vehicle in a Closed Track**

**Dissertação de Mestrado**

Dissertation presented to the Postgraduate Program in Electric Engineering of the Departamento de Engenharia Elétrica, PUC-Rio as partial fulfillment of the requirements for the degree of Mestre em Engenharia Elétrica.

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## **Determinação e Controle da Trajetória Ótima de um Veículo Terrestre em Traçado Fechado Pré-definido**

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## Resumo

Ribeiro, Sergio; Tanscheit, Ricardo; Speranza Neto, Mauro. **Determinação e Controle da Trajetória Ótima de um Veículo Terrestre em um Traçado Fechado Pré-Definido**. Rio de Janeiro, 2009. 98p. Dissertação de Mestrado – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

A determinação de uma trajetória ótima não é uma tarefa simples, uma vez que ela é diretamente dependente dos limites de aceleração suportada por cada veículo. Essa pesquisa aborda um método de otimização baseado em algoritmos genéticos que identifica a trajetória que um carro deve percorrer para completar uma pista pré-definida no menor tempo. Considerando um modelo veicular de Partícula Orientada, o método otimiza os perfis de aceleração que levam o veículo a percorrer a trajetória de menor tempo. Adicionalmente, projeta-se um controlador fuzzy para emular o comportamento de um ser humano na direção do veículo ao longo da trajetória ótima. Para alimentar o controlador, foram testados dois métodos de geração de erro: o Erro Presente da Trajetória e o Erro Futuro da Trajetória (FBTE), que é a medida de posição do carro quanto a sua tendência de movimento. Resultados obtidos com controladores clássicos, como o PDD, são confrontados com os fornecidos pelo controlador fuzzy alimentado pelo procedimento de geração de Erro Futuro de Trajetória (FBTE).

## Palavras-chave

Dinâmica Veicular. Modelos de Veículos Terrestres. Otimização. Trajetória Ótima. Controle Fuzzy. Algoritmos Genéticos.

## Abstract

Ribeiro, Sergio; Tanscheit, Ricardo (Advisor); Speranza Neto, Mauro  
**Optimal Trajectory Definition and Control for a Terrestrial Vehicle in a Closed Track.** Rio de Janeiro, 2009. 98p. MSc. Dissertation – Departamento de Engenharia Elétrica, Pontifícia Universidade Católica do Rio de Janeiro.

The definition of the minimum time trajectory in a track is not obvious, since it is directly dependent on the acceleration limits that the vehicle can withstand. This paper presents an optimization method based on Genetic Algorithms that identifies the path that a car must follow in order to complete a given circuit in minimum time. By considering an Oriented Particle model, the method optimizes the acceleration profiles that drive the vehicle along the trajectory in minimum time. In addition, a fuzzy controller is designed to emulate the behavior of a human driver controlling a high speed car along the optimized trajectory. Two different error generation procedures were tested as controller inputs: the Present Trajectory Error and the Future-based Trajectory Error (FBTE), which gives information on the car's tendency of movement. Results obtained with other controllers in the same application, such as the PDD, are compared to those provided by the fuzzy controller fed by the FBTE procedure.

## Key Words

Vehicular Kinematics. Models of Terrestrial Vehicles. Optimization. Optimal Trajectory. Fuzzy Control. Genetic Algorithms.



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## List of Symbols

$a_{BMaz}$	Maximum Braking Acceleration
$a_{LMax}$	Maximum Lateral Acceleration
$a_{Max}$	Maximum Accelerations' array
$a_N$	Normal Acceleration
$a_{N0}$	Peak Turn Acceleration
$\bar{a}_{N0}(i)$	Average Peak Lateral Acceleration for ith track stretch
$a_{reg}$	Array of points indexes
$a_T$	Tangent Acceleration
$a_{T0}$	Peak Traction Acceleration
$a_{T1}$	Peak Braking Acceleration
$a_{TMax}$	Maximum Traction Acceleration
$a_y$	Normal Acceleration
$b_d$	Front axel size
$b_t$	Rear axel size
$d$	Dinamic traveled track distance
$d_{Acc}$	Accomplished distance before the car leaves the track limits
$d_f$	Loose gape of the Steering Wheel
$d_{FB}$	Minimum braking distance
$d_{FT}$	Distance where the maximum speed is achieved
$d_{N0,1,2}$	Characteristic distances of the Normal Acceleration Profile
$d_T$	Entire track distance
$d_{T0,1,2}$	Characteristic distances of the Tangent Acceleration Profile
$E_a$	Orientation Error
$E_p$	Position Error
$i_{CP}$	Closest point index
$i_{last}$	Previous iteration index
$K_d$	Gain between Steering Wheel angle and actual Wheel angle
$l$	Distance between the axles

$L(i)$	Straight Line Length for the $i$ th track stretch
$l_d$	Distance between front shaft and the vehicle's Center of Mass
$l_t$	Distance between rear shaft and the vehicle's Center of Mass
$l_w$	Lane width
$n_{FS}$	Number of forward steps considered
$n_{gap}$	Number of points that compose the search gap
$P_{FE}$	Friction Ellipse penalty
$R(i)$	Curve Radius for the $i$ th track stretch
$R_t$	Curve radius of rear shaft
$t_i$	Time at the instant $i$
$V$	Linear Velocity
$v_x$	Horizontal Components of the velocity
$v_y$	Vertical Components of the velocity
$w_{FS}$	Forward steps weights array
$x$	Horizontal position coordinate in a Local Reference System
$x_C(i)$	Horizontal coordinate of the car on the Center Line reference system
$x_T(i)$	Horizontal coordinate of starting point for the $i$ th track stretch
$y$	Vertical position coordinates in a Local Reference System
$y_C(i)$	Vertical coordinate of the car on the Center Line reference system
$y_T(i)$	Vertical coordinate of starting point for the $i$ th track stretch
$\alpha(i)$	Curve Angle for the $i$ th track stretch
$\alpha_v$	Angle of Attack
$\beta(i)$	Orientation of starting point for the $i$ th track stretch
$\delta_d$	Right transformation of the steering wheel angle
$\delta_e$	Left transformation of the steering wheel angle
$\delta_i$	Step size
$\theta$	Angle between the vehicle $x$ -axis and the
$\rho$	Curve radius of the Center of Mass
$\tau$	Decay constant of exponential in Acceleration Profiles
$\omega$	Angular Velocity

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