4
Uniformly Informative Signals

In this section, we specialize the model to a uniformly informative private signal. More specifically, we assume that the private signals, x_i, carry the same amount of information across every possible message sent by the Central Banker:

$$x_i|\theta, I_i, s_n = \begin{cases} \theta \text{ if } I_i = 1 \\ x_i \sim U[\theta_{n-1}, \theta n], x_i \perp \theta \text{ if } I_i = 0 \end{cases}.$$

The assumption here is that $g_n = 1 \; \forall n$, so that the price setters can never infer from x_i whether $I_i = 0$ or $I_i = 1$ *irrespective* of the message sent by the Central Banker. While restrictive, this allows us to fully characterize the set of communication equilibria.

Indeed, in the appendix, we show that the equilibrium of the pricing game is as follows

Proposition 7: Given a statement s_n made by the Central Bank when $\theta \in [\theta_{n-1}, \theta_n]$, and private signals $\{x_i\}$, an equilibrium in the pricing game takes the form of

$$p_i = \left(\frac{q(1-r)}{1-rq^2}\right) x_i + \left(1 - \frac{q(1-r)}{1-rq^2}\right) y_n \quad \forall\, i \in [0,1],$$

where

$$y_n \equiv E(\theta|s_n) = \frac{\theta_n + \theta_{n-1}}{2}.$$

It follows that the average price level is

$$\overline{p}|\theta, s_n = kq\theta + (1-qk)y_n$$

while the price variability is

$$Var(p_n|s_n, \theta) = k^2[q(1-q)(\theta - y_n)^2 + (1-q)\frac{1}{3}(\frac{\theta_n - \theta_{n-1}}{2})^2].$$

Using these expressions, one has that the Central Bank's expected payoff, given a state θ and a statement s_n, is

$$(\overline{p}|s_n - \theta + c)^2 + \beta Var(p_n|s_n, \theta).$$

Given that, it is easy to compute the indifference condition that characterizes the set of equilibria. The counterpart of proposition 3.3 for the present setting is:

Proposition 8: Given (c, q, r, β), there exists $M(c, q, r, \beta) \in \mathbb{N}$ communication equilibria, such that, $\forall N \leqslant M(\cdot),\ N \in \mathbb{N}$, the partitions $\{[\theta_{n-1}, \theta_n]\}_n$ are defined by cut-off states $\{\theta_n\}$ that satisfy the following difference equation:

$$\theta_{n+1} - 2\theta_n + \theta_{n-1} - 4c\eta = 0$$

where

$$\eta \equiv \frac{(1-qk)}{(1-qk)^2 + \beta k^2(1-q)(q+1/3)}$$

Under the uniform informativeness assumption, the set of all possible equilibria is completely indexed by ηc, so that η indexes the degree of informativeness that an equilibrium may have [1]. The larger η, the lesser informative the communication policy from the Central Bank. Therefore, we can derive the properties of the communication equilibria by simply analyzing the behavior of η. Throughout the analysis, we focus on the most informative equilibrium.

We first establish the counterpart of proposition 3.3.

Proposition 9: If the Central Banker does not care about the price dispersion ($\beta = 0$) , as the agents information gets almost perfectly precise ($q \to 1$,) the unique communication equilibrium leads to a *single* statement s being made, irrespective of the state θ, and the coordination motive ($r \in (0, 1)$).

Proof: First note that $\forall\, r \in (0, 1)$, $\frac{\partial k}{\partial q} > 0$. Further more, when $\beta = 0$, $\eta = 1/(1 - qk)$. By the definition of k, $\lim_{q\to 1} k = 1$ and $\lim_{q\to 0} k = 0$ for all $r \in (0, 1)$. Therefore, we have:

$$\lim_{q\to 0} \eta|_{\beta=0} = 1 \quad and \ \lim_{q\to 1} \eta|_{\beta=0} = +\infty$$

Additionally:

$$\frac{\partial \eta}{\partial q}\,|_{\beta=0} = \frac{k + q\frac{\partial k}{\partial q}}{(1-qk)^2} > 0$$

[1]In fact, every equilibrium in our setting corresponds to an equilibrium of the communication game given in (5) with a bias that is adjusted by η.

Therefore, as $q \to 1$, η increases monotonically, converging to $+\infty$. ■

Much as in the previous section, if the Central Banker doesn't care about price dispersion ($\beta = 0$), and the price setters' information becomes precise ($q \to 1$), the only equilibrium is such that no information is conveyed. The difference here is that such results holds irrespective of the degree of complementarity among price setters (r).

The behavior of η is represented in figure 4.1.

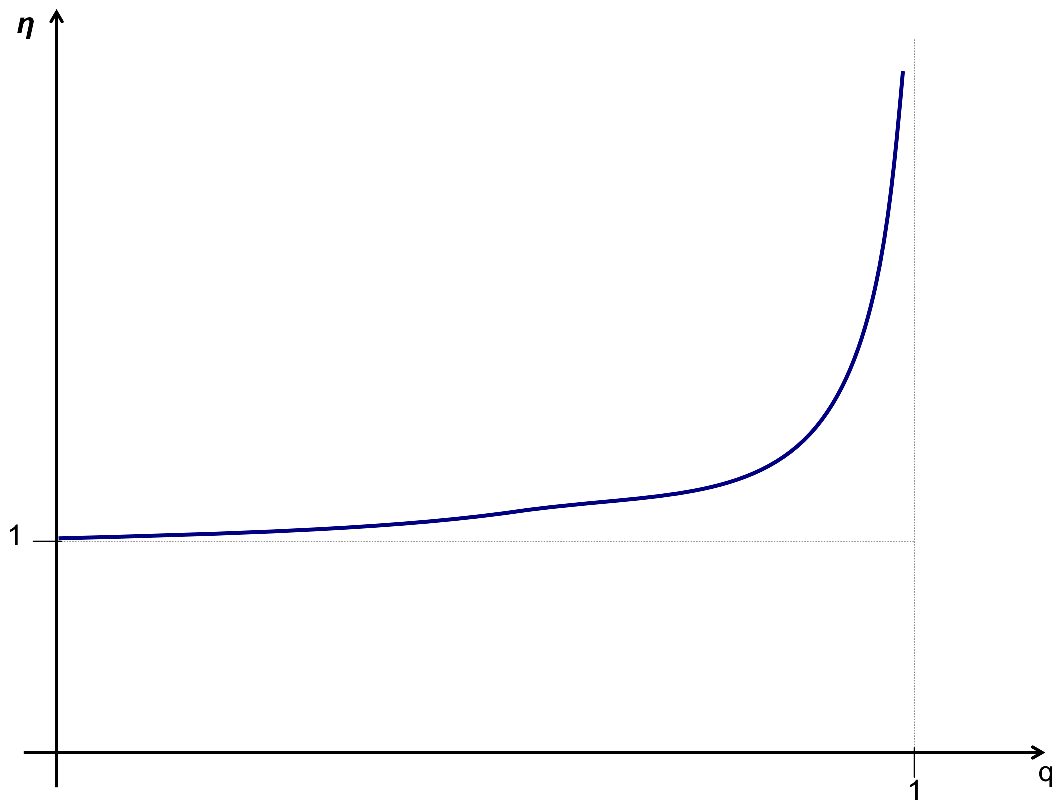

Figura 4.1: Behavior of η when $\beta = 0$

We now address the question of how the communication policy changes as the fundamentals (q, β, r) change. First, note that, no matter how much the Central Banker cares about the price dispersion ($\beta \in [0, +\infty)$), as the agents information gets completely uninformative ($q \to 0$), the game converges to that of (5), and so $\eta \to 1$. On the other hand, as the firm's information gets almost perfectly precise ($q \to 1$):

$$\lim_{q \to 1} \eta \equiv \bar{\eta} = \frac{3(1 + \frac{r}{(1-r)^2})}{2\beta}. \tag{4-1}$$

For a given value of the coordination motive (r), there is a large enough β such that $\bar{\eta} < 1$. In words the misalignment of incentives decreases when the information of the firms is very precise and the Central Banker cares significantly about the price dispersion. Much as before, the reasoning is simple: the influence of the Central Banker on the average price is almost none, so he can credibly transmit more information in equilibrium.

For intermediate values of information precision ($q \in (0,1)$), η will assume the value of 1 only when:

$$\beta(1-r) = \frac{(1-q^2)}{(1-q)(q+1/3)} \tag{4-2}$$

The left hand side of (4-2) is always between 0 and $+\infty$ as $r \in (0,1)$ and $\beta \in [0,+\infty)$. As for the right hand side of (4-2), it is easy to see that it's partial derivative with respect to q is always negative (therefore it is strictly decreasing function of q) and it can assume values in (3/2,3) as $q \in (0,1)$.

Therefore, if the Central Banker puts a lot of weight on price dispersion, $\eta < 1$ for whatever precision of the firm's information. This means the Banker can always communicate more information. On the other hand, if the price setters have a large enough coordination motive (r), $\eta > 1$ for whatever precision of the firm's information, and less information is transmitted.

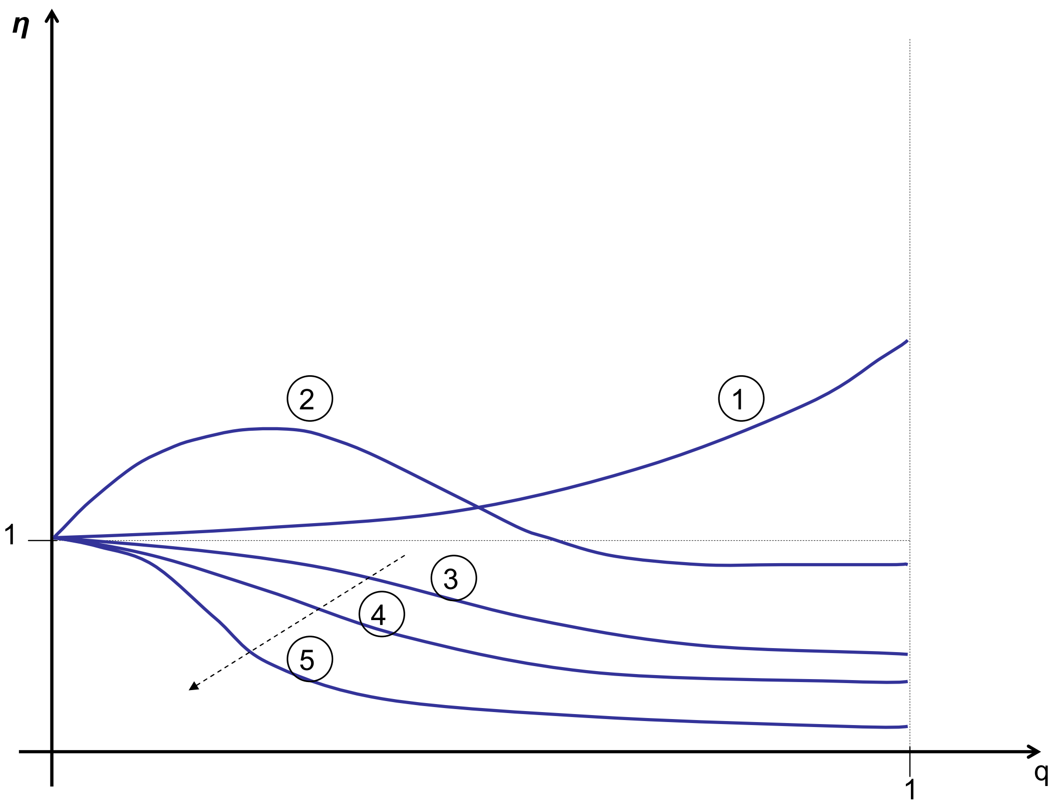

Figura 4.2: Behavior of η as β increases

In Figure 4.2 we show the behavior of η as a function of q. For a given value of r, as β increases the curves follow the sequence depicted above. An increasing aversion to price dispersion on the part of the Central Banker has a monotonic impact upon the amount of information transmitted. The more the Central Banker cares about price dispersion, the more information gets transmitted in equilibrium. In the limit, as β tends to $+\infty$, η converges to:

$$\eta(q) = \begin{cases} 1 & \text{if } q = 0 \\ 0 & \text{if } q \in (0, 1] \end{cases}$$

The above result is the same as in Proposition 3.3. So is the intuition.

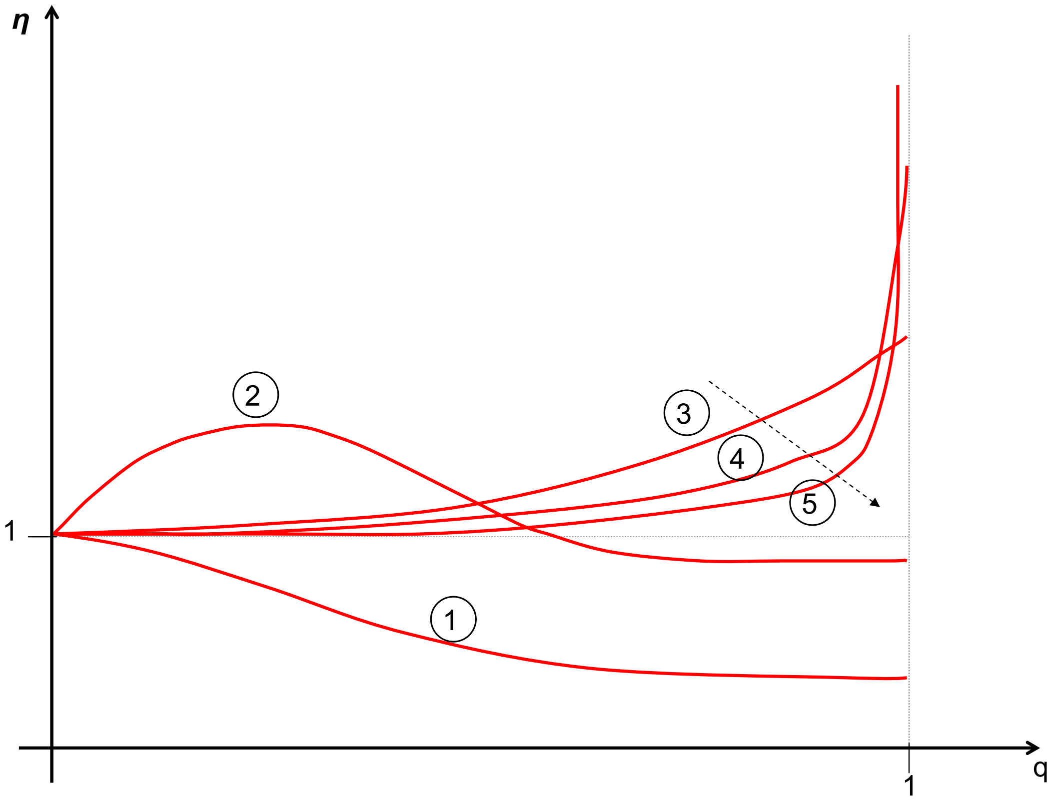

Figura 4.3: Behavior of η as r increases

In Figure 4.3 we show the behavior of η as a function of q. For a given value of β, as r increases the curves follow the sequence depicted. As can be seen, the effect of the coordination motive (r) is non-monotonic. For a given β and information precision q, as r increases, the amount of information transmitted diminishes up to a certain point, when it starts to increase again as η converges back to 1. In the limit, when $r \sim 1$, η as a function of q converges to:

$$\eta(q) = \begin{cases} 1 & \text{if } q \in [0, 1) \\ +\infty & \text{if } q = 1 \end{cases}$$

That is, the coordination motive is so strong that the communication between the Central Banker and the firms is bounded only by the interest misalignment (c), as in (5). The result is similar to that of Proposition 3.3, in which the increasing concern of the firms about the average price allows the Central Banker to credibly transmit *more* information.