7
Other results

7.1
Groups with non-trivial center acting on surfaces

Consider a finitely generated subgroup G of $\mathrm{Diff}(M)_0$

Corollary 7.1.1. *If f is a non-periodic central element of G that leaves invariant a probability measure whose support is compact and not contained in $\mathrm{Fix}(f)$, then f is not a distortion element in G. In other words,*

$$\liminf_{n\to\infty} \frac{|f^n|}{n} > 0,$$

where $|\cdot|$ stands for a word norm.

Proof. Let h be an element of G. Since f is central, if x is a fixed point for f,

$$fh(x) = hf(x) = h(x),$$

showing that $h(\mathrm{Fix}(f)) = \mathrm{Fix}(f)$. This allows us to consider h restricted to S and to lift it to $H : \mathbb{D} \to \mathbb{D}$ (as usual, we consider an identity lift). By lifting G entirely, we get a group of $\tilde{G}$ of diffeomorphisms of the disk $\mathbb{D}$ (which commute with the deck transformations).

Let μ be a f-invariant probability as in the statement of this corollary. Since the support of μ is compact and G is finitely generated, there exists a finite upper bound C for $d(x, Hx)$, for almost every x and every H in $\tilde{G}$. If f were a distortion element, we would have

$$\lim_{n\to\infty} \frac{d(x, F^n(x))}{n} \leq \liminf_{n\to\infty} \frac{C|F^n|}{n} = 0,$$

contradicting Theorem 6.3.1. □

7.2
Positive linear growth in dimension 3

The use of the Brouwer translation arc theorem prevents us from directly applying all methods of the text in dimensions higher than 2. However, by reexamining the proofs of Lemmas 5.3.2 and 5.3.3, we have the following result for dimension 3.

Let M be a geometrically finite 3-manifold, i.e., a 3-manifold whose convex core has finite volume, and $f : M \to M$ a C^1 diffeomorphism isotopic to the identity (relatively to $\mathrm{Fix}(f)$). Suppose that $M \backslash \mathrm{Fix}(f) = \mathbb{H}^3/\Gamma$, where Γ is free and finitely generated.

Theorem 7.2.1. *If f preserves a probability measure whose support is compact and not contained in* $\mathrm{Fix}(f)$ *and the identity lift $F : \mathbb{H}^3 \to \mathbb{H}^3$ has no recurrent points, then $d(x, F^n(x))$ has positive linear growth, where d stands for the hyperbolic metric on $\mathbb{H}^3$.*

Proof. We define the set E as before. The proof of Lemma 5.3.2 (Case 1) remains valid. In the proof of Lemma 5.3.3, since Brouwer's theorem can no longer be used, we obtain a recurrent point instead of a fixed point. Since F does not have recurrent points, Case 2 is not possible. The proof of Lemma 6.2.3 also remains valid. □