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Conclusion

Along this dissertation we developed a formulation through which we extend geometric quantities to non-smooth objects. In particular we address the delicate task to define a tangent object at a singularity. Although there exists many ways to approximate such tangent spaces, most of them are rather complex and intricate. Starting from classical geometric theory where submanifolds are parameterized through differential applications, we propose to use distributions in order to differentiate raw continuous parameterizations. Although there exists a great difference between differential and singular geometry, our main objective was to build up a direct geometric formulation to reduce that difference.

The $\mathcal{D}$-geometric notions were designed via distribution theory precisely for that purpose. As distributions generalize functions, we used them to substitute classical parameterizations in order to obtain a new type of parameterizations. Building up on the concept of immersion, we managed to generalize some fundamentals tools of differential geometry. $\mathcal{D}$-immersions generalize classical immersions, in the sense that the distribution associated to an immersion is a $\mathcal{D}$-immersion. Moreover they strictly extend immersions as we exhibit non-trivial $\mathcal{D}$-immersions such as graphs of L^1 functions.

$\mathcal{D}$-submanifolds would generalize smooth submanifolds in the sense that $\mathcal{D}$-immersions associated to parameterizations of a smooth submanifold M define the $\mathcal{D}$-submanifold associated to M. However we have not managed to extend classical submanifolds as we thought we could do. A great difficulty remains when trying to extend our results from C^1 to C^0 and define flexible notions for compatible $\mathcal{D}$-immersions. This happens here mainly because of the proper distribution theory which does not allow C^1 test functions. Meanwhile this work provided answers and lead us to new questions. We further experimented these notions applying them to tangent cone approximations and curvature estimations.

Other properties of $\mathcal{D}$-immersions might be discovered and we may hope that $\mathcal{D}$-submanifolds could someday extend classical differential submanifolds. We ended the fourth chapter by introducing a new notion of tangent cone, in our attempt to study singularities. It is beyond the scope of this dissertation to proceed to a deeper analysis of this definition but it is a stimulating problem for future works. In particular, we would like to further study the relations between different definitions of tangent cones especially in the light of applications to singularities analysis.