Pontifícia Universidade Católica
do Rio de Janeiro

David Rey

Distributions and Immersions

PUC-Rio - Certificação Digital Nº 0610731/CA

MSc Thesis

Thesis presented to the Post–graduate Program in Applied Mathematics of the Mathematics Department, PUC–Rio as partial fulfillment of the requirements for the degree of Master in Applied Mathematics

Adviser: Prof. Thomas Lewiner

Rio de Janeiro
November 2007

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Rio de Janeiro — November 29, 2007

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David Rey received his B.Sc from Université Montpellier II in Electrical Engineering, with minor in Opto-Electronics. He worked during a semester at the Gávea Sensor laboratory inside the Pontifícia Universidade Católica do Rio de Janeiro on fiber optic sensors, before entering the Mathematics department in 2006.

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Abstract

Rey, David; Lewiner, Thomas. **Distributions and Immersions**. Rio de Janeiro, 2007. 60p. MScThesis — Department of Mathematics, Pontifícia Universidade Católica do Rio de Janeiro.

The challenge of studying shapes has led mathematicians to create powerful abstract concepts, in particular through Differential Geometry. However, differential tools do not apply to simple shapes like cubes. This work is an attempt to use modern advances of the Analysis, namely Distribution Theory, to extend differential quantities to singular objects. Distributions generalize functions, while allowing infinite differentiation. The substitution of classical immersions, which usually serve as submanifold parameterizations, by distributions might thus naturally generalize smooth immersion. This leads to the concept of $\mathcal{D}$-immersion. This work proves that this formulation actually generalizes smooth immersions. Extensions to non-smooth of immersions are discussed through examples and specific cases.

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Keywords

Differential Geometry. Immersions. Distribution Theory. Geometric Singularities. Discrete Differential Geometry.

Resumo

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Os desafios de estudar formas levaram matemáticos a criar abstrações, em particular através da geometria diferencial. Porém, formas simples como cubos não se adequam a ferramentas diferenciáveis. Este trabalho é uma tentativa de usar avanços recentes da análise, no caso a teoria das distribuições, para estender quantidades diferenciáveis a objetos singulares. Como as distribuições generalizam as funções e permitem derivações infinitas, a substituição das parametrizações de subvariedades clássicas por distribuições poderia naturalmente generalizar as subvariedades suaves. Isso nos leva a definir $\mathcal{D}$-imersões. Esse trabalho demonstra que essa formulação, de fato, generaliza as imersões suaves. Extensões para outras classes de subvariedades são discutidas através de exemplos e casos particulares.

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Palavras–chave

Teoria de Morse. Teoria de Forman. Topologia Computacional. Geometria Computacional. Modelagem Geométrica. Matemática Discreta.

Contents

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Quelles sont les trouvailles qui sont des déconvertes et celles qui sont des inventions? La découverte est la trouvaille d'un objet extérieur à nous, qui, même si nous ne l'avons connu que récemment, a toujours existé et existera après nous, et sur lequel notre possibilité de choix est minime. Au contraire, l'invention est la trouvaille d'un objet nouveau, qui n'existait pas avant la trouvaille, et sur laquelle nous avons une grande liberté de choix. [...]

En outre, l'invention des nombres complexes est venue à un certain moment de l'histoire de l'Occident, mais, si cela ne s'était pas produit à ce moment là, elle serait sûrement venue plus tard. Les espaces vectoriels de dimensions ≥ 4, les espaces de Banach et Hilbert, l'inversion et la transformation par polaires réciproques, les distributions, les ondelettes, l'ordinateur, sont des inventions. Mais, une fois trouvés les espaces de Hilbert, la théorie spectrale est une découverte. Une fois trouvées les distributions, leur transformation de Fourier est une découverte. La grotte est une découverte, la hutte une invention, la laine est une découverte, le tissage une invention. On peut jouer longtemps à ce petit jeu de société pas très profond, et tout ce qui vient d'être dit est contestable.

Mais cela montre d'une part que découverte et invention s'entremêlent, qu'elles peuvent être relatives aux objets les plus élementaires comme les plus savants, et qu'il n'y a pas de différence essentielle entre les mathématiques et les autres sciences.

Laurent Schwartz, *Un mathématicien aux prises avec le siècle.*

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