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A
Expressão analítica dos coeficientes do Sistema Composto

Este apêndice apresenta as expressões que relacionam os coeficientes α_i do sistema composto em função dos coeficientes $\boldsymbol{\gamma'}$ do sistema não-linear e dos coeficientes $\boldsymbol{\delta}$ do sistema de pré-distorção.

$$\alpha_1 = \delta_1\ \gamma_1' \tag{A-1}$$

$$\alpha_3 = \delta_1\ \delta_1\ \delta_1^*\ \gamma_3' \ +\ \delta_3\ \gamma_1' \tag{A-2}$$

$$\begin{aligned}\alpha_5 = &\ \delta_1\ \delta_1\ \delta_3^*\ \gamma_3' \ +\ \delta_1\ \delta_3\ \delta_1^*\ \gamma_3' \ +\ \delta_1\ \delta_1^2\ \delta_1^{*2}\ \gamma_5' \\ &+\ \delta_3\ \delta_1\ \delta_1^*\ \gamma_3' \ +\ \delta_5\ \gamma_1'\end{aligned} \tag{A-3}$$

$$\begin{aligned}\alpha_7 = &\ \delta_1\ \delta_1\ \delta_5^*\ \gamma_3' \ +\ \delta_1\ \delta_3\ \delta_3^*\ \gamma_3' \ +\ \delta_1\ \delta_5\ \delta_1^*\ \gamma_3' \\ &+\ \delta_1\ \delta_1^3\ \delta_1^{*3}\ \gamma_7' \ +\ \delta_3\ \delta_1\ \delta_3^*\ \gamma_3' \ +\ \delta_3\ \delta_3\ \delta_1^*\ \gamma_3' \\ &+\ \delta_3\ \delta_1^2\ \delta_1^{*2}\ \gamma_5' \ +\ \delta_5\ \delta_1\ \delta_1^*\ \gamma_3' \ +\ \delta_7\ \gamma_1'\end{aligned} \tag{A-4}$$

$$\begin{aligned}\alpha_9 = &\ \delta_1\ \delta_1\ \delta_7^*\ \gamma_3' \ +\ \delta_1\ \delta_3\ \delta_5^*\ \gamma_3' \ +\ \delta_1\ \delta_5\ \delta_3^*\ \gamma_3' \\ &+\ \delta_1\ \delta_7\ \delta_1^*\ \gamma_3' \ +\ \delta_1\ \delta_1^2\ \delta_3^{*2}\ \gamma_5' \ +\ \delta_1\ \delta_3^2\ \delta_1^{*2}\ \gamma_5' \\ &+\ \delta_3\ \delta_1\ \delta_5^*\ \gamma_3' \ +\ \delta_3\ \delta_3\ \delta_3^*\ \gamma_3' \ +\ \delta_3\ \delta_5\ \delta_1^*\ \gamma_3' \\ &+\ \delta_3\ \delta_1^3\ \delta_1^{*3}\ \gamma_7' \ +\ \delta_5\ \delta_1\ \delta_3^*\ \gamma_3' \ +\ \delta_5\ \delta_3\ \delta_1^*\ \gamma_3' \\ &+\ \delta_5\ \delta_1^2\ \delta_1^{*2}\ \gamma_5' \ +\ \delta_7\ \delta_1\ \delta_1^*\ \gamma_3'\end{aligned} \tag{A-5}$$

$$\begin{aligned}
\alpha_{11} = {} & \delta_1\ \delta_3\ \delta_7^*\ \gamma_3' \ + \ \delta_1\ \delta_5\ \delta_5^*\ \gamma_3' \ + \ \delta_1\ \delta_7\ \delta_3^*\ \gamma_3' \\
& + \ \delta_3\ \delta_1\ \delta_7^*\ \gamma_3' \ + \ \delta_3\ \delta_3\ \delta_5^*\ \gamma_3' \ + \ \delta_3\ \delta_5\ \delta_3^*\ \gamma_3' \\
& + \ \delta_3\ \delta_7\ \delta_1^*\ \gamma_3' \ + \ \delta_3\ \delta_1^2\ \delta_3^{*2}\ \gamma_5' \ + \ \delta_3\ \delta_3^2\ \delta_1^{*2}\ \gamma_5' \\
& + \ \delta_5\ \delta_1\ \delta_5^*\ \gamma_3' \ + \ \delta_5\ \delta_3\ \delta_3^*\ \gamma_3' \ + \ \delta_5\ \delta_5\ \delta_1^*\ \gamma_3' \\
& + \ \delta_5\ \delta_1^3\ \delta_1^{*3}\ \gamma_7' \ + \ \delta_7\ \delta_1\ \delta_3^*\ \gamma_3' \ + \ \delta_7\ \delta_3\ \delta_1^*\ \gamma_3' \\
& + \ \delta_7\ \delta_1^2\ \delta_1^{*2}\ \gamma_5'
\end{aligned} \tag{A-6}$$

$$\begin{aligned}
\alpha_{13} = {} & \delta_1\ \delta_5\ \delta_7^*\ \gamma_3' \ + \ \delta_1\ \delta_7\ \delta_5^*\ \gamma_3' \ + \ \delta_1\ \delta_1^2\ \delta_5^{*2}\ \gamma_5' \\
& + \ \delta_1\ \delta_3^2\ \delta_3^{*2}\ \gamma_5' \ + \ \delta_1\ \delta_5^2\ \delta_1^{*2}\ \gamma_5' \ + \ \delta_1\ \delta_1^3\ \delta_3^{*3}\ \gamma_7' \\
& + \ \delta_1\ \delta_3^3\ \delta_1^{*3}\ \gamma_7' \ + \ \delta_3\ \delta_3\ \delta_7^*\ \gamma_3' \ + \ \delta_3\ \delta_5\ \delta_5^*\ \gamma_3' \\
& + \ \delta_3\ \delta_7\ \delta_3^*\ \gamma_3' \ + \ \delta_5\ \delta_1\ \delta_7^*\ \gamma_3' \ + \ \delta_5\ \delta_3\ \delta_5^*\ \gamma_3' \\
& + \ \delta_5\ \delta_5\ \delta_3^*\ \gamma_3' \ + \ \delta_5\ \delta_7\ \delta_1^*\ \gamma_3' \ + \ \delta_5\ \delta_1^2\ \delta_3^{*2}\ \gamma_5' \\
& + \ \delta_5\ \delta_3^2\ \delta_1^{*2}\ \gamma_5' \ + \ \delta_7\ \delta_1\ \delta_5^*\ \gamma_3' \ + \ \delta_7\ \delta_3\ \delta_3^*\ \gamma_3' \\
& + \ \delta_7\ \delta_5\ \delta_1^*\ \gamma_3' \ + \ \delta_7\ \delta_1^3\ \delta_1^{*3}\ \gamma_7'
\end{aligned} \tag{A-7}$$

$$\begin{aligned}
\alpha_{15} = {} & \delta_1\ \delta_7\ \delta_7^*\ \gamma_3' \ + \ \delta_3\ \delta_5\ \delta_7^*\ \gamma_3' \ + \ \delta_3\ \delta_7\ \delta_5^*\ \gamma_3' \\
& + \ \delta_3\ \delta_1^2\ \delta_5^{*2}\ \gamma_5' \ + \ \delta_3\ \delta_3^2\ \delta_3^{*2}\ \gamma_5' \ + \ \delta_3\ \delta_5^2\ \delta_1^{*2}\ \gamma_5' \\
& + \ \delta_3\ \delta_1^3\ \delta_3^{*3}\ \gamma_7' \ + \ \delta_3\ \delta_3^3\ \delta_1^{*3}\ \gamma_7' \ + \ \delta_5\ \delta_3\ \delta_7^*\ \gamma_3' \\
& + \ \delta_5\ \delta_5\ \delta_5^*\ \gamma_3' \ + \ \delta_5\ \delta_7\ \delta_3^*\ \gamma_3' \ + \ \delta_7\ \delta_1\ \delta_7^*\ \gamma_3' \\
& + \ \delta_7\ \delta_3\ \delta_5^*\ \gamma_3' \ + \ \delta_7\ \delta_5\ \delta_3^*\ \gamma_3' \ + \ \delta_7\ \delta_7\ \delta_1^*\ \gamma_3' \\
& + \ \delta_7\ \delta_1^2\ \delta_3^{*2}\ \gamma_5' \ + \ \delta_7\ \delta_3^2\ \delta_1^{*2}\ \gamma_5'
\end{aligned} \tag{A-8}$$

$$\begin{aligned}
\alpha_{17} = {} & \delta_1\ \delta_1^2\ \delta_7^{*2}\ \gamma_5' \ + \ \delta_1\ \delta_3^2\ \delta_5^{*2}\ \gamma_5' \ + \ \delta_1\ \delta_5^2\ \delta_3^{*2}\ \gamma_5' \\
& + \ \delta_1\ \delta_7^2\ \delta_1^{*2}\ \gamma_5' \ + \ \delta_3\ \delta_7\ \delta_7^*\ \gamma_3' \ + \ \delta_5\ \delta_5\ \delta_7^*\ \gamma_3' \\
& + \ \delta_5\ \delta_7\ \delta_5^*\ \gamma_3' \ + \ \delta_5\ \delta_1^2\ \delta_5^{*2}\ \gamma_5' \ + \ \delta_5\ \delta_3^2\ \delta_3^{*2}\ \gamma_5' \\
& + \ \delta_5\ \delta_5^2\ \delta_1^{*2}\ \gamma_5' \ + \ \delta_5\ \delta_1^3\ \delta_3^{*3}\ \gamma_7' \ + \ \delta_5\ \delta_3^3\ \delta_1^{*3}\ \gamma_7' \\
& + \ \delta_7\ \delta_3\ \delta_7^*\ \gamma_3' \ + \ \delta_7\ \delta_5\ \delta_5^*\ \gamma_3' \ + \ \delta_7\ \delta_7\ \delta_3^*\ \gamma_3'
\end{aligned} \tag{A-9}$$

$$
\begin{aligned}
\alpha_{19} = {} & \delta_1\, \delta_1^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_1\, \delta_3^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_1\, \delta_5^3\, \delta_1^{*3}\, \gamma'_7 \\
& +\; \delta_3\, \delta_1^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_3^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_5^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_3\, \delta_7^2\, \delta_1^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_7\, \delta_7^{*}\, \gamma'_3 \;+\; \delta_7\, \delta_5\, \delta_7^{*}\, \gamma'_3 \\
& +\; \delta_7\, \delta_7\, \delta_5^{*}\, \gamma'_3 \;+\; \delta_7\, \delta_1^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_3^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_7\, \delta_5^2\, \delta_1^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_1^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_7\, \delta_3^3\, \delta_1^{*3}\, \gamma'_7
\end{aligned}
\tag{A-10}
$$

$$
\begin{aligned}
\alpha_{21} = {} & \delta_1\, \delta_3^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_1\, \delta_5^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_1\, \delta_7^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_3\, \delta_1^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_3\, \delta_3^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_3\, \delta_5^3\, \delta_1^{*3}\, \gamma'_7 \\
& +\; \delta_5\, \delta_1^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_3^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_5^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_5\, \delta_7^2\, \delta_1^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_7\, \delta_7^{*}\, \gamma'_3
\end{aligned}
\tag{A-11}
$$

$$
\begin{aligned}
\alpha_{23} = {} & \delta_3\, \delta_3^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_5^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_7^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_5\, \delta_1^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_5\, \delta_3^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_5\, \delta_5^3\, \delta_1^{*3}\, \gamma'_7 \\
& +\; \delta_7\, \delta_1^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_3^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_5^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_7\, \delta_7^2\, \delta_1^{*2}\, \gamma'_5
\end{aligned}
\tag{A-12}
$$

$$
\begin{aligned}
\alpha_{25} = {} & \delta_1\, \delta_5^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_1\, \delta_7^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_1\, \delta_1^3\, \delta_7^{*3}\, \gamma'_7 \\
& +\; \delta_1\, \delta_3^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_1\, \delta_5^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_1\, \delta_7^3\, \delta_1^{*3}\, \gamma'_7 \\
& +\; \delta_5\, \delta_3^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_5^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_7^2\, \delta_3^{*2}\, \gamma'_5 \\
& +\; \delta_7\, \delta_1^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_7\, \delta_3^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_7\, \delta_5^3\, \delta_1^{*3}\, \gamma'_7
\end{aligned}
\tag{A-13}
$$

$$
\begin{aligned}
\alpha_{27} = {} & \delta_3\, \delta_5^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_7^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_3\, \delta_1^3\, \delta_7^{*3}\, \gamma'_7 \\
& +\; \delta_3\, \delta_3^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_3\, \delta_5^3\, \delta_3^{*3}\, \gamma'_7 \;+\; \delta_3\, \delta_7^3\, \delta_1^{*3}\, \gamma'_7 \\
& +\; \delta_7\, \delta_3^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_5^2\, \delta_5^{*2}\, \gamma'_5 \;+\; \delta_7\, \delta_7^2\, \delta_3^{*2}\, \gamma'_5
\end{aligned}
\tag{A-14}
$$

$$
\begin{aligned}
\alpha_{29} = {} & \delta_1\, \delta_7^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_5^2\, \delta_7^{*2}\, \gamma'_5 \;+\; \delta_5\, \delta_7^2\, \delta_5^{*2}\, \gamma'_5 \\
& +\; \delta_5\, \delta_1^3\, \delta_7^{*3}\, \gamma'_7 \;+\; \delta_5\, \delta_3^3\, \delta_5^{*3}\, \gamma'_7 \;+\; \delta_5\, \delta_5^3\, \delta_3^{*3}\, \gamma'_7 \\
& +\; \delta_5\, \delta_7^3\, \delta_1^{*3}\, \gamma'_7
\end{aligned}
\tag{A-15}
$$

$$
\begin{aligned}
\alpha_{31} = {} & \delta_1\, \delta_3^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_1\, \delta_5^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_1\, \delta_7^3\, \delta_3^{*3}\, \gamma_7' \\
& +\; \delta_3\, \delta_7^2\, \delta_7^{*2}\, \gamma_5' \;+\; \delta_7\, \delta_5^2\, \delta_7^{*2}\, \gamma_5' \;+\; \delta_7\, \delta_7^2\, \delta_5^{*2}\, \gamma_5' \\
& +\; \delta_7\, \delta_1^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_3^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_5^3\, \delta_3^{*3}\, \gamma_7' \\
& +\; \delta_7\, \delta_7^3\, \delta_1^{*3}\, \gamma_7'
\end{aligned}
\tag{A-16}
$$

$$
\begin{aligned}
\alpha_{33} = {} & \delta_3\, \delta_3^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_3\, \delta_5^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_3\, \delta_7^3\, \delta_3^{*3}\, \gamma_7' \\
& +\; \delta_5\, \delta_7^2\, \delta_7^{*2}\, \gamma_5'
\end{aligned}
\tag{A-17}
$$

$$
\begin{aligned}
\alpha_{35} = {} & \delta_5\, \delta_3^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_5\, \delta_5^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_5\, \delta_7^3\, \delta_3^{*3}\, \gamma_7' \\
& +\; \delta_7\, \delta_7^2\, \delta_7^{*2}\, \gamma_5'
\end{aligned}
\tag{A-18}
$$

$$
\begin{aligned}
\alpha_{37} = {} & \delta_1\, \delta_5^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_1\, \delta_7^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_3^3\, \delta_7^{*3}\, \gamma_7' \\
& +\; \delta_7\, \delta_5^3\, \delta_5^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_7^3\, \delta_3^{*3}\, \gamma_7'
\end{aligned}
\tag{A-19}
$$

$$
\alpha_{39} = \delta_3\, \delta_5^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_3\, \delta_7^3\, \delta_5^{*3}\, \gamma_7' \tag{A-20}
$$

$$
\alpha_{41} = \delta_5\, \delta_5^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_5\, \delta_7^3\, \delta_5^{*3}\, \gamma_7' \tag{A-21}
$$

$$
\alpha_{43} = \delta_1\, \delta_7^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_5^3\, \delta_7^{*3}\, \gamma_7' \;+\; \delta_7\, \delta_7^3\, \delta_5^{*3}\, \gamma_7' \tag{A-22}
$$

$$
\alpha_{45} = \delta_3\, \delta_7^3\, \delta_7^{*3}\, \gamma_7' \tag{A-23}
$$

$$
\alpha_{47} = \delta_5\, \delta_7^3\, \delta_7^{*3}\, \gamma_7' \tag{A-24}
$$

$$
\alpha_{49} = \delta_7\, \delta_7^3\, \delta_7^{*3}\, \gamma_7' \tag{A-25}
$$

B
Autocorrelação e Densidade Espectral de potência da envoltória complexa do sinal OFDM

A envoltória complexa do sinal OFDM com relação a uma frequência f_c, escreve-se

$$\tilde{m}(t) = \sum_{i=1}^{N} \sum_{k=-\infty}^{\infty} \sqrt{2}\, d_{ik}\, p(t+\epsilon-kT)\, e^{j[\frac{2\pi i}{T}(t+\epsilon)+\theta]} \tag{B-1}$$

onde d_{ik} é uma variável aleatória complexa que corresponde ao sinal transmitido no k-ésimo intervalo da i-ésima portadora, $p(t)$ é o pulso formatador com energia unitária e é igual para todas as portadoras, T é a duração do símbolo, ϵ é uma variável aleatória uniforme em $\left[-\frac{T}{2}, \frac{T}{2}\right)$ e θ é uma variável aleatória uniforme em $[0, 2\pi)$.

Opcionalmente $\tilde{m}(t)$, pode ser escrito como

$$\tilde{m}(t) = \sum_{k=-\infty}^{\infty} \tilde{m}_k(t)\, p(t+\epsilon-kT) \tag{B-2}$$

onde

$$\tilde{m}_k(t) = \sum_{i=1}^{N} \sqrt{2}\, d_{ik}\, e^{j[\frac{2\pi i}{T}(t+\epsilon)+\theta]} \tag{B-3}$$

$\tilde{m}_k(t)$ refere-se à envoltória complexa do sinal transmitido no k-ésimo intervalo, aqui denominado símbolo OFDM.

A Função Autocorrelação do processo estocástico $\tilde{m}_k(t)$ é definida por:

$$R_{\tilde{m}_k}(t_1, t_2) = \mathbb{E}\left[\tilde{m}_k(t_1)\tilde{m}_k^*(t_2)\right] \tag{B-4}$$

considerando-se (B-3) tem-se

$$\begin{aligned} R_{\tilde{m}_k}(t_1,t_2) &= \mathbb{E}\left[\left(\sum_{i=1}^{N}\sqrt{2}\,d_{ik}e^{j[\frac{2\pi i}{T}(t_1+\epsilon)+\theta]}\right)\left(\sum_{\ell=1}^{N}\sqrt{2}\,d_{\ell k}^{*}\,e^{-j[\frac{2\pi\ell}{T}(t_2+\epsilon)+\theta]}\right)\right] \\ &= 2\,\mathbb{E}\left[\left(\sum_{i=1}^{N}\sum_{\ell=1}^{N} d_{ik}d_{\ell k}^{*}\,e^{j\frac{2\pi}{T}[it_1-\ell t_2+(i-\ell)\epsilon]}\right)\right] \end{aligned} \tag{B-5}$$

como já foi dito anteriormente as variáveis aleatórias ϵ e d_{ik} são estatisticamente independentes, sendo assim podemos escrever (B-5) como

$$R_{\tilde{m}_k}(t_1,t_2) = \sqrt{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\mathbb{E}\left[d_{ik}d_{\ell k}^{*}\right]\mathbb{E}\left[e^{j\frac{2\pi}{T}(i-\ell)\epsilon}\right]e^{j\frac{2\pi}{T}(it_1-\ell t_2)} \tag{B-6}$$

Note que, como ϵ é uniforme em $\left[-\frac{T}{2},\frac{T}{2}\right)$,

$$\mathbb{E}\left[e^{j\frac{2\pi}{T}(i-\ell)\epsilon}\right] = \begin{cases} 0, & i\neq\ell; \\ \\ 1, & i=\ell. \end{cases} \tag{B-7}$$

sendo assim a Função Autocorrelação de $\tilde{m}_k(t)$ se escreve

$$R_{\tilde{m}_k}(t_1,t_2) = \sqrt{2}\sum_{i=1}^{N}\mathbb{E}\left[|d_{ik}|^2\right]e^{j\frac{2\pi i}{T}(t_1-t_2)} \tag{B-8}$$

considere que $E_s = \mathbb{E}\left[|d_{ik}|^2\right]$, pois sabe-se que $E_s = \mathbb{E}\left[|s_\ell|^2\right]$. Por tanto tem-se

$$R_{\tilde{m}_k}(t_1,t_2) = 2\,E_s\sum_{i=1}^{N}e^{j\frac{2\pi i}{T}(t_1-t_2)}ou \tag{B-9}$$

Sabe-se que a densidade espectral de potência de um processo estacionário no sentido amplo (demostrado em [13]) é dada pela Transformada de Fourier de sua função autocorrelação. Assim a partir de (B-9) e fazendo $\tau = t_1 - t_2$, tem-se

$$\begin{aligned} S_{\tilde{m}_k}(f) &= 2\,E_s\,\mathcal{F}\left[\sum_{i=1}^{N}e^{j[\frac{2\pi i}{T}(\tau)]}\right] \\ &= 2\,E_s\sum_{i=1}^{N}\mathcal{F}\left[e^{j[\frac{2\pi i}{T}(\tau)]}\right] \end{aligned} \tag{B-10}$$

como

$$\mathcal{F}\left[e^{j[\frac{2\pi i}{T}(\tau)]}\right] = \delta\left(f-\frac{i}{T}\right) \tag{B-11}$$

A Densidade Espectral de Potência do sinal $\tilde{m}_k(t)$ é dada por (B-12)

$$S_{\tilde{m}_k}(f) = 2\ E_s \sum_{i=1}^{N} \delta\left(f - \frac{i}{T}\right) \tag{B-12}$$

a partir de (B-12) podemos afirmar que a densidade espectral de potência do sinal $\tilde{m}_k(t)$ é dada por um trem de impulsos de área E_s espaçados de $1/T$ conforme ilustrado na Figura B.1

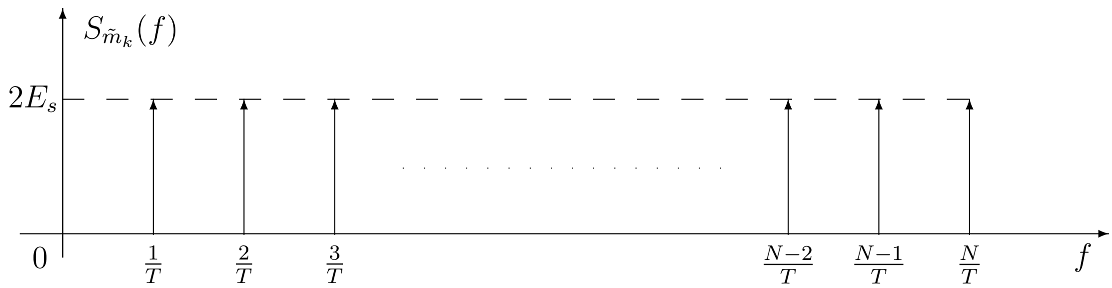

Figura B.1: Densidade Espectral de Potência do sinal $\tilde{m}_k(t)$

Considerando-se que a Função Autocorrelação normalizada de $\tilde{m}_k$ é definida como

$$\begin{aligned} \bar{R}_{\tilde{m}_k}(\tau) &= \frac{R_{\tilde{m}_k}(\tau)}{R_{\tilde{m}_k}(0)} \\ &= \frac{R_{\tilde{m}_k}(\tau)}{2E_s N} \end{aligned} \tag{B-13}$$

a densidade espectral de potência normalizada de $\tilde{m}_k(t)$ definida como a transformada de fourier de $\bar{R}_{\tilde{m}_k}(\tau)$, se escreve

$$\bar{S}_{\tilde{m}_k}(f) = \frac{1}{N} \sum_{i=1}^{N} \delta\left(f - \frac{i}{T}\right) \tag{B-14}$$

Se sabe também por [13] que

$$R_{\tilde{m}}(\tau) = \frac{1}{T} R_{\tilde{m}_k}(\tau)\, [p(t) * p(-t)]\,(\tau) \tag{B-15}$$

A partir de (B-15) a densidade espectral de potência do processo $\tilde{m}(t)$ é dada por

$$S_{\tilde{m}}(f) \;=\; \mathcal{F}\left[R_{\tilde{m}}(\tau)\right] \tag{B-16}$$

ou ainda, utilizando-se algumas propriedades da Transformada de Fourier e substituindo (B-15) em (B-16), tem-se

$$\begin{aligned} S_{\tilde{m}}(f) &= \mathcal{F}\left[\frac{1}{T}R_{\tilde{m}_k}(\tau)\left[p(t)*p(-t)\right](\tau)\right] && \text{(B-17)}\\ &= \frac{1}{T}\mathcal{F}\left[R_{\tilde{m}_k}(\tau)\right]\left\{\mathcal{F}\left[p(\tau)\right]\mathcal{F}\left[p(-\tau)\right]\right\} && \text{(B-18)}\\ &= \frac{1}{T}S_{\tilde{m}_k}(f)*P(f)P^*(f) && \text{(B-19)}\end{aligned}$$

onde $P(f) = \mathcal{F}\left[p(\tau)\right]$, ou ainda,

$$S_{\tilde{m}}(f) = \frac{1}{T}S_{\tilde{m}_k}(f)*\left|P(f)\right|^2 \tag{B-20}$$

considerando-se (B-12), esta Densidade Espectral de Potência se escreve

$$S_{\tilde{m}}(f) = \frac{2Es}{T}\sum_{i=1}^{N}\left|P\left(f-\frac{i}{T}\right)\right|^2 \tag{B-21}$$

Define-se a Densidade Espectral de Potência normalizada como sendo a Transformada de Fourier de $\bar{R}_{\tilde{m}}(\tau)$, tem-se

$$\bar{S}_{\tilde{m}}(f) = \frac{S_{\tilde{m}}(f)}{P_{\tilde{m}}} \tag{B-22}$$

onde $S_{\tilde{m}}(f)$ é dado por (B-21) e $P_{\tilde{m}}$ é dado por

$$\begin{aligned} P_{\tilde{m}} &= \frac{R_{\tilde{m}_k}(0)}{T}\\ &= \frac{2EsN}{T} && \text{(B-23)}\end{aligned}$$

assim podemos reescrever a densidade espectral de potência normalizada como

$$\bar{S}_{\tilde{m}}(f) = \frac{1}{N}\sum_{i=1}^{N}\left|P\left(f-\frac{i}{T}\right)\right|^2 \tag{B-24}$$