5
Numerical illustrations

In the illustrations developed here, the market demand expectations known by each company are represented by instances of exponential probability-density functions, whose means are denored by β_X and β_Z, respectively, for Buyer's and Supplier's market demands. Two settings will be studied, which are equal except the mean of the probability-density function that represents Supplier's market demand. These settings were chosen for examining the effect that Supplier's market demand has on the desirability of the contract. The parameters that characterize the manufacturing settings are given in Table 5 and these two settings will be denoted by $S1$ and $S2$.

		Setting 1	Setting 2
Buyer's market-demand mean	β_X	200	200
Supplier's market-demand mean	β_Z	100	300
Product market price	p_p	2000	2000
Material market price	p_m	1000	1000
Raw material market price	p_{rm}	500	500
Buyer's capacity cost-function coefficient	a_B	10	10
Supplier's capacity cost-function coefficient	a_S	1	1
Buyer's production variable cost	v_p	100	100
Supplier's production variable cost	v_m	50	50
Buyer's production margin*	π_B	900	900
Supplier's production margin*	π_S	450	450

$^*: thecompanysellingitsproducttothespotmarket.$

Table 5.1: Manufacturing setting parameters.

In this chapter, only the final expressions for the companies' expected operational profits will be presented. The terms involved in those profits, under independent planning (IP), contract situation (given ζ) and central planning (CP), are determined in Appendix D. Though the companies' capacity problems admit unique optimal solution and implicit in the expressions for the first-order optimality condition, these non-linear problems were solved by computational programs in AIMMS. That global program and their subprograms are exposed in Appendix E, as well as the results obtained by them are presented in tabular way.

In order to have a global view of the results obtained for both settings, which will be commented along this chapter, the following Table 5.2 summarizes the results for the companies' optimal capacities and expected profits for the two benchmarks – independent planning ($C_k, EP_k,\ k \in \{B, S\}$, and $EP_{IP} = EP_B + EP_S$) and central planning ((C_B^*, C_S^*) and EP_{CP}^*). Also, the minimum and maximum values for the companies' results obtained ($C_{k|\zeta^*}$ and $EP_{k|\zeta^*},\ k \in \{B, S\}$, and $EP_{D|\zeta^*} = EP_{B|\zeta^*} + EP_{S|\zeta^*}$, with $\zeta^* \in \Omega_{VOC}^{Si}$) from the set of initial contracts $\Omega_\circ$ considered (see Section 5.3) are presented. The minimum and maximum values for the performance measures obtained from Ω° are presented in that table. These measures are the companies' surplus ($\delta_{k|\zeta^*}, k \in \{B, S\}$) and dyad's improvement and efficiency ($\eta_{D|\zeta^*}$ and $\xi_{D|\zeta^*}$), which describe the impact that the type of contract proposed has on the companies, individually and jointly.

	Setting 1			Setting 2		
	Value	Minimum	Maximum	Value	Minimum	Maximum
C_B	37.34	–	–	37.34	–	–
C_B^*	44.72	–	–	42.10	–	–
$C_{B\|\zeta^*}$	–	41.10	46.12	–	40.55	44.40
C_S	90.77	–	–	140.75	–	–
C_S^*	111.15	–	–	153.28	–	–
$C_{S\|\zeta^*}$	–	109.55	111.76	–	152.85	153.92
EP_B	16712.57	–	–	16712.57	–	–
$EP_{B\|\zeta^*}$	–	16714.90	25785.25	–	16718.64	22307.80
EP_S	18605.51	–	–	30743.79	–	–
$EP_{S\|\zeta^*}$	–	18722.58	27801.76	–	30748.40	36332.44
EP_{IP}	35318.08	–	–	47456.36	–	–
EP_{CP}	44516.68	–	–	53065.33	–	–
$EP_{D\|\zeta^*}$	–	44349.36	44516.67	–	52999.84	53065.33
$\delta_{B\|\zeta^*}$	–	0.01%	54.29%	–	0.04%	33.48%
$\delta_{S\|\zeta^*}$	–	0.63%	49.43%	–	0.01%	18.18%
$\eta_{D\|\zeta^*}$	–	25.57%	26.04%	–	11.68%	11.82%
$\xi_{D\|\zeta^*}$	–	99.62%	100.00%	–	99.88%	100.00%

Table 5.2: Results for the companies' optimal capacities and expected profits, where $\zeta^* \in \Omega_{VOC}^{Si}$ for $i \in \{1, 2\}$.

This chapter is organized in four sections. Section 5.1 treats the benchmarks –independent and central planning– for both settings. In Section 5.2 is illustrated the contract situation under a given contract for each setting. Section 5.3 approaches the contract problem over a set of initial contracts, whose results allow to do the analysis of the contract proposed as discussed in Section 5.4.

5.1
Benchmarks – Independent and central planning

5.1.1
The companies under independent planning

Since Buyer's and Supplier's medium-term production-capacity cost-functions are characteristic of each company, they are the same ones under independent planning and under the contract. Recalling that, in this work, those cost-functions are considered to be quadratic, and defining them by $\Psi_k(C) = a_k \cdot C^2$, the marginal production-capacity cost is given by $\frac{d}{dC}\Big(\Psi_k(C)\Big) = 2a_k \cdot C$, $k \in \{B, S\}$. The simplified expressions for Buyer's and Supplier's expected operational profit-functions are presented, respectively, by Expression (5-1) and Expression (5-2). Buyer's and Supplier's expected operational profit-functions, under independent planning, and the capacity cost-functions are sketched in Figure 5.1 and Figure 5.2, respectively, for Setting 1 and Setting 2.

$$E_X\Big[\Pi_B(C,x)\Big] = \pi_B \cdot \beta_X \left(1 - e^{-\frac{C}{\beta_X}}\right) \tag{5-1}$$

$$E_Z\Big[\Pi_S(C,z)\Big] = \pi_S \cdot \beta_Z \left(1 - e^{-\frac{C}{\beta_Z}}\right) \tag{5-2}$$

Buyer's and Supplier's marginal expected operational profit-functions, in terms of the capacity C, are given, respectively, by Expression (5-3) and Expression (5-4), which are shown together to their marginal capacity cost-functions in Figure 5.3 and Figure 5.4, respectively, for Setting 1 and Setting 2.

$$\frac{d}{dC}\left(E_X\Big[\Pi_B(C,x)\Big]\right) = \pi_B \cdot e^{-\frac{C}{\beta_X}} \tag{5-3}$$

$$\frac{d}{dC}\left(E_Z\Big[\Pi_S(C,z)\Big]\right) = \pi_S \cdot e^{-\frac{C}{\beta_Z}} \tag{5-4}$$

According to the results presented in Table 5.2, Buyer's and Supplier's optimal capacity decisions are, respectively, $C_B = 37.34$ and $C_S = 90.77$ for

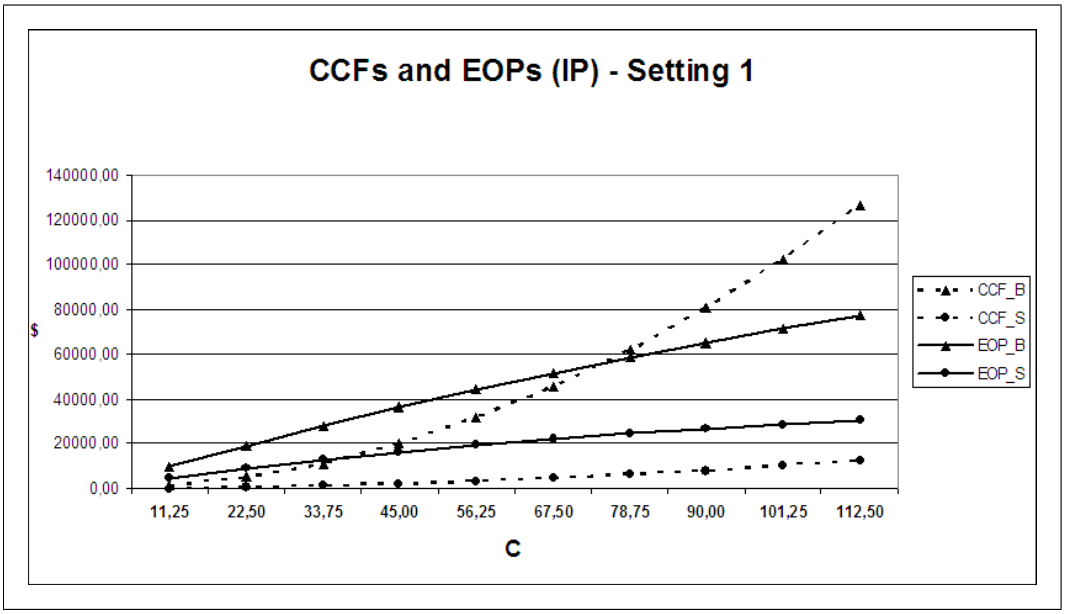

Figure 5.1: Capacity cost-functions and expected operational profit-functions under independent planning for Setting 1.

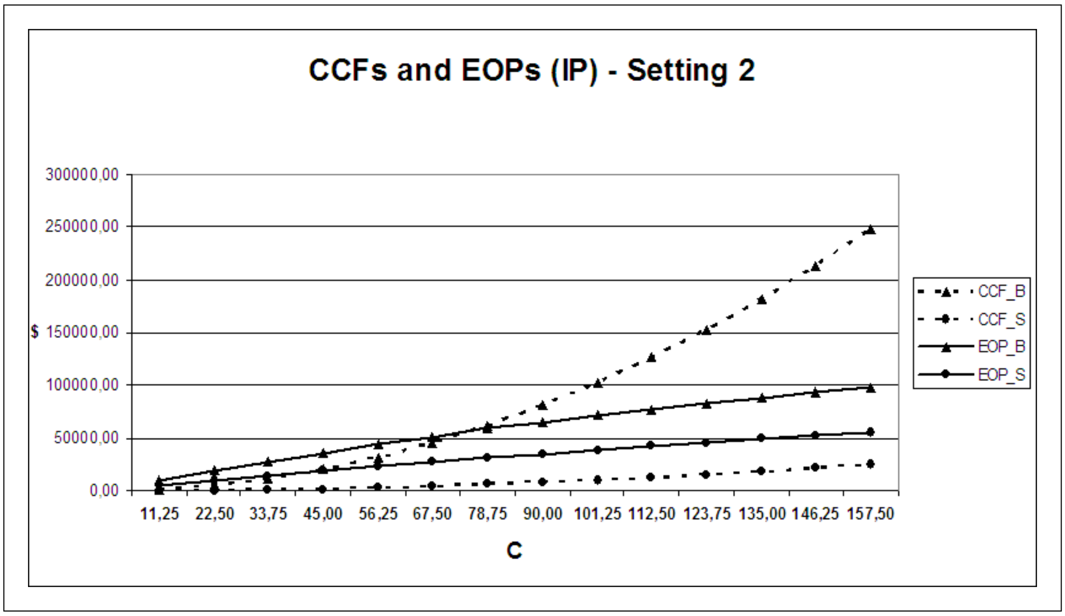

Figure 5.2: Capacity cost-functions and expected operational profit-functions under independent planning for Setting 2.

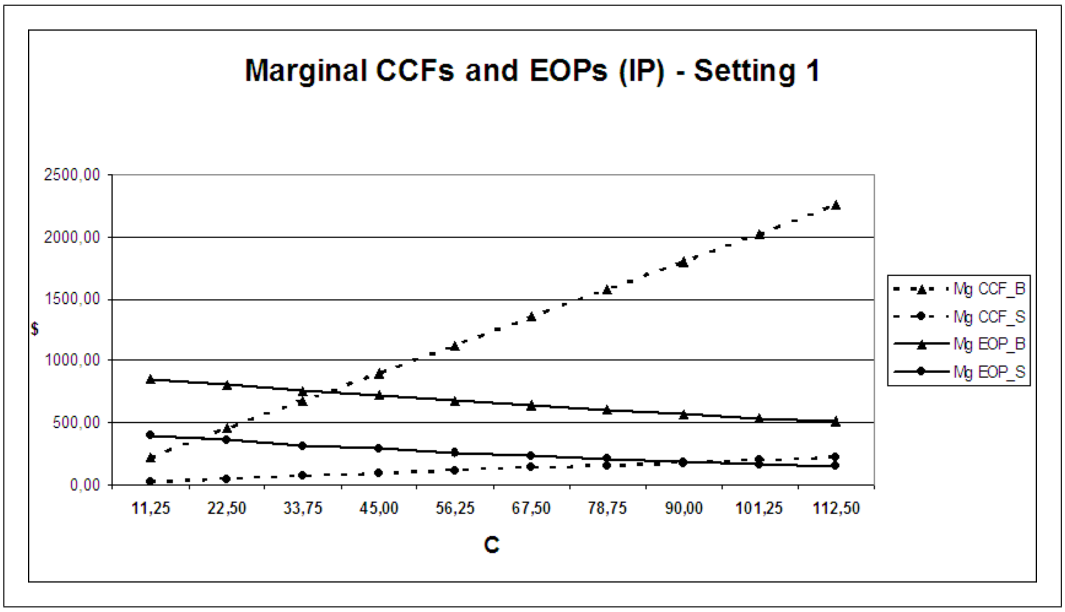

Figure 5.3: Marginal capacity cost-functions and marginal expected operational profit-functions under independent planning for Setting 1.

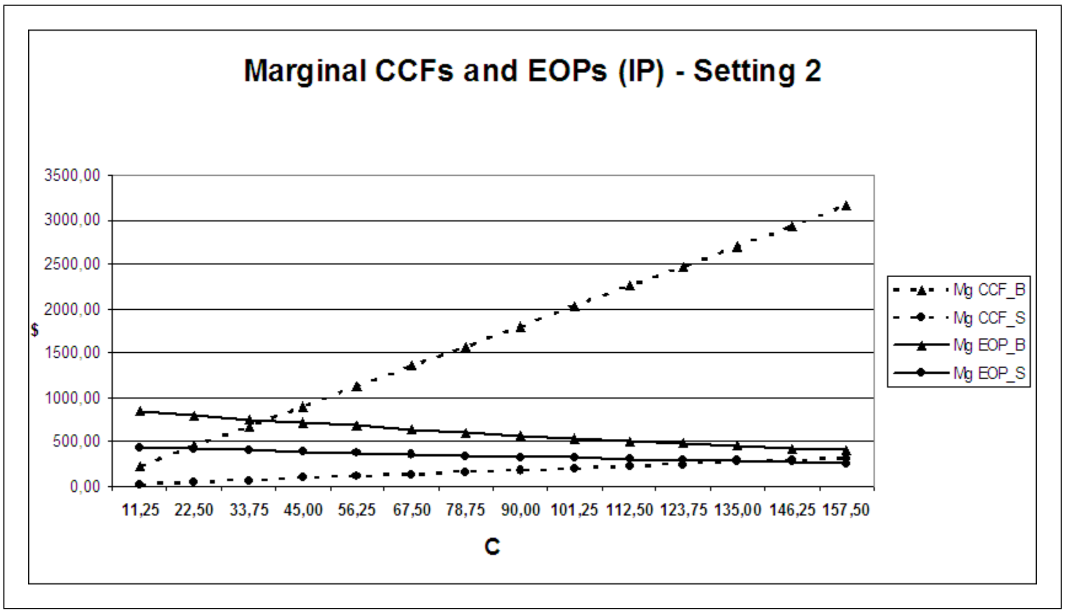

Figure 5.4: Marginal capacity cost-functions and marginal expected operational profit-functions under independent planning for Setting 2.

Setting 1. Grafically, those capacity-values correspond to the intersection between the marginal expected operational profit-function and marginal capacity cost-function, respectively, for Buyer and Supplier, which is appreciated in Figure 5.3. Analogously, the results obtained for Setting 2, that is, $C_B = 37.34$ and $C_S = 140.75$ are observated in Figure 5.4.

5.1.2 The companies coordinated by central planning

If companies act as producing units coordinated by a single central planning entity, the entity's production-capacity cost-function under central planning is given by $\Psi_B(C_B) + \Psi_S(C_S) = a_B \cdot C_B^2 + a_S \cdot C_S^2$ and, then, the marginal capacity cost-function is given by $\frac{d}{dC}\Big(\Psi_B(C_B) + \Psi_S(C_S)\Big) = 2a_B \cdot C_B + 2a_S \cdot C_S$.

The simplified expression of the expected operational profit for this benchmark situation is given by Expression (5-5). The companies' optimal capacity decisions obtained by central planning, according to the results presented in Table 5.2, are $C_B^* = 44.72$ and $C_S^* = 111.15$ for Setting 1, while $C_B^* = 42.10$ and $C_S^* = 153.28$ for Setting 2.

$$E_{X,Z}\Big[\Pi_{CP}(C_B, C_S, x, z)\Big] = \pi_B \cdot \beta_X \left(1 - e^{-\frac{C_B}{\beta_X}}\right) + \pi_S\Big[(\beta_X + \beta_Z) -$$
$$- \begin{cases} \frac{\beta_X^2}{\beta_X - \beta_Z} \cdot e^{-\frac{C_S}{\beta_X}} + \frac{\beta_Z^2}{\beta_X - \beta_Z} \cdot e^{-\frac{C_S}{\beta_Z}}\Big] & , \; C_S \leq C_B \\ \\ (-C_S + \beta_Z + C_B + \beta_X)\, e^{-\frac{C_B}{\beta_X}} - \frac{\beta_Z^2}{\beta_X - \beta_Z} \cdot e^{-\frac{C_S}{\beta_Z}} \left(e^{\left(\frac{1}{\beta_Z} - \frac{1}{\beta_X}\right)C_B} - 1\right)\Big] - \\ \quad -e^{-\frac{C_B}{\beta_X}} \left((C_S - C_B) - \beta_Z \left(1 - e^{-\frac{C_S - C_B}{\beta_Z}}\right)\right) & , \; C_S > C_B \end{cases} \tag{5-5}$$

5.2 The contract situation – Given a contract

To consider the contracts $\zeta_{S1} = (47.02, 0.10, 131.38)$ and $\zeta_{S2} = (43.88, 0.07, 71.47)$, respectively, for Setting 1 and Setting 2. Again, since the company k's production-capacity cost-functions is given by $\Psi_k(C) = a_k \cdot C^2$, its marginal production-capacity cost-function is $\frac{d}{dC}\Big(\Psi_k(C)\Big) = 2a_k \cdot C$, $k \in \{B, S\}$. The simplified expressions for the Buyer's and Supplier's expected operational profit-functions under the contract ζ are given, respectively, by

Expression (5-6) and Expression (5-7), which are sketched together with their capacity cost-functions in Figure 5.5 and Figure 5.6, repectively, for Setting 1 and Setting 2.

$$E_X\left[\Pi_{B|\varsigma}(C,x)\right] = \begin{cases} \left(\pi_B + d\cdot p_m + t\right)\cdot \beta_X\left(1-e^{-\frac{C}{\beta_X}}\right) - t\cdot R & , \ C \le R \\ \pi_B \cdot \beta_X\left(1-e^{-\frac{C}{\beta_X}}\right) + \left(d\cdot p_m + t\right)\cdot \beta_X \cdot & \\ \quad \cdot \left(1-e^{-\frac{R}{\beta_X}}\right) - t\cdot R & , \ C > R \end{cases} \tag{5-6}$$

$$\begin{aligned} E_{Y,Z}\left[\Pi_{S|\varsigma}(C,y,z)\right] &= \pi_S\Big[(\beta_X+\beta_Z) - (-C+\beta_Z+K+\beta_X)\, e^{-\frac{K}{\beta_X}} - \frac{\beta_Z^2}{\beta_X-\beta_Z}\cdot \\ &\quad \cdot e^{-\frac{C}{\beta_Z}}\left(e^{\left(\frac{1}{\beta_Z}-\frac{1}{\beta_X}\right)K} - 1\right)\Big] - \left(C - K - \beta_Z\left(1-e^{-\frac{C-K}{\beta_Z}}\right)\right) e^{-\frac{K}{\beta_X}} + \\ &\quad + \begin{cases} -\left(d\cdot p_m + t\right)\cdot \beta_X\left(1-e^{-\frac{C}{\beta_X}}\right) + t\cdot R & , \ K = y_{max} \ , \ y_{max} \le R \\ -\left(d\cdot p_m + t\right)\cdot \beta_X\left(1-e^{-\frac{R}{\beta_X}}\right) + t\cdot R & , \ K = R \ \ , \ y_{max} > R \end{cases} \end{aligned} \tag{5-7}$$

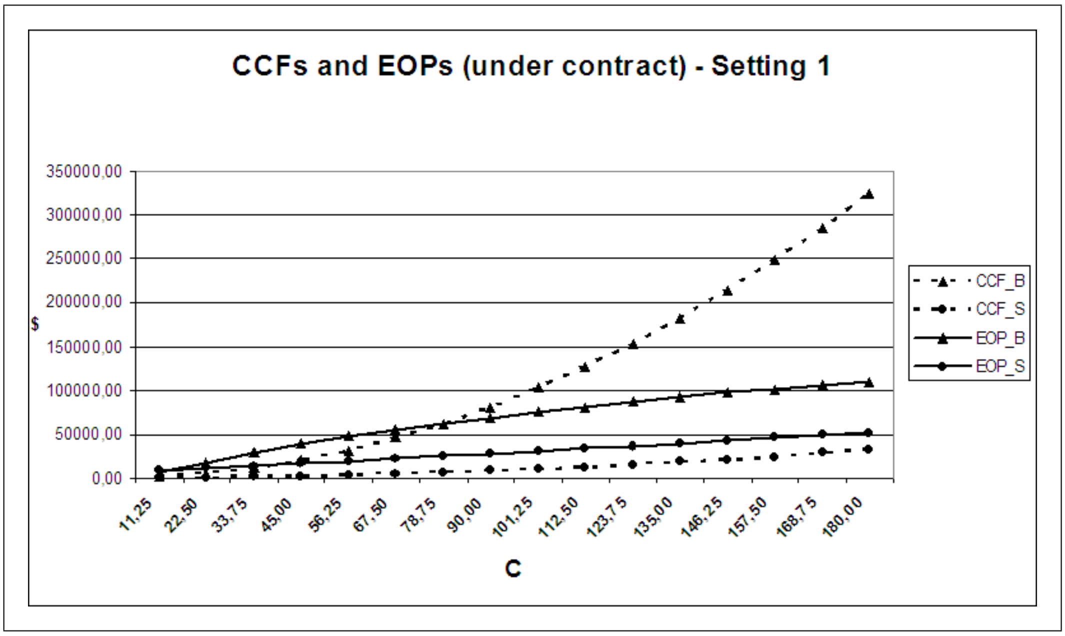

Figure 5.5: Capacity cost-functions and expected operational profit-functions under the contract ζ_{S1}.

Buyer's and Supplier's marginal expected operational profit-functions, in terms of the capacity C, are given, respectively, by Expression (5-8) and

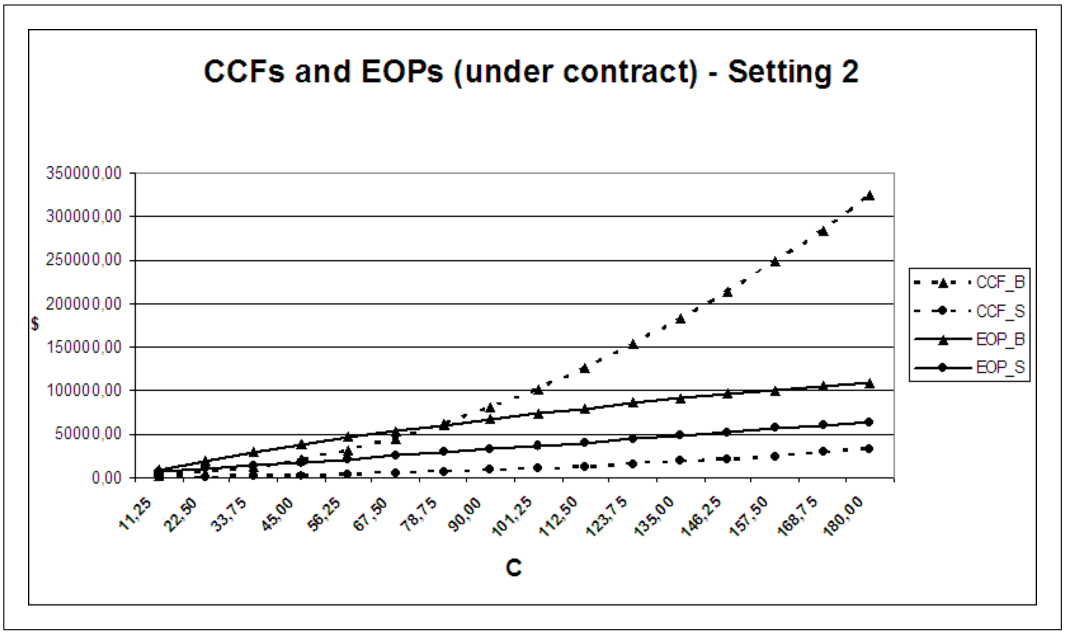

Figure 5.6: Capacity cost-functions and expected operational profit-functions under the contract ζ_{S2}.

Expression (5-9), where $K = y_{max}$ if $y_{max} \leq R$, or $K = R$ if $y_{max} > R$, in Expression (5-9). Those profit-functions are scketch together with the capacity cost-functions in Figure 5.7 and Figure 5.8, respectively, for Setting 1 and Setting 2.

$$\frac{d}{dC}\left(E_X\left[\Pi_{B|\zeta}(C,x)\right]\right) = \begin{cases} (\pi_B + d \cdot p_m + t)\ e^{-\frac{C}{\beta_X}} & , C \leq R \\ \pi_B \cdot e^{-\frac{C}{\beta_X}} & , C > R \end{cases} \tag{5-8}$$

$$\frac{d}{dC}\left(E_{Y,Z}\left[\Pi_{S|\zeta}(C,y,z)\right]\right) = \pi_S \left[e^{-\frac{K}{\beta_X}} e^{-\frac{C-K}{\beta_Z}} + \frac{\beta_Z}{\beta_X - \beta_Z} e^{-\frac{C}{\beta_Z}} \left(e^{\left(\frac{1}{\beta_Z} - \frac{1}{\beta_X}\right)K} - 1\right)\right] \tag{5-9}$$

Grafically, it is observed that $C_{B|\zeta_{S1}} \in [45.00, 56.25]$ and $C_{S|\zeta_{S1}} \in [101.25, 123.75]$ for Setting 1, while $C_{B|\zeta_{S2}} \in [33.75, 56.25]$ and $C_{S|\zeta_{S2}} \in [146.25, 168.75]$ for Setting 2. Actually, the contracts ζ_{S1} and ζ_{S2} are viable and optimal contracts, respectively, for Setting 1 and Setting 2. In fact, they were obtained from the inicial contract $\zeta_\circ = (22.50, 0.10, 100.00)$ and the companies' optimal capacity decisions obtained by computational programs made in AIMMS are $C_{B|\zeta_{S1}} = 45.24$, $C_{S|\zeta_{S1}} = 111.38$, $C_{B|\zeta_{S2}} = 42.22$ and $C_{S|\zeta_{S2}} = 153.31$.

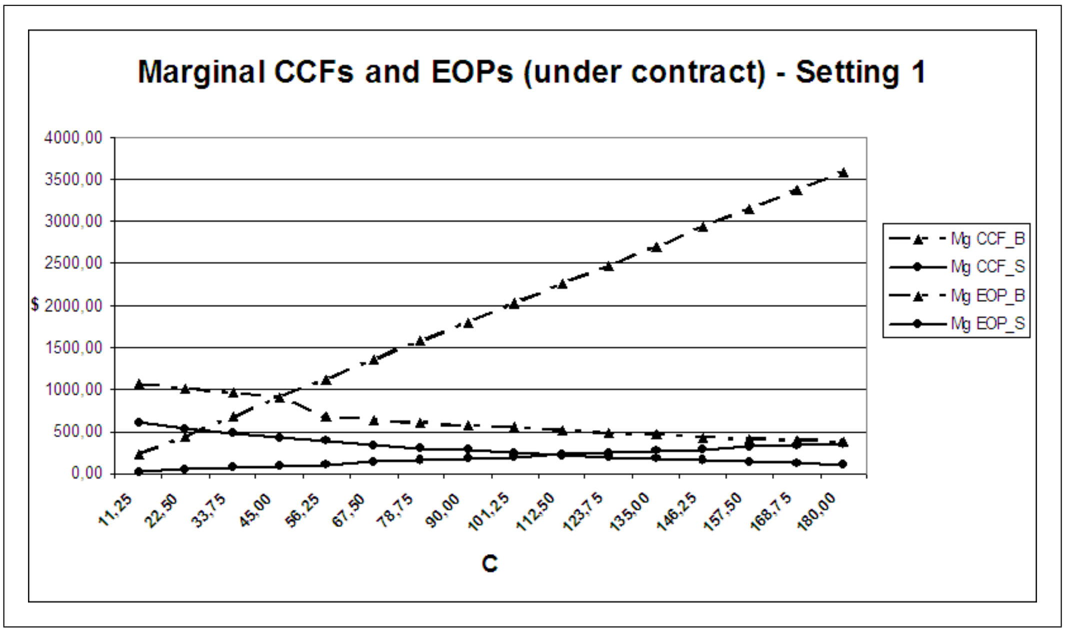

Figure 5.7: Marginal capacity cost-functions and marginal expected profits under the contract ζ_{S1}.

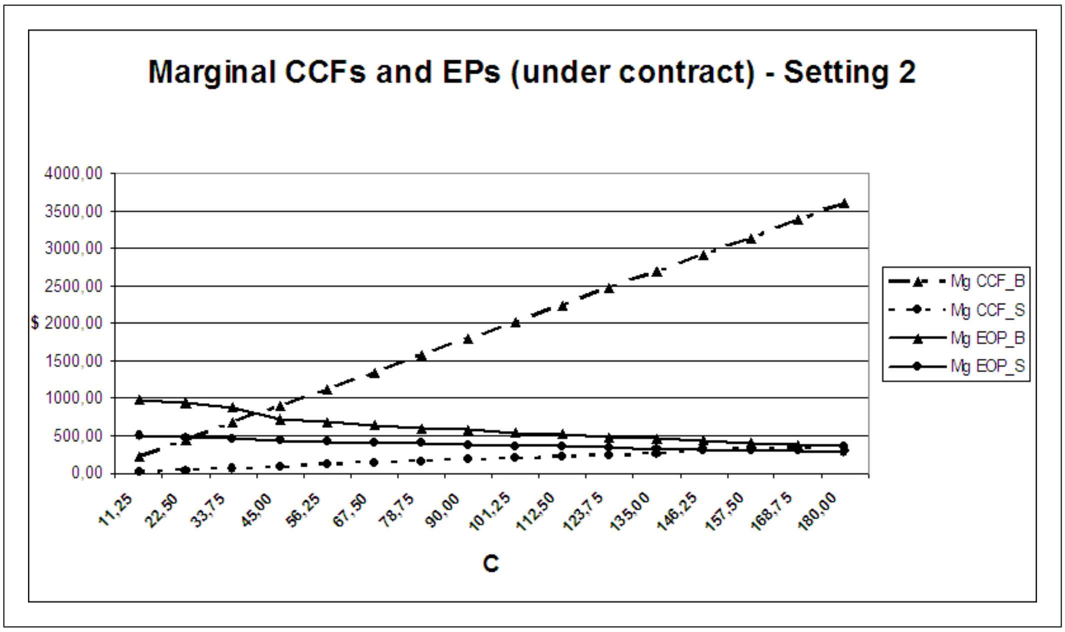

Figure 5.8: Marginal capacity cost-functions and marginal expected profits under the contract ζ_{S2}.

5.3
The contract problem – Given a set of initial contracts

The analysis of contract, according to the approach defined in Chapter 3, is carried out over a set of viable and optimal contracts. For that, the contract problem is solved for each contract in a discrete set of initial contracts with parameters in a regularly spaced grid. So, viable and optimal contracts are obtained from the initial contracts by applying the optimization procedure. The computational programs made in AIMMS are presented in Appendix E, as well as the tables of results that describe the viable and optimal contracts obtained by it. In those tables, $\zeta_\circ = (R_\circ, d_\circ, t_\circ)$ represents the initial contract, while $\zeta^* = (R^*, d^*, t^*)$ the viable and optimal contract obtained from that contract.

The set of initial contracts $\Omega_\circ$ will be defined by a grid considering a regular partition for each one of the three dimensions of the semi-bounded continuous parameter space Ω, which is defined by Equation (4-19) (see Section 4.3). For that, the material market price (p_m) is considered as an upper bound for the penalty parameter (t). In these illustrations, that partition will be considered with ten elements in each axis. So, the analysis of the contract will be carried out taking into account the contract instances that are viable and optimal among the ones obtained from the 1000 initial contracts. Table 5.3 summarizes the lower and upper bounds for the sets $\Omega_\circ$ and Ω_{VOC} for both settings, where Ω_{VOC} is the set comprised by viable and optimal contracts. Note that the same set $\Omega_\circ$ is used for both settings. The optimization procedure, first, finds optimal contracts and, then, the ones that, also, are viable are chosen for forming the set Ω_{VOC}.

		Setting 1		Setting 2	
Set		Lower bound	Upper bound	Lower bound	Upper bound
$\Omega_\circ$	R	0	225	0	225
	d	0	1	0	1
	t	0	1000	0	1000
Ω_{VOC}	R	41.10	110.48	41.03	153.60
	d	0.06	0.31	0.04	0.20
	t	0.07	1000.00	1.58	1000.00

Table 5.3: Bounds for the sets of contracts for Setting 1 and Setting 2.

5.4

Discussion – Analysis of the contract for the settings

The evaluation for the contracts in the set Ω_{VOC} for Setting 1, which is obtained from the set of initial contracts $\Omega_\circ$ considered, the Buyer's surplus ($\delta_{B|\zeta^*}$) is in the interval $[0.01\%, 54.29\%]$, while Supplier's surplus ($\delta_{S|\zeta^*}$) is in the interval $[0.63\%, 49.43\%]$, where ζ^* represents an any viable and optimal contract. That means, both companies have the possibility of increasing considerably their expected profits under the contract proposed.

Similar results are obtained for Setting 2, but with a lower maximum improvement for the companies. In fact, in this setting, Supplier faces a market demand with a larger mean than in Setting 1. So, it is hoped that the contract has a less significative impact in than in Setting 1. Since Supplier expects a larger demand, the Buyer's order is less relevant to her. Even so, in this instance, the contract proposed continues being interesting for Buyer. According to the results presented in Table 5.2, the Buyer's surplus varies in the interval $[0.04\%, 33.48\%]$, while the Supplier's one varies in the interval $[0.01\%, 18.18\%]$. However, the viable and optimal contracts lead to an improvement for the dyad of 11.75% approximately, which is not to be disregarded, while that efficieny level reached in the dyad is 99.94% approximately.

For Setting 1, the improvement of the dyad's expected profit ($\eta_{D|\zeta^*}$) resulted to be the interval $[25.57\%, 26.04\%]$, while the range for the dyad's efficiency level ($\xi_{D|\zeta^*}$), which is reached by viable and optimal contracts obtained from $\Omega_\circ$, is in the interval $[99.62\%, 100.0\%]$. While, for Setting 2, those ranges are given, respectively by the intervals $[11.68\%, 11.82\%]$ and $[99.88\%, 100.00\%]$.

The ranges for the companies' surplus, in both settings, show that there are contracts that distribute differently the dyad's gain obtained by them. Therefore, if the improvement aspired by the companies is in those ranges, then it is possible to establish a coordinating-contract by some negotiation process, which depends on the companies' bargain power.

The numeric error in reaching the full coordination can be due to approximation in the evaluation for the correspondent expected profits in the iterations carried out for finding the optimal contract from a given initial contract. And, also, they can be derived from the modeling assumption that considers to Buyer not buying from Supplier his additional material requirement (see Section 3.5.1).