5 Applications

After presenting and discussing several theoretical and methodological issues, I entirely dedicate some practical examples to this chapter. In those, two methods from chapter 4 - namely: the reduced restricted Kalman filtering and the restricted Kalman prediction equations - are considered, implemented and evaluated. The remaining of this chapter is structured as follows. Section 5.1[1] presents an application in time-varying factor modeling for dynamic style analysis, in which an accounting restriction on the coefficients is tackled by the reduced restricted Kalman filtering. In section 5.2[2], time-varying econometric models are considered for the estimation and interpretation of the dynamic exchange rate pass-through over Brazilian price indexes; again, the reduced restricted Kalman filtering is key for testing some economic hypothesis imposed under two specific restrictions. And, in section 5.3, the material concerning restricted predictions from section 4.3 is conveniently implemented for obtaining predictions of quarterly GNP that must be somehow consistent with the annual GNP (that is: for each year, the sum of quarterly GNP data is *restricted* to equal the annual GNP).

The models to be discussed in the sequel have been implemented using the Ox 3.0 language (cf. Doornick, 2001) with occasional use of the Ssfpack 3.0 library for linear state space modeling (Koopman *et al.*, 2002). My computer was an Athlon XP 2200 Mhz, with 378 Mb-RAM. The computational efficiency of the estimations are separately analyzed and discussed in the appropriate sections. All the estimations were carried under the *exact maximum likelihood estimation* and the *exact initial Kalman filter* (cf. Durbin and Koopman, 2001, chapters 5 and 7).

[1] I am in great debt with Luciano Vereda, who was the sole responsible for the economic interpretations of the proposed models and of the corresponding empirical results.

[2] Most of the conception, theory, and results and their interpretations have been carried out by Rafael Martins de Souza and Luiz Felipe Pires Maciel, to whom I am grateful.

5.1
Case I: Semi-strong dynamic style analysis

5.1.1
Motivation

Depending on the type of an investment fund under investigation, detailed information on the actual portfolio composition is not usually available. *Return-based style analysis*, or simply *style analysis*, is a statistical method for the estimation/approximation of the unknown composition of an investment fund portfolio. Standard practice of style analysis only uses the so-called *external information*, which is represented by the fund returns and some market indexes returns, and is implemented by the *asset class factor model* (cf. Sharpe, 1988 and 1992). Later, this has been modified by the add of an intercept term (cf. de Roon, Nijman and ter Horst, 2004), as follows:

$$R_t^P = \alpha + \beta_1 R_{t1} + \beta_2 R_{t2} + \cdots + \beta_m R_{tm} + \varepsilon_t. \tag{5-1}$$

Assumptions: R_t^P is the portfolio return; $R_t = (R_{t1}, R_{t2}, \ldots, R_{tm})'$ represents some asset class indexes returns, which should satisfy the assumptions of *exhaustiveness*, of *mutual exclusiveness* and of *different behavior* (cf. Sharpe, 1988 and 1992); $\beta_1, \beta_2, \ldots, \beta_m$ are the unknown allocations/exposures which are sometimes supposed to satisfy an accounting constraint, known as the *portfolio restriction*[3]: $\sum_{i=1}^{m} \beta_i = 1$; α is the *Jensen's measure* or *Jensen's alpha* (cf. de Roon, Nijman and ter Horst, 2004), and represents the idiosyncratic fund return, i.e., it measures how much the fund aggregates - or looses - by means of its selectivity strategies[4]; and ε_t is a typical random error process with finite second moments.

Even though being a much used tool in investment analysis, model (5-1) has a drawback: it ignores the fact that asset class exposures and selectivity *do change* over time, reflecting the very plausible and possible reallocations of the assets by the portfolio manager - an idea that was also evoked in Pizzinga and Fernandes (2006) and in Swinkels and Van der Sluis (2006). Later, in Pizzinga *et al.* (2008) a class of *semi-strong*[5] style analysis models was proposed, the

[3] There is also a *short-sale restriction*, which is sometimes considered and is implemented by forcing non-negativeness of $\beta_1, \beta_2, \ldots, \beta_m$. But, as this restriction is not always meaningful (*e.g.*, most of hedge funds take positions in derivative markets), this is not adopted here.

[4] In fact, the *actual* Jensen's measure emerges in the context of equilibrium models, such as the CAPM or the APT model - cf. Elton *et al.* (2006) - or the multi-factor model of Carhart (1997). However, I shall retain the intercept term of (5-1) as the selectivity measure, since this and the former are used to achieve the same ends of the measure proposed in Jensen (1968).

[5] This means that only the portfolio restriction is imposed; cf. the style analysis taxonomy proposed by de Roon, Nijman and ter Horst (2004).

exposures and Jensen's measure of which were both made stochastically time-varying as a (vector) random walk. This represented a direct generalization of the static model (5-1), and its estimation was carried out by an appropriate restricted linear state space model. Empirical illustrations were presented using return series of Brazilian US Dollar/Real exchange rate funds. Among several points, there was clearly a visual evidence that the time-varying exposures onto US Dollar-Real exchange rate markets behaved under different autoregressive regimes, one of those directly associated to the 2002 Brazilian presidential election, a period of some political turbulence and high volatility.

This sections's exercise aims at, firstly, uncovering evidence on switching regimes for the time-varying exposures of Brazilian US Dollar-Real exchange rate funds under an econometrically more compelling way and, secondly, interpreting the estimated exposures from the "more appropriate" model. In it, I empirically evaluate several sounding dynamics: (1)random walk; (2)simply autoregressive; (3)autoregressive with abrupt switching regimes; and (4)nonlinear under a general smoothing transition function. The elected regime switching variable for the 3rd and the 4th models is the AR(1)-GARCH(1,1) volatility of US Dollar/Real exchange rate. As it will be seen, the adoption of time-varying portfolio-restricted exposures following such processes makes it necessary to use the reduced restricted Kalman filtering of section 4.2.

5.1.2 Competing models

I now present the analytical expressions of several *time-varying asset class factor models* for *semi-strong dynamic style analysis*. In what follows, the reducing method from the subsection 4.2.2 has been evoked in order to make the portfolio restriction attainable.

Let me first obtain the expression corresponding to the portfolio restriction on the state vector, which I shall denote in this section by γ_t and whose coordinates represent the exposures and the Jensen's measure. To do this, we use steps 1 and 2 of the algorithm of subsection 4.2.2:

$$
\begin{aligned}
1 &= [1\,1\ldots 1\;0]\,(\beta_{t1},\beta_{t2},\ldots,\beta_{tm},\alpha_t)' \\
\Rightarrow \quad 1 &= \beta_{t1} + [1\ldots 1\;0]\,(\beta_{t2},\ldots,\beta_{tm},\alpha_t)' \\
\Rightarrow \beta_{t1} &= 1 - [1\ldots 1\;0]\,(\beta_{t2},\ldots,\beta_{tm},\alpha_t)' \\
\Rightarrow \gamma_{t,1} &= 1 - [1\ldots 1\;0]\,\gamma_{t,2}.
\end{aligned}
$$

I now move on to the measurement equation of the reduced model by making use of step 3 of the algorithm in conjunction with the last equality obtained above:

$$\begin{aligned} R_t^c &= R_{t1}\beta_{t,1} + [R_{t2}\dots R_{tm}\ 1]\,(\beta_{t2},\dots,\beta_{tm},\alpha_t)' + \varepsilon_t \\ &= R_{t1} - R_{t1}\,[1\dots 1\ 0]\,(\beta_{t2},\dots,\beta_{tm},\alpha_t)' + [R_{t2}\dots R_{tm}\ 1]\,(\beta_{t2},\dots,\beta_{tm},\alpha_t)' + \varepsilon_t \\ \Rightarrow R_t^c - R_{t1} &= [R_{t2}-R_{t1}\dots R_{tm}-R_{t1}\ 1]\,(\beta_{t2},\dots,\beta_{tm},\alpha_t)' + \varepsilon_t \\ \Rightarrow R_t^c - R_{t1} &= [R_{t2}-R_{t1}\dots R_{tm}-R_{t1}\ 1]\,\gamma_{t,2} + \varepsilon_t. \end{aligned}$$

Finally, combining a rather encompassing state equation with the expression above, I arrive at the following general structure:

$$\begin{gathered} R_t^P - R_{t,1} = [R_{t2}-R_{t1}\dots R_{tm}-R_{t1}\ \ 1]\,\gamma_{t,2} + \varepsilon_t,\ \varepsilon_t \sim NID(0,\sigma^2 X_t) \\ \gamma_{t+1,2} = diag\left(T_t^\beta, 1\right)\gamma_{t,2} + \eta_t \ \ ,\ \ \eta_t \sim NID(0,Q) \\ \gamma_{t,1} = 1 - [1\dots 1\ 0]\,\gamma_{t,2}. \end{gathered} \tag{5-2}$$

I enumerate some features of model (5-2). Firstly, it should be reinforced that the last coordinate of γ_t is the time-varying Jensen's measure α_t and the remaining coordinates are the time-varying exposures $\beta_{t1}, \beta_{t2}, \dots, \beta_{tm}$. Secondly, as can be directly seen from the second line of (5-2), the Jensen's measure follows a random walk And thirdly, X_t in the measurement error's variance is some nonnegative variable that must respond for occasional heteroscedastic behavior, and Q can be set full[6].

The decision towards the random walk for the evolution of the Jensen's measure deserves some justification. Although such choice seems too simple and perhaps "unrealistic" in first glance, three reasons support such recognition. The first is that of *parsimony* and *simplicity*, as there is no additional clue to guide one in choosing a more complex transition equation. The second is the allowance for the possibility of fundamental selectivity changes along time due to non-stationarity. The third comes from the next result:

Proposition 3 *For the model in* (5-2)*, set* $\mathcal{F}_\infty \equiv \sigma\left\{R_s^P : s \geq 1\right\}$ *and* $\hat{a}_{t|\infty} \equiv E\left(\alpha_t|\mathcal{F}_\infty\right)$*. Then,* $\limsup_{t\longrightarrow+\infty} \hat{a}_{t|\infty} = +\infty$ *and* $\liminf_{t\longrightarrow+\infty} \hat{a}_{t|\infty} = -\infty$ $\mathcal{P} - a.s.$

Proof: Without loss of generality, suppose the underlying probability space is *complete*. Take an arbitrary $i \in \{1,\dots,m\}$. According to Chung (2001), ch.8, I have $\limsup_{t\longrightarrow+\infty} \alpha_t = +\infty$ $\mathcal{P} - a.s.$ It means that there is a subsequence

[6] The impacts on the exposures, represented by the components of η_t, "communicate" among themselves. Note that it would be unreasonable to assume that investment decisions (and hence the exposures) are related only by the portfolio restriction (which is an accounting constraint), since they reflect the same underlying shocks. In other words, shocks that lead investors to augment their exposures onto some asset classes can also make them decide to reduce their positions on others.

α_{t_j} such that $\lim_{j\longrightarrow+\infty}\alpha_{t_j} = +\infty \ \mathcal{P} - a.s.$ Taking such subsequence as nondecreasing, I obtain

$$\limsup_{t\longrightarrow+\infty} \hat{a}_{t|\infty} \geq \lim_{j\longrightarrow+\infty}\hat{a}_{t_j|\infty} = +\infty \quad \mathcal{P} - a.s,$$

where the equality follows from the Monotone Convergence Theorem for conditional expectations (cf. Chung, 2001, ch.9). The liminf case is dealt with under the same fashion. □

The interpretation: although non-stationary, the choice of a random walk has the advantage that, for"large" series, the smoothed Jensen's measure must intercept the time x-line infinitely often $\mathcal{P} - a.s.$ (in other simple words: with probability 1, the estimated Jensen's measures *never explode*).

The remaining part of model (5-2)'s specification lies on the transition sub-matrix $T_t^{\beta} \equiv diag\,(\phi_{t2}, \ldots, \phi_{tm})$, which drives the evolution of the unrestricted block of time-varying exposures in $\gamma_{t,2}$. Let me first enumerate the possibilities I am going to investigate empirically and, in the sequel, give appropriate rationalities to each of them:

1. *Random walk* (*RW*): $T_t^{\beta} \equiv I_{(m-1)\times(m-1)}$.

2. *Purely autoregressive* (*AR*): $T_t^{\beta} \equiv diag\,(\phi_2, \ldots, \phi_m)$, where $|\phi_i| < 1$ for all i.

3. *Autoregressive with abrupt switching regimes*: some diagonal entries of T_t^{β} take the form $\phi_{i1} + \phi_{i2}d_{ti}$, where $d_{ti} = 1$ if some exogenous variable z_t assumes certain values and $d_{ti} = 0$ otherwise.

4. *Nonlinear under a general smoothing transition function*: some diagonal entries of T_t^{β} take the form $\phi_{i,1}+\phi_{i,2}z_t+\phi_{i,3}z_t^2$, where z_t is some exogenous variable.

The first model is clearly the most parsimonious and had already been used by Pizzinga *et al.* (2008) within this same style analysis framework[7]. In the occasion, two Brazilian US Dollar/Real exchange rate funds had their time-varying Jensen's measures and exposures analyzed. As already told, exposures estimated under this framework seem to follow two different patterns, one during the months near the 2002 Brazilian presidential election

[7]According to Swinkels and van der Sluis (2006), this specification should be used if one believes that exposures can increase or decrease over time when responding to shocks (that turn out to exert a permanent effect). In contrast, if one believes that exposures can deviate for some time from normal (or "steady-state") levels but will forcefully come back to them (which means that shocks exert a transitory effect), then a variant like the second model should be used.

(when exposures onto US Dollar/Real markets appeared to be more erratic and less persistent), and the other during the remaining months (in which exposures were much more stable). This "stylized fact" was interpreted as suggesting that linear models should be abandoned in favor of more sophisticated ones (specially regime switching models), in which the consequences from changes in the decision making process (which possibly varies with the state of the economy, perhaps with market volatility) could be better captured.

The second model captures situations in which managers try to target "steady-state" exposures. When compared with the first model, the number of parameters grows by $m-1$ autoregressive coefficients. Since the eigenvalues of T_t^{β} have absolute values strictly smaller than 1 (one), non-stationarity and/or "explosive" behaviors for the exposures are ruled out, something that brings some inferential attractiveness. Besides, this second model can be understood as a bridge to the nonlinear third and fourth versions.

The third and fourth models undoubtedly add complexity to the process of parameter estimation but are justified by their ability of capturing the state-dependent behavior of managers and investors (which generates the aforementioned possibility of multiple regimes in exposures' dynamics). One might recall that the nonlinear processes[8] used here are respectively the threshold autoregressive (TAR) model and a general smoothing transition autoregressive (STAR) model, in which the second-order polynomial on z_t is an attempt to approximate a more general "smooth" transition function. For a comprehensive treatment of these types of switching-regime proposals outside the state space framework, see Enders (2004). Once these dynamics are postulated to the state equation, parameter estimation can be accomplished under the usual paradigm of maximizing the prediction error decomposition form of the likelihood (see, for example, Harvey, 1989, ch.3; and Durbin and Koopman, 2001, ch.7).

5.1.3 Model Selection

As there are four alternatives to describe the time-varying exposures, I must discuss how to decide in practice which model seems to be the most appropriate. I actually adopt the following selection mechanism:

– Likelihood-ratio (LR) tests to validate or to refuse the nonlinear proposals (3) and (4).

[8]Even though being *nonlinear processes* (*i.e.*, there is no corresponding Gaussian nor *i.i.d* Wold decomposition - cf. Brockwell and Davis, 1991 and 2003), these choices for the state equation still provide us with a Gaussian "linear" space model.

– Information criteria, such as AIC and BIC.

– Predictive power by comparing Pseudo R^2 and MSE measures.

– Diagnostic tests over the standardized innovations.

The listed strategies had been fully discussed in Harvey (1989), ch. 5. Here, the null for the LR test shall be H_0: *"The parameters associated to the switching regimes are all zero"*. Consequently, our test aims at comparing the "reduced" model (2) to the "complete" model (3) or (4). There are strong theoretical evidences that, asymptotically, $LR = 2\left[logL_{Max,Comp} - logL_{Max,Red}\right] \sim \chi^2_k$, where k is the number of parameters set to zero under the null, since at least the reduced model maintains the standards for good properties of maximum likelihood estimation[9] (cf. Pagan, 1980).

5.1.4 Reducing versus Augmenting

This subsection is dedicated to some discussion concerning the type of restricted Kalman filtering that would be the most appropriate to obtain portfolio-restricted estimated exposures. As discussed in previous chapters, two major possibilities could *a priori* be evoked for such task, and so far we have concentrated only in the details of the *reduced* restricted Kalman filtering. But, as we will show, the use of the *augmented* restricted Kalman filtering considerably limits the choices of models for the exposures' evolution. Observe that, even though Propositions 1 and 2 from section 4.2 could be evoked here, I shall make use of the rather special structure of the portfolio restriction in order to uncover more drawbacks of the augmented restricted Kalman filtering.

The time-varying asset class factor model corresponding to the augmenting approach shall have its measurement equation displayed as

$$\begin{pmatrix} R_t^P \\ 1 \end{pmatrix} = \begin{pmatrix} R_t' & 1 \\ 1\cdots 1 & 0 \end{pmatrix} \gamma_t + \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}, \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix} \sim NID\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 X_t & 0 \\ 0 & 0 \end{pmatrix} \right). \tag{5-3}$$

The state equation retains its general form as presented in the second line of (5-2), but with the dimension increased by an unit. In addition, I use the following presupposition for the remaining of this section:

Assumption 8 *The state equation associated to model (5-3) is such that:*

1. $\forall t \geq 1$ *and* $\forall i = 1, \ldots, m : \sum_{j \neq i} (\phi_{ti} - \phi_{tj})\, E(\beta_{tj}) = 0$;

[9]Analytical and/or Monte Carlo investigations for the LR test about its asymptotic properties would deserve some special attention, but I rather leave this important issue for future research.

2. $\forall i = 1, \ldots, m+1 : \sigma_i^2 \equiv Var(\eta_{ti}) = 0 \Rightarrow Var(\gamma_{1i}) = 0$.

Note that, even though the first statement of Assumption 8 looks artificial in first glance, this includes standard zero-mean setups for the initial state vector as particular cases, whatever diffuse or non-diffuse (in other words: it is more stringent to suppose the initial state vector γ_1 is at least integrable with unconditional expectation given by $a_1 \equiv E\left[(\beta_{11}, \ldots, \beta_{1m}, \alpha_1)'\right] = (0_{1\times m}, a_{1,m+1})'$).

The next two propositions, which are related to the augmented model (5-3) reveal that the augmented restricted Kalman filtering looses much flexibility in terms of the possible choices for the state equation.

Proposition 4 *For all t, each diagonal entry of the matrix T_t^β must be equal to one. That is, $T_t^\beta \equiv T^\beta = I_{m\times m}$.*

Proof: Take an arbitrary $t \geq 1$. From the second line of the measurement equation (5-3), I obtain the portfolio restriction

$$\beta_{t1} = 1 - \beta_{t2} - \cdots - \beta_{tm}. \tag{5-4}$$

From the state equation given in (5-2), I have

$$\beta_{t+1,1} = \phi_{t1}\beta_{t1} + \eta_{t1}. \tag{5-5}$$

Now, put (5-4) into (5-5) and make use of the state equation again to get

$$1 - \phi_{t2}\beta_{t2} - \cdots - \phi_{tm}\beta_{tm} - \sum_{j=2}^{m} \eta_{tj} = \phi_{t1} - \phi_{t1}\beta_{t2} - \cdots - \phi_{t1}\beta_{tm} + \eta_{t1}, \tag{5-6}$$

which is equivalent to

$$(1 - \phi_{t1}) + (\phi_{t1} - \phi_{t2})\,\beta_{t2} + \cdots + (\phi_{t1} - \phi_{tm})\,\beta_{tm} = \sum_{j=1}^{m} \eta_{tj}. \tag{5-7}$$

Taking unconditional expectations on both sides of (5-7) and evoking the first item of Assumption 8, I finally get $1 - \phi_{t1} = 0$. The other exposures are dealt with analogously. □

The message is clear. If one chooses the augmented Kalman filtering for estimating the time-varying asset class factor model under the portfolio restriction, there is no possibility left but a random walk evolution for *every coordinate* the state vector. Therefore, proposals 2, 3 and 4 listed in subsection 5.1.2, or any other non-random walk specification, whether time-varying or fixed, would not be even checkable for a given data set. Note that Proposition 1 from section 4.2 would at the most reveal that only one coordinate of the state vector is a random walk.

The second proposition, stated below, rules out any possibility of time-varying exposures under contemporaneously independent errors $\eta_{t,1}, \ldots, \eta_{t,m}$:

Proposition 5 *Let Q^{β} be the covariance matrix associated to the exposures' random error. If $Q^{\beta} \equiv diag\left(\sigma_1^2, \ldots, \sigma_m^2\right)$, then $Q^{\beta} = O$.*

Proof: From the portfolio restriction imposed in the second line of (5-2), β_t is in fact a singular random vector. This is, by Proposition 4, equivalent to $max\left\{rank\left(Q^{\beta}\right), rank\left(P_1^{\beta}\right)\right\} < m$, where P_1^{β} is the block of P_1 associated to the initial exposures $(\beta_{11}, \ldots, \beta_{1m})'$. Then, as Q^{β} is diagonal, it must follow that there exists $i \in \{1, \ldots, m\}$ such that $\sigma_i^2 = 0$, which in turn implies in $Var\left(\beta_{1i}\right) = 0$ (cf. the second item of Assumption 8). But, as $\beta_{ti} = 1 - \sum_{j \neq i} \beta_{tj}$, I must have

$$0 = Var\left(\beta_{ti}\right) = \sum_{j \neq i} Var\left(\beta_{tj}\right), \tag{5-8}$$

where both equalities follow from the state equation being a random walk (cf. Proposition 4) and from Q^{β} being diagonal. The conclusion from (5-8) is that $\sigma_j^2 = 0$ for all $j = 1, \ldots, m$. □

From this last result, one should learn that the adoption of the augmented restricted Kalman filtering also forces one to always consider exposures whose impacts, which are represented by the components of η_t, are correlated. Such limitation, like the former, certainly does not arise under the reduced restricted Kalman filtering.

5.1.5 Empirical results

The asset class indexes were the CDI (the average rate charged in overnight transactions between depository institutions), the US Dollar/Real exchange rate (in percentage points) and observed variations in two financial indicators, Quantum Cambial and Quantum Fixed Income[10]. The data comprise 209 observations on weekly returns from 2001 to 2004 and were obtained from *Quantum Axis* (www.quantumfundos.com.br). Two US Dollar/Real exchange rate funds inside the Brazilian industry were considered[11]: HSBC Cambial FIF and Itau Matrix US Hedge FIF.

[10]The Quantum Fixed Income indicator approximately tracks variations in the market price of a certain fund's share, whose objectives are such that its market value increases whenever the six-month swap rate decreases. The Quantum Cambial indicator, in turn, follows variations in the market price of another fund's share, whose value increases whenever the premium from a swap contract - the so called DI-Dollar - decreases. For additional discussion on these two indicators, see Varga (1999).

[11]Since the fund Itau Matrix US Hedge FIF has been bought by another fund - namely Itau B Cambial FI - at the end of 2004, I made the corresponding estimation with the data

The covariance matrix Q (cf. the state equation in (5-2)) was considered full in all the estimations[12]. The X_t heteroscedastic variable (cf. the measurement equation of (5-2)) was chosen to be the US Dollar/Real AR(1)-GARCH(1,1)[13] volatility in the analyzed period, and its standardized version was used as the z_t switching regime variable for the exposures onto the US Dollar/Real exchange rate and the Quantum Cambial. The dummy variable d_t from the TAR specification takes 1 whenever $z_t \geq 1.3$, and 0 otherwise. This calibration was chosen to capture the period of high volatility, which took place from the last week of September 2002 (which is located around the 90th observation) to the third week of February 2003 (which is located around the 110th observation)[14]. Figure 5.1, which helps recognizing these patterns, also illustrates what happened throughout the period. Note that the 2nd half of 2002 was marked by a confidence crisis that surged on the eve of the Brazilian presidential elections. This crisis found a very fertile ground to grow due to fears about the macroeconomic policies that could be followed by the candidate who was leading the polls, Luis Inacio Lula da Silva. When agents perceived that Lula administration would not change economic fundamentals like the floating exchange rate and inflation targeting regimes, expectations about future economic developments became favorable, financial market indicators turned positive and volatility dropped.

If one makes careful inspection on the information depicted in tables 5.1 and 5.2, several points emerge. Looking first at computational efficiency, it is clear that the computational times, even though larger for the nonlinear proposals, remain essentially negligible. This could be of great value, should one try to use/implement these dynamic style-analysis proposals in practice.

Stepping further, one should note that the predictive power from the competing proposals gives us no clue about which model is the most adequate - it seems that, for these particular estimations, all models can reproduce the data almost under similar capabilities (cf. Pseudo R^2 and MSE measures). Also, the use of AIC and BIC criteria is of no help in deciding which model should be considered.

The diagnostic tests in the last three lines of these tables[15], though, uncover important aspects. They actually tell that, in terms of model basic

until the first week of 2003 November (149 observations). Additional information can be obtained at the National Association of Investment Banks (ANBID) (www.anbid.com).

[12]But it must be remarked that, from what was discussed in subsection 5.1.3, there would be no problem in attempting diagonal specifications under the reduced restricted Kalman filtering, on which the estimations are based.

[13]The corresponding implementation has been made using EViews (www.eviews.com)

[14]I could have tried to estimate the value of the limiar, but it would demand more periods of high volatility in the data.

[15]The three tests were applied to the standardized innovations.

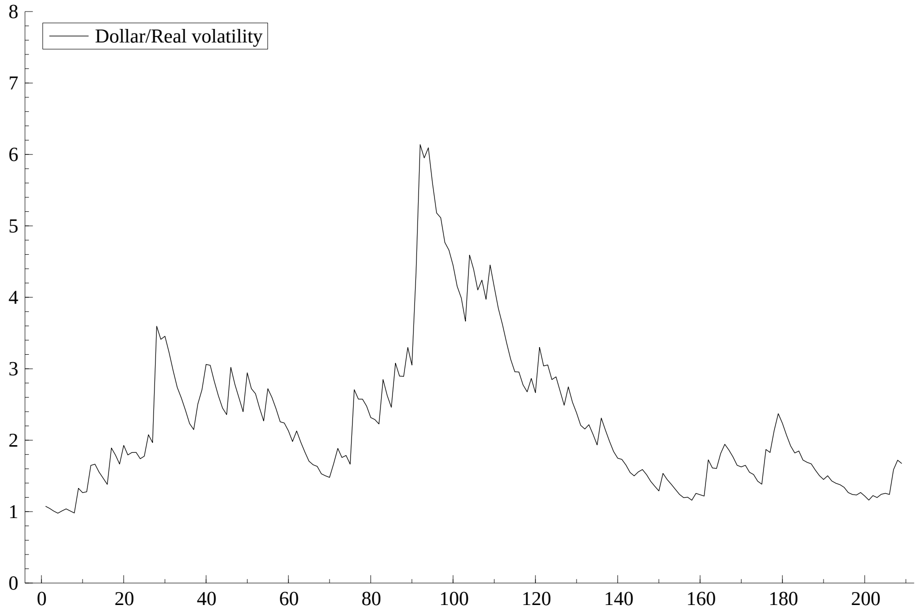

Figura 5.1: US Dollar/Real volatility obtained under the AR(1)-GARCH(1,1) model.

Tabela 5.1: Results from the estimations with HSBC FIF Cambial.

Attribute	RW	AR	TAR	$STAR$
Log-likelihood	-149.462	-148.124	-143.421	-143.657
Computational time	0.61	3.85	6.76	10.66
Pseudo R^2	0.905	0.907	0.905	0.905
MSE	0.549	0.534	0.545	0.550
AIC	1.516	1.561	1.554	1.576
BIC	1.756	1.801	1.858	1.912
Linearity LR test	-	-	9.406 (0.009)	8.935 (0.063)
Ljung-Box test (30 lags)	54.984 (0.004)	52.910 (0.006)	61.172 (0.001)	51.236 (0.009)
Homoscedasticity F test	0.435 (0.010)	0.481 (0.02)	0.384 (0.003)	0.442 (0.011)
Jarque-Bera test	18.694 (0.000)	18.478 (0.000)	60.247 (0.000)	11.182 (0.004)

assumptions, the $STAR$ proposal systematically behaves better than the others. This is an indication that, in the analyzed period, exchange rate exposures were driven by some switching regime nonlinear process.

Finally, looking at the results from the LR linearity test, there is evidence, at least under a 10% significance level, that the $STAR$ specification is supported by the data.

Taking into account these findings, there is no option left but to accept, amongst the four considered proposals, the $STAR$ model as the best description for the exposures onto the exchange rate markets.

Figures 5.2 and 5.3 depict time plots for the restricted Kalman smoothing estimates of Jensen's measure and of the exposures onto U.S. Dollar/Real exchange rate spot markets and Quantum Cambial. Visual inspection suggests that the investment strategy followed by the managers of HSBC FIF Cambial

Tabela 5.2: Results from the estimations with Itau Matrix US Hedge FIF.

Attribute	*RW*	*AR*	*TAR*	*STAR*
Log-likelihood	-243.201	-242.417	-234.786	-233.996
Computational time	0.33	0.77	11.1	5.44
Pseudo R^2	0.697	0.679	0.761	0.684
MSE	3.008	3.029	2.366	3.009
AIC	3.489	3.479	3.430	3.446
BIC	3.793	3.782	3.814	3.871
Linearity LR test	-	-	15.262 (0.000)	16.841 (0.002)
Ljung-Box test (30 lags)	32.144 (0.361)	37.576 (0.161)	31.757 (0.379)	37.376 (0.166)
Homoscedasticity F test	0.670 (0.210)	0.899 (0.738)	0.944 (0.857)	0.779 (0.434)
Jarque-Bera test	6.286 (0.043)	4.242 (0.120)	7.412 (0.024)	2.618 (0.270)

is such that exposures onto U.S. Dollar/Real exchange rate spot markets were negligible throughout the sample, except during the period of higher volatility, when a significant long position was taken. Furthermore, exposures onto Quantum Cambial were always significant, wandering around a share of approximately 75% of the portfolio throughout the period. This outcome probably reflects preventive measures taken by fund managers during the crisis, which protected the portfolio against the losses caused by the decrease in the market value of dollar-indexed bonds issued by the Brazilian government. Managers of Itau Matrix US Hedge FIF, in turn, followed an investment strategy in which the exposures onto U.S. Dollar/Real exchange rate spot markets wandered around 75%-80% of the portfolio throughout the sample (even though the large confidence intervals observed during the period avoid ascertaining this); on the other hand, exposures onto Quantum Cambial were negative and significant at several occasions.

I now look at information extracted by the model on selectivity skills by analyzing the time path of Jensen's measure. The graphs on the top of Figures 5.2 and 5.3 suggest that managers of HSBC FIF Cambial and Itau Matrix US Hedge FIF revealed a slight tendency of generating gains during the period marked by the confidence crisis. One can understand these facts concerning HSBC and Itau funds by recalling that it is precisely during periods of increased volatility that managers have significant profit opportunities by engaging in high-frequency operations (e.g., day-trade transactions).

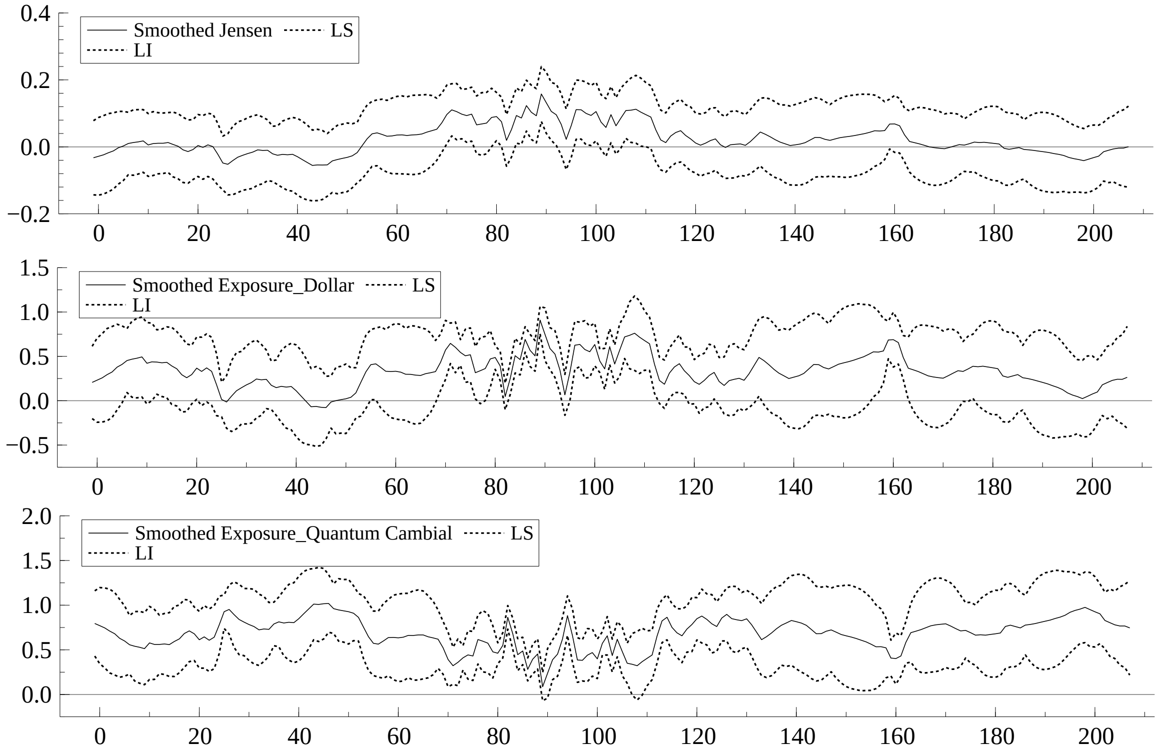

Figura 5.2: Smoothed $STAR$ exposures and Jensen's measure for the HSBC FIF Cambial with respective 95% confidence intervals.

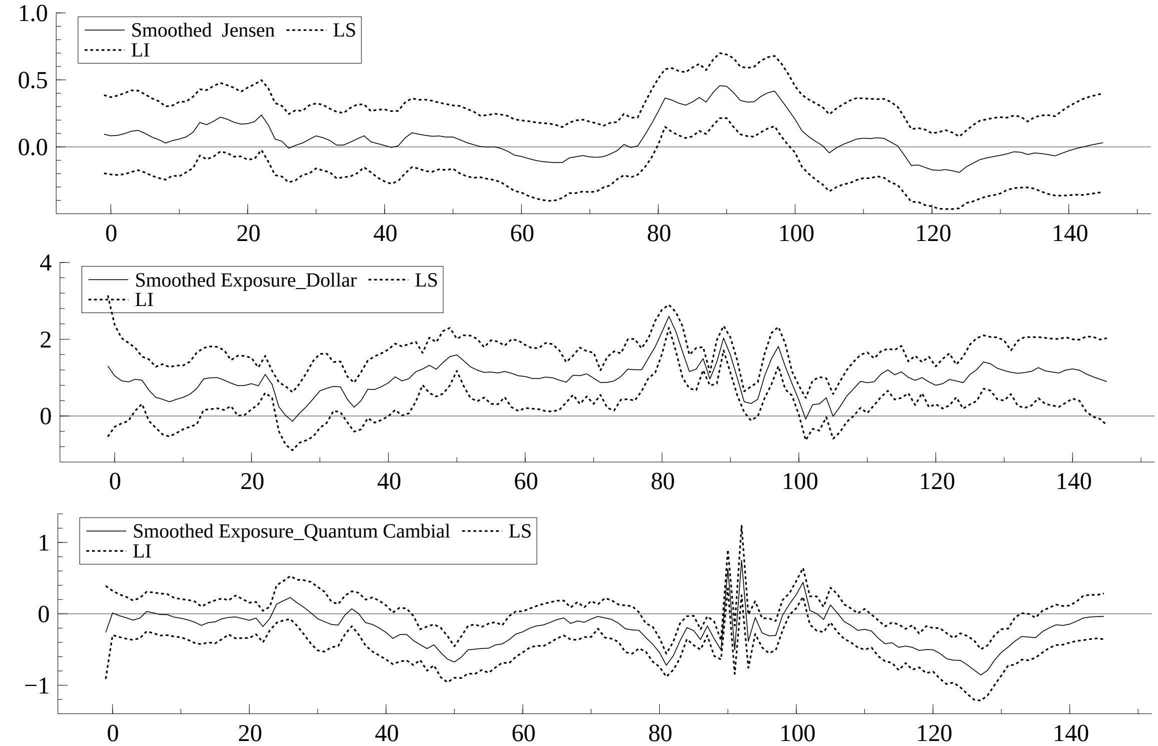

Figura 5.3: Smoothed $STAR$ exposures and Jensen's measure for the Itau Matrix US Hedge FIF with respective 95% confidence intervals.

Tabela 5.3: Pairwise correlation summary for the smoothed exposures onto exchange rate markets.

Fund	Correlation
HSBC FIF Cambial	-0.9592
Itau Matrix US Hedge FIF	-0.6230

Table 5.3, which shows pairwise relations for smoothed exposures onto U.S. Dollar/Real exchange rate and Quantum Cambial, allows a better understanding of the funds' behavior. Each series was tested individually for unit root (cf. Enders, 2004, ch.4) and all results indicated stationarity. In the sequel, pairwise correlations were estimated. The information in Table 5.3 suggests that the positions in the U.S. Dollar/Real exchange rate spot market and Quantum Cambial were negatively related, reflecting the fact that managers engaged in hedge operations to avoid a devaluation in their funds' shares. These results can even reveal a tendency to incur in some degree of leverage. One can see this by looking at what would happen if the U.S. Dollar/Real exchange rate has increased during the period (event that actually happened). Note that Itau Matrix US Hedge FIF would profit from this movement from two sources: (i) the long position in the U.S. Dollar/Real exchange rate market and; (ii) the short position in the Quantum Cambial. This outcome can be understood by recalling that there is a positive relationship between the U.S. Dollar/Real exchange rate and the premium in the DI-Dollar swap contract.

5.2 Case II: Estimation of dynamic exchange rate past-through

5.2.1 Motivation

In this section, linear state space models are proposed to estimate the pass-through of Brazilian price indexes against the US Dollar/Real exchange rate from 1996 to 2005. The methodological framework encompasses the reduced restricted Kalman filtering from section 4.2, which permits verifying the plausibility of some economic hypothesis.

There are three main targets. The first is to decide whether models of null (or of full) pass-through are acceptable to the price indexes investigated here. The second is to carry out likelihood ratio tests for the significance of some economic exogenous variables, which shall be termed determinants in this paper and are theoretically associated with the pass-through. The third is to analyze the behavior of the estimated pass-through from the best models.

Basic concepts and some review of the literature

In an open economy, domestic prices can be affected by external shocks, whether from currency relative prices adjustment or from movements of international supply and demand. The exchange rate is a quite volatile economic variables in macroeconomic policy. How much the exchange rate affects the economy? One of the faster channels is into prices. This channel is called *(exchange rate) pass-through*. There are few studies for this effect in Brazil in which the response of the prices to a change in exchange rate is suitably tackled.

The importance of past-through estimation has increased since the adoption of inflation targeting regime (cf. Fraga, Goldfajn and Minella, 2003), and the recognition that it is crucial for inflation forecasting. In addition to these motivations above, there is some evidence of a time-varying pass-through, even though only few studies have considered this assumption. Indeed, as Parsley (1995) points out, the stability of exchange rate pass-through is not well tested in common econometric specifications of pass-through equations.

Pass-Through Determinants

According to Menon (1996), Taylor (2000), and Campa and Goldberg (2002), the main drivers of price sensibility to exchange rate changes can be inferred. In face of literature with macroeconomic approach, the pass-through depends on: *inflation persistence*, *openness degree of the economy*, the *output gap*, and *real exchange rate disalignments*. From the standpoint of disaggregated analysis, the exchange rate pass-through is also associated with *the competition degree of each industry* and with *firm's market power* (with the elasticity price-demand).

5.2.2
The model setting and inference

I now present the state space model for the exchange rate pass-through for a given index price as follows:

$$\Delta log\ p_t = \sum_{k=1}^{m} \beta_{kt} \Delta log\ e_{t-k} + \psi_0 + \psi_1 \Delta log(ap_t) + \epsilon_t, \quad \epsilon_t \sim NID(0, \sigma^2) \quad (5\text{-}9)$$

$$\beta_{t+1} = \beta_t + \gamma_1 \Delta log(IPA_t)\mathbf{1}_{q\times 1} + \gamma_2 \Delta log(ip_t)\mathbf{1}_{q\times 1} + \gamma_3 \Delta log(re_t)\mathbf{1}_{q\times 1} \\ + \gamma_4 \Delta log(o_t)\mathbf{1}_{q\times 1} + \xi_t, \quad \xi_t \sim NID(0, Q) \tag{5-10}$$

The former equation linearly relates the observed monthly log-variation of price to the log-variation of exchange rate until time $t-m$ and to an exogenous variable, the American price index, ap_t. The coefficients of $\Delta log\ e_{t-k}$ in equation (5-9) are the state coordinates, which represent the *components* of the past-through (the sum of them is termed *long run past-through*) and whose dynamics are given in equation (5-10), which also sets the impact from the following determinants: IPA series that represents the inflationary environment; ip_t is the industrial production index, re_t is the exchange rate disalignment, and o_t is the openness of the economy. The matrix $Q_{m\times m}$ is set diagonal, even though the components from the past-through (*i.e.* the state coordinates) do maintain degrees of dependency due to the presence of common determinants in the state equation.

The reduced restricted Kalman filtering has to be evoked in order to make the restrictions of *full* past-through ($\sum_{i=1}^{m} \beta_{it} = 1$) and of *null* past-through ($\sum_{i=1}^{m} \beta_{it} = 0$) attainable. The completeness of the exchange rate passing-through (the first restriction) means that all the variation of the exchange rate is passed to the domestic prices. This is key for Economic Theory standpoint, since it means that the PPP hypothesis is acceptable. On the other hand, the acceptance that null exchange rate passing-through model is the most adequate scenario implies the exchange rate movements do not have any effect in the domestic prices, and so, the monetary authority needs not be concerned with exchange rate movements to make monetary policy with such price indexes.

Besides checking the hypotheses of completeness (or absence) of exchange rate passing-through, another purpose of this application is to identify the most adequate number of lags of the exchange rate, that is the value of m. For such, quite the same steps listed in subsection 5.1.3 shall be used.

Finally, the significance of the parameters ψ_0, ψ_1, γ_1, γ_2, γ_3 and γ_4 will be tested under a likelihood ratio (LR) testing approach. Since both the reduced and the complete model maintain the standards for good properties of maximum likelihood estimation (cf. Pagan, 1980), it follows that, asymptotically, $LR \equiv 2\left[logL_{Max,Comp} - logL_{Max,Red}\right] \sim \chi_1^2$, in which $logL_{Max,Red}$ represents the maximum of the log-likelihood for a model with a particular explanatory variable dropped from the specification.

5.2.3

Empirical results

The analyzed data contain monthly observations from August of 1999 to January of 2007 of the Brazilian wholesale price index (IPA), the Brazilian consumer price index (IPC), the American price index, the exchange rate between the Brazilian Real and the American Dollar, the Brazilian industrial production index and a measure of openness, which is the sum of imports and exports as a proportion of GNP. The decision of using data since August of 1999 is justified by the inflation target system adopted by the Banco Central (institution corresponding to the American Federal Reserve in Brazil) in June of 1999. The data has been obtained from IPEA Data (www.ipeadata.gov.br), and each estimation has taken less than 2 seconds, something that highlights the computational efficiency of the adopted state space framework.

Overall IPA

The most adequate model for the IPA series is the model with 7 lags on the exchange rate. Even though only the 4 first states have a confidence interval that does not contain zero, this decision has been based on the lack of serial correlation for the residuals. Figure 5.4 shows the evolution of the coefficients along time. The $PseudoR^2 = 0.64$ suggests that the model provides a reasonable adjustment for the IPA. The long run pass-through given in Figure 5.5 has some variation when we compare the beginning of the sample to the end with a edge at the 2002, the year of elections preceding the Lula's administration in Brazil, a period of great volatility in the exchange rate.

The restricted models were estimated to verify whether the hypothesis of null and full exchange rate pass-through have some support from the data. The information criteria shown in table 5.4 do not provide any evidence that these extremes allow a better fit. The LR significance tests are given in table 5.5. The p-values reveal no evidence that the proposed determinants help to explain the behavior of the pass-through.

Criterium	unrestricted	$\sum_{i=1}^{7} \beta_{i,t} = 0$	$\sum_{i=1}^{7} \beta_{i,t} = 1$
AIC	3.000	3.746	4.326
BIC	3.583	4.274	4.854

Tabela 5.4: IPA information criteria of the unrestricted and the restricted models.

First level IPA disaggregation

In order to evaluate the disaggregation effects on the exchange rate pass-through, its estimation was considered for some groups of products. The first

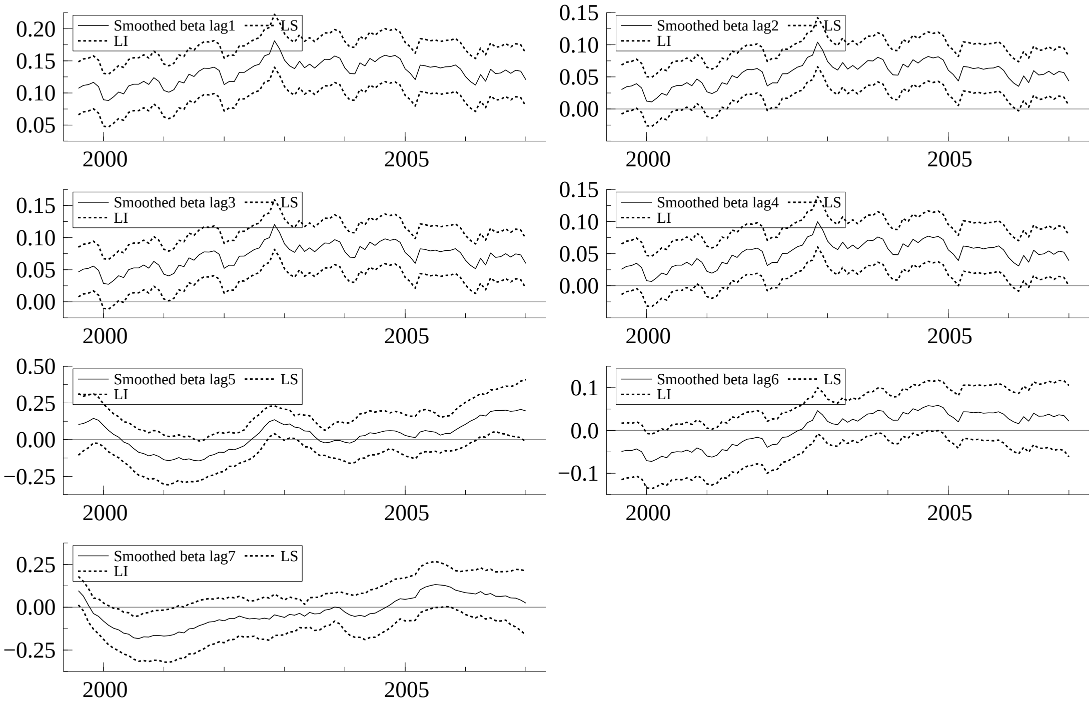

Figura 5.4: IPA smoothed betas.

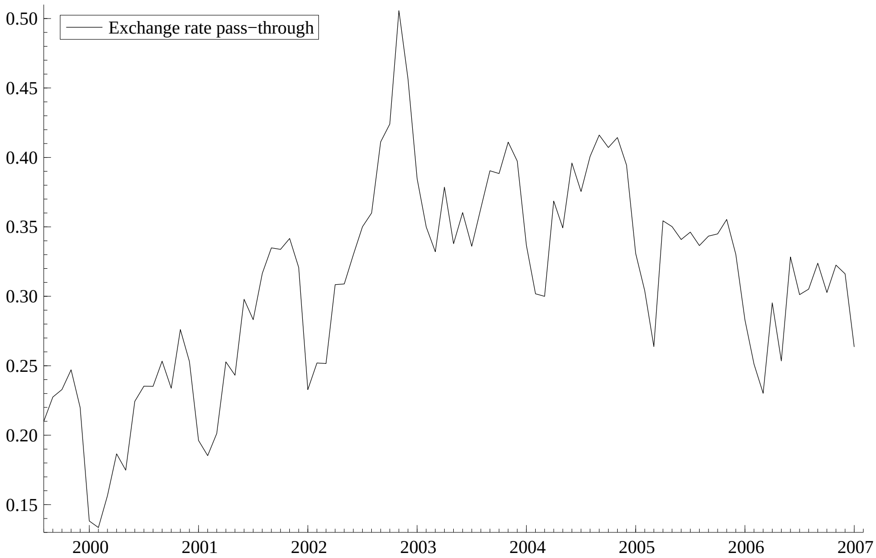

Figura 5.5: IPA long run exchange rate pass-through.

ψ_1	γ_1	γ_2	γ_3	γ_4
0.001	0.000	0.001	0.001	0.000
(1.000)	(1.000)	(1.000)	(0.505)	(1.000)

Tabela 5.5: IPA estimated parameters and corresponding p-values in parenthesis.

level of disaggregation splits the overall IPA into two main groups: consumption and production goods.

The more adequate model for the IPA consumption series has only a lag of the exchange rate, since it has the lower information criteria values and its residuals shows no serial correlation. The $PseudoR^2 = 0.615434$ provides evidence in favor of goodness-of-fit. Since the decision of having only one lag for the exchange rate, the short and long run exchange rate pass-through are the same. Its variation over time can be seen at figure 5.6. During the year of 2002, the exchange rate pass-through presented higher values compared to the rest of the sample period, probably due to the same explanations already given. Also, there is some indication of seasonal patterns, since the pass-through seems to be close to zero in the very begging of each year.

As shown in table 5.6, the LR significance tests reveal that three proposed determinants are supported by the data. One might also observe that the inertial parameter ψ_1 is statistically significant for the measurement equation.

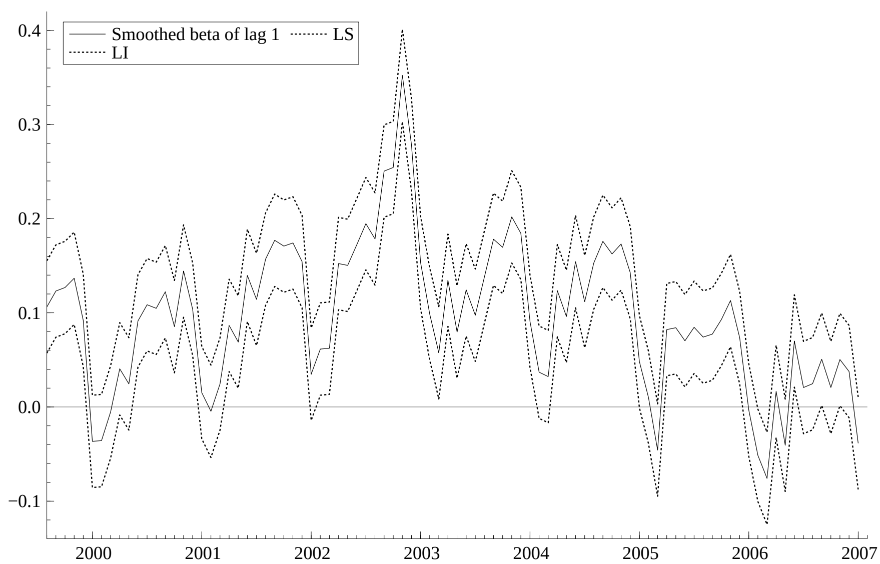

Figura 5.6: IPA consumption smoothed betas.

As it happened to the IPA consumption series, the most adequate model to the IPA production series was the model with only one lag of exchange rate pass-through. Again, the high value of the $PseudoR^2 = 0.729$ provides

ψ_1	γ_1	γ_2	γ_3	γ_4
0.561	-0.0004	0.012	0.005	-0.002
(0.000)	(0.000)	(0.061)	(0.006)	(0.347)

Tabela 5.6: IPA consumption estimated parameters and corresponding p-values in parenthesis.

us some confidence that the model fits the data in a proper way. The pass-through variation over time can be seen in figure 5.7. This remarks some aspects similar to those found in the previous analysis, except for the lack of evidence on seasonality.

The LR significance tests shown in table 5.7 provide us with two statistically significant determinants. Still, the inertial parameter ψ_1 is again statistically significant at the measurement equation.

Figura 5.7: IPA production smoothed betas.

ψ_1	γ_1	γ_2	γ_3	γ_4
0.590	-0.001	0.002	0.002	-0.001
(0.000)	(0.699)	(0.000)	(0.000)	(0.823)

Tabela 5.7: IPA production estimated parameters and corresponding p-values in parenthesis.

IPC

The model adjusted with 2 lags of the exchange rate shows that the IPC seems to be not responding to the exchange rate movements. As can be seen in figure 5.8, the states corresponding to all lags are varying around zero within the whole sample period. The long-run pass-through presented in figure 5.9 is also oscillating around zero. This shall be taken as the first symptom of absence of passing-through, and this is reinforced by the application of the restricted Kalman filtering, since the model which has the null pass-through restriction has the best information criteria; see table 5.8.

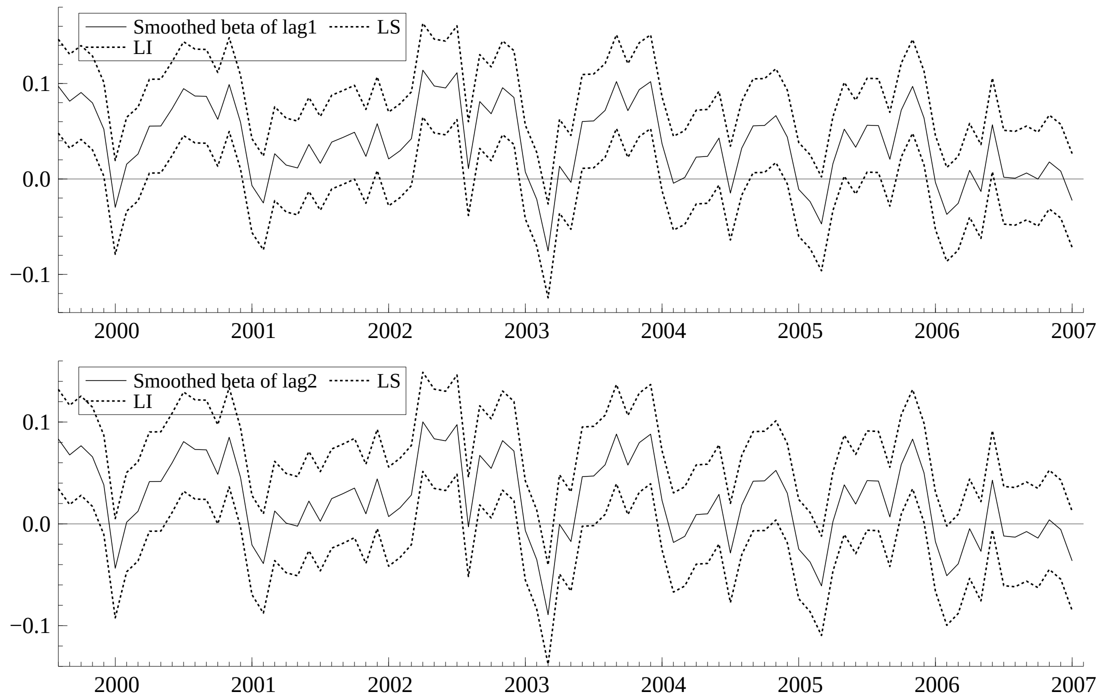

Figura 5.8: IPC smoothed betas.

Criterium	unrestricted	$\sum_{i=1}^{2} \beta_{i,t} = 0$	$\sum_{i=1}^{2} \beta_{i,t} = 1$
AIC	2.838	2.801	5.291
BIC	3.144	3.051	5.541

Tabela 5.8: IPC information criteria of the unrestricted and the restricted models.

5.3 Case III: GNP benchmarking estimation and prediction

5.3.1 Motivation

I close the applications of this Thesis by facing GNP quarterly prediction. Here is the setting. There are two series, a quarterly series of GNP which is subject to measurement error and an annual total series of the same economic variable that is "accurately" recorded. The goal is to produce a quarterly GNP

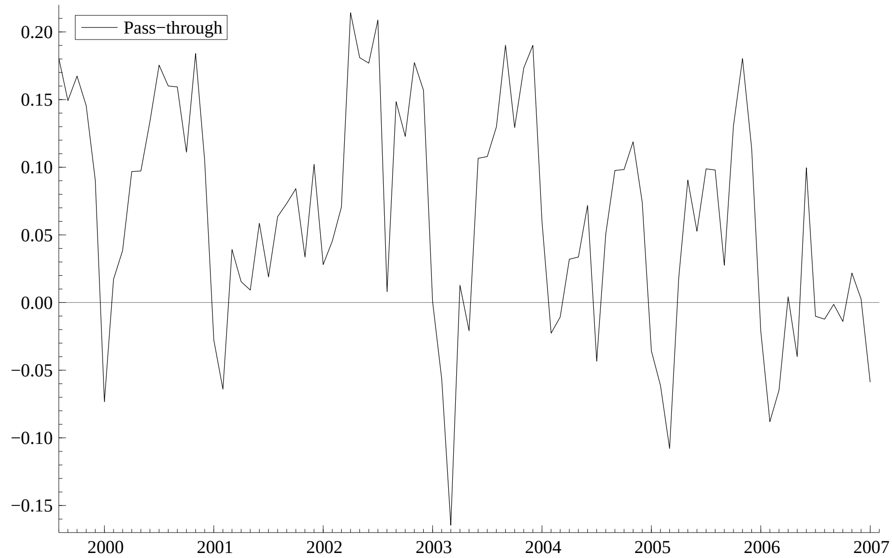

Figura 5.9: IPC long run pass-through.

free from those measurement errors, and this is supposed to be accomplished by conveniently using the information from the annual totals. This is in fact a *benchmarking* problem and its general formulation was examined in a rather comprehensive state space fashion by Durbin and Quenneville (1997). Later, Durbin and Koopman (2001), ch.3, quickly revisited the corresponding state space forms.

5.3.2 Model setup

Here the focus is to make predictions under this benchmarking framework, which shall generate quarterly predictions from the GNP free from measurement error and under *consistency* (that is, the estimated quarterly GNP must sum up to the annual totals GNP). For this purpose, I use the restricted Kalman predictor of section 4.3 with an alternative state space form that evinces the consistency restriction. This representation is an augmented state space model, the augmentations of which only appear in time periods multiple of 4 (four): that is, in these time periods the information from the annually totals GNP is attached to the measurement equation (this is making use of the time- and size-varying flexibility of the augmenting restricted Kalman filtering!). In this sense, the measurements would be Y_t if we "are not" in $4i$, and would be $(Y_t, X_t)'$ if we "are" in $4i$, where Y_t represents some quarterly GNP, X_t represents some totally GNP of some year and $i = 1, 2, \ldots$. The state vector would be $\alpha_t \equiv (\mu_t, \mu_{t-1}, \mu_{t-2}, \mu_{t-3}, \gamma_t, \gamma_{t-1}, \gamma_{t-2}, \gamma_{t-3}, \varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \xi_t)'$, where μ_t is a local level, γ_t is a dummy seasonal effect, ε_t is a Gaussian white noise irregular component and ξ_t represents the $AR(1)$ measurement error. In addition, one must set $H_t \equiv 0$, $d_t \equiv 0$ and $c_t \equiv 0$. Finally, check below the Z_t matrices for this alternative restricted state space form:

$$Z_t = \begin{cases} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \; if\ t \neq 4i,\ i = 1, 2, \ldots \\ \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \; if\ t = 4i,\ i = 1, 2, \ldots \end{cases}$$

The matrices $T_t \equiv T$, $R_t \equiv R$ and $Q_t \equiv Q$ are obvious and are omitted to save space. Observe that the specification is purely based on the structural modeling framework (see Harvey, 1989, chapter 2) for the quarterly GNP series. Also see that the *theoretical* consistency correction acts in the time indexes multiple of four. From Theorem 1 and from the computational algorithm described in the end of section 4.3, the *empirical* consistency correction is achieved in-sample and out-of-sample periods (the latter would be the prediction) whenever the Kalman updating and smoothing equations actuate on a series extended in the way proposed in 4-14 (notice that $q_t = X_t$ for every t multiple of 4).

5.3.3
Empirical results

To illustrate the proposed benchmarking prediction model, I applied it to the Brazilian GNP series constructed by the methodology proposed in Cerqueira *et al.* (2007). The very original series had been obtained from IBGE (*www.ibge.gov.br*) and IPEADATA (*www.ipeadata.gov.br*). I estimated the model with 140 observations ranging from the first quarter of 1960 to the fourth quarter of 1994, a 21.7-second task. In the sequel, I used the restricted Kalman predictor to the next two years using the annual totals of 1995 and 1996 to get the consistency restrictions satisfied. Table 5.9 presents the prediction results. The reader can easily confirm that the predicted quarterly GNP is consistent with the annual totals.

Tabela 5.9: Results of the benchmarking prediction.

Year/Quarter	1st	2nd	3rd	4th	Annual total
1995	1,066	1,153	1,129	1,096	4,444
1996	1,030	1,165	1,167	1,139	4,502