# 1
# Introduction

## 1.1
## Motivation

In some relevant practical situations from areas like economics, finance, actuary, engineering etc, some state space models (Harvey, 1989; Brockwell and Davis, 1991; and Durbin and Koopman, 2001) would make more sense if estimated under some meaningful restrictions on the state vector.

Some examples:

– One may consider state space models to conduct time-varying econometric models where well-established economic restrictions on the coefficients should be at least attested.

– Statistical models raised from physical considerations sometimes make sense only if considered under symmetry constraints on their parameters, whenever they are fixed or stochastically varying (cf. Pizzinga, Ruggeri and Guedes, 2005).

– In the claims reserving issue, some dynamic models for *runoff data* (cf. de Jong and Zehnwirth, 1983) may have columns and/or nonnegativity restrictions in the development/delay effect.

– Dynamic factor models for portfolio on-line recovering should be at least subject to accounting restrictions (eg. the portfolio allocations must add up to one, for every time period; cf. Pizzinga and Fernandes, 2006).

And, whenever one attempts to perform such constrained state estimation, some questions naturally arise. How should one implement this constrained estimation? When should this estimation be done? Which statistical properties do these methods of estimation share? Which theoretical and/or computational complications could emerge? Are all possible types of restrictions handleable? Are the imposed restrictions checkable for their plausibility under a specific method? And what could be said about the initialization of the recursions from these restricted estimations?

This Doctorate Thesis concentrates on additional developments concerning the *restricted Kalman filtering*, appropriate to problems that demand *linear* restrictions in the setting of *linear* state space models. By ranging from theoretical results and new methods to illustrative applications, the contributions address most - if not all - of the previous questions.

In the sequel, I will detail each one of the presumed novelties of the Thesis. But, first, let me review some literature on the subject.

## 1.2 A glimpse at the literature

There are essentially two directions the literature about linear state space models under restrictions has taken, one more "statistical-like"and another more "engineering-like". One very interesting point I have noticed was the absence of cross-references between these two fields. The result: an unavoidable overlapping between contributions coming from both "worlds".

### 1.2.1 Statistical papers

From the statistical/econometric side, I understand that Doran (1992) is the most seminal paper on the subject, in which the restricted Kalman filtering by augmentation was proved, under lots of matrix operations, for update and smoothing equations. At the end of his paper, Doran also made an attempt to further extend his approach for cases of nonlinear restrictions, but at least first-order differentiable. During the next five years, Doran have published two more papers. In Doran (1996), his previous approach was used in a problem of estimating Australian provincial populations according to annual national population. And, in Doran and Rambaldi (1997), the same approach was once more evoked to solve the problem of estimating time-varying econometric models (demand systems to be exact) also under time-varying and quite interpretable restrictions; in that same paper, the authors also discussed the relevant question concerning numerical optimization for the maximum likelihood estimation of unknown parameters. Some of those Doran's three important papers have been mentioned in the literature - mainly the very first one -, and his approach has been revisited as well. See for instance the book by Durbin and Koopman (2001), subsection 6.5.

Another more recent work on the subject is the paper due to Pandher (2002), which again cited Doran in his bibliographic review. In his paper, Pandher was fully concerned about forecasting multivariate time series under linear restrictions. His approach differed from the one proposed by Doran, although we can note again some augmentation strategy under a structural modeling framework. Pandher also gave some results on statistical efficiency of the state prediction under restrictions and on observability of the state vector under his method. At last, Pandher recognized another previous and related work when he cites Leybourne (1993), who had tackled univariate state space models under time invariant restrictions over a random walk state vector.

### 1.2.2 Engineering papers

Now, I take a quick look at engineering articles on restricted Kalman filtering. For instance, I can cite the papers by Wen and Durrant-Whyte (1991), Massicotte *et al.* (1995) and Geeter *et al.* (1997), none of which has referred to or has been referred to by the statisticians previously revisited in subsection 1.2.1. But two specific and much more recent papers deserve a little bit of attention.

The first paper was written by Simon and Chia (2002), who derived at least two different versions of the restricted Kalman filtering by alternative perspectives, even though making use of the same type of solution, the Lagrange multipliers approach. Their first version of the Kalman filtering is, in fact, almost the same as that originally obtained by Doran (1992) for the updating equations. So, this serves as an example of the aforementioned overlapping among the literature. Simon and Chia also presented five theorems that uncover good properties of their developed restricted Kalman filtering; four of them relate to mean square error efficiency. The methodological/theoretical part of the paper is closed by a discussion on how nonlinear restrictions could be encompassed by their restricted Kalman filtering. And the outcome is another material previously released by Doran (1992), although apparently not perceived by Simon and Chia.

The second paper, by Simon and Simon (2004), clearly continues the first paper's *leitmotiv* by trying to incorporate inequality constraints in the Kalman filtering. The authors solved the task by using quadratic programming and the interesting find was that the problem actually remains as a special case of imposing equality constraints.

## 1.3 This Thesis's contributions

The specific contributions of my research consist of investigation and development of the following topics:

1. A more general and more elegant proof of the restricted Kalman filtering (updating and smoothing equations) which uses *Hilbert space geometry*. This new proof is compared with those previous from Doran (1992).

2. An alternative proof based on Kalman recursions of the restricted Kalman filtering. The importance of this alternative proof lies on its idea of *re-writing the augmented model* in a useful and equivalent way, which would be the building block for other methods and results to be also presented.

3. An alternative and "constructive"proof of the *statistical efficiency* from the restricted Kalman filtering, which is compared with the proof already presented in my Master dissertation (cf. Pizzinga, 2004).

4. An alternative approach for imposing time-invariant restrictions to the estimation of random-walk state vectors.

5. A comparison between the restricted Kalman filtering and the restricted *recursive* least squares, and the establishment of the equivalence between both techniques under a particular albeit relevant case.

6. Development and implementation of a new restricted Kalman filtering under a *reduced modeling approach*, followed by detailed confrontations with the usual restricted Kalman filtering by augmentation.

7. Development of a *restricted Kalman predictor* which is applicable to general situations and which encompasses the method by Pandher (2002) as a particular case.

8. An alternative, "parametric", very short and quite general proof of the restricted Kalman filtering under a *conditional expectation* framework, followed by comparisons with previous demonstrations.

9. The proof that the initial exact Kalman smoother (cf. Durbin and Koopman, 2001, ch. 5) still yields restricted smoothed state vectors within the "diffuse" period, whenever applied to an appropriate augmented model.

10. A practical illustration in finance, in which a dynamic factor model under a linear and interpretable restriction is used to understand the style of Brazilian exchange rate funds.

11. An application in macroeconomics, in which dynamic models for exchange rate past-through are proposed and estimated with Brazilian price indexes.

12. A practical illustration in macroeconomics, in which a univariate benchmarking model, recognized as a linear state space model under restrictions, is used to predict the Brazilian GNP.

If I have to classify the above contributions into three kinds, I might do as follows:

- Contributions 1, 2, 3, 5, 8 and 9 bear *theory*;
- Contributions 4, 6 and 7 bear *methods*;
- Contributions 10, 11 and 12 offer three *applications*.

## 1.4 Thesis's organization

This Thesis is arranged as follows. Chapter 2 revisits the essentials of linear state space models and the Kalman filtering. Chapter 3 is entirely dedicated to the theoretical issues concerning the imposition of linear restrictions to the Kalman equations, in which new proofs for already established results are given and, in addition, new results are derived. Chapter 4 focuses on some new methods that can be potentially useful in situations of linear state space modeling under linear restrictions on the state vector. Chapter 5 offers the aforementioned applications in finance and macroeconomics, which illustrate the performance of some methods discussed in chapter 4. Finally, chapter 6 closes the Thesis by suggesting some additional research topics.