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A
Artigos publicados

Neste apêndice encontra-se a lista de publicações decorrentes da pesquisa realizada durante a presente tese de doutorado.

Artigos em Periódicos

1. M. S. Novo, L. C. da Silva, and F. L. Teixeira, "Finite volume modeling of borehole electromagnetic logging in 3-D anisotropic formations using coupled scalar-vector potentials", *IEEE Antennas and Wireless Propagation Letters*, vol. 6, pp. 549-552, 2008.

2. M. S. Novo, L. C. da Silva, and F. L. Teixeira, "Comparison of coupled-potentials and field-based finite-volume formulations for modeling of borehole electromagnetic tools", *IEEE Geoscience and Remote Sensing Letters*. (To appear)

Artigos em Congressos

1. M. S. Novo, L. C. da Silva, and F. L. Teixeira,"Analysis of electromagnetic well-logging tools for oil and gas exploration using finite volume techniques" *International Microwave and Optoelectronics Conference - IMOC 2007*, Salvador, BA, Brazil, 2007.

2. M. S. Novo, F. L. Teixeira, and L. C. da Silva, "Implementação de condições de contorno tipo CFS-PML na modelagem eletromagnética de instrumentos de sondagem petrolífera", *2006 XII Simpósio Brasileiro de Microondas e Optoeletrônica & VII Congresso Brasileiro de Eletromagnetismo*, Minas Gerais, MG, Brazil, Aug. 07-10, 2006.

3. M. S. Novo, F. L. Teixeira, and L. C. da Silva, "Finite volume analysis of LWD electromagnetic tool response in complex borehole environments using scalar and vector potentials", *2006 URSI North American Radio Science Meeting Digest*, Albuquerque, NM, USA, July 9-14, 2006.

4. M. S. Novo, F. L. Teixeira, and L. C. da Silva,"A nonuniform finite volume technique for modeling LWD electromagnetic tools in inhomoge-

neous earth media", *XI International Conference on Ground Penetrating Radar*, Columbus, OH, USA, June 19-22, 2006.

5. M. S. Novo, L. C. da Silva, and F. L. Teixeira, "Application of CFS-PML to finite volume analysis of EM well-logging tools in multilayered geophysical formations", *Proceedings of the CEFC2006 - IEEE Conference on Electromagnetic Field Computation*, Miami, FL, USA, April 30-May 3, 2006.

B
Expressões dos elementos da matriz do sistema - Modelo isotrópico - Formulação por campos

Neste apêndice são apresentadas as expressões dos elementos não nulos da matriz do sistema [C][X]=[B], decorrente da aplicação do método dos volumes finitos em meios isotrópicos. Neste modelo as equações de Maxwell são resolvidas através da formulação por campos desenvolvida na Seção 3.3.2.

Os elementos C_{mn} são dados por:

– Para $m = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

onde $i = 1, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$.

1. Para $n = i + (j-1)N_\rho + (k-3)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k-1/2)}\rangle}\right) \tag{B-1}$$

2. Para $n = i + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\right) \tag{B-2}$$

3. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j+1/2,k)}\rangle}+\frac{1}{\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\right) \\ & + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k-1/2)}\rangle}+\frac{1}{\langle\mu_{(i+1/2,j,k+1/2)}\rangle}\right) \\ & - \rho_{i+1/2}\Delta\varphi\Delta z\left[\langle\sigma_{(i+1/2,j,k)}\rangle-(i\omega)\langle\epsilon_{(i+1/2,j,k)}\rangle\right] \end{aligned} \tag{B-3}$$

4. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\right) \tag{B-4}$$

5. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k+1/2)}\rangle}\right) \tag{B-5}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-6}$$

7. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-2)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-7}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_i}{(i\omega)\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-8}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-9}$$

10. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} \tag{B-10}$$

11. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} \tag{B-11}$$

12. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} \tag{B-12}$$

13. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+I+(j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} \tag{B-13}$$

– Para $m = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$ onde $i = 2,\cdots,N_\rho; j = 1,\cdots,N_\varphi; k = 2,\cdots,N_z$

1. Para $n = (i-1)+(j-1)N_\rho+(k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \tag{B-14}$$

2. Para $n = i+(j-1)N_\rho+(k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-15}$$

3. Para $n = (i-1)+jN_\rho+(k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \tag{B-16}$$

4. Para $n = i+jN_\rho+(k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \tag{B-17}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2}-\rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{(i,j+1/2,k-1/2)}\rangle} \tag{B-18}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i-1}}{(i\omega)\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \tag{B-19}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z \rho_i}{(i\omega)\langle \mu_{(i-1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2} \right) + \frac{\Delta z \rho_i}{(i\omega)\langle \mu_{(i+1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2} \right) + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k-1/2)} \rangle} - (\rho_{i+1/2} - \rho_{i-1/2})\Delta z (\langle \sigma_{(i+1/2,j,k+1/2)} \rangle - (i\omega)\langle \epsilon_{(i+1/2,j,k+1/2)} \rangle) \tag{B-20}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + I + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta z \rho_{i+1}}{(i\omega)\langle \mu_{(i+1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2} \right) \tag{B-21}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{B-22}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k-1/2)} \rangle \rho_i \Delta \varphi} \tag{B-23}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k-1/2)} \rangle \rho_i \Delta \varphi} \tag{B-24}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k+1/2)} \rangle \rho_i \Delta \varphi} \tag{B-25}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k+1/2)} \rangle \rho_i \Delta \varphi} \tag{B-26}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 1, \cdots, N_z$

1. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{(i-1/2,j,k+1/2)} \rangle} \tag{B-27}$$

2. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{(i+1/2,j,k+1/2)} \rangle} \tag{B-28}$$

3. Para $n = (i-1) + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{(i-1/2,j,k+1/2)} \rangle} \tag{B-29}$$

4. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{(i+1/2,j,k+1/2)} \rangle} \tag{B-30}$$

5. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{B-31}$$

6. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{B-32}$$

7. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j-1/2,k+1/2)} \rangle} \tag{B-33}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{B-34}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j-1/2,k+1/2)} \rangle} \tag{B-35}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-2) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{(i-1/2,j,k+1/2)} \rangle} \tag{B-36}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$\begin{aligned} C_{mn} = & \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j-1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \\ & + \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{(i+1/2,j,k+1/2)} \rangle} + \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{(i-1/2,j,k+1/2)} \rangle} \\ & - \Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle \right) \end{aligned} \tag{B-37}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{(i+1/2,j,k+1/2)} \rangle} \tag{B-38}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{B-39}$$

Os elementos B_n são dados por:

$$B_n = \begin{cases} I_0, \text{ se } n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) \\ \qquad\qquad +(k-2)(N_\rho - 1)N_\varphi; \\ 0, \text{ caso contrário.} \end{cases}$$

onde I_0 é a amplitude da corrente de excitação. $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$

C
Expressões dos elementos da matriz do sistema - Modelo isotrópico - Formulação por potenciais

Neste apêndice são apresentadas as expressões dos elementos não nulos da matriz do sistema [C][X]=[B], decorrente da aplicação do método dos volumes finitos em meios isotrópicos. Neste modelo as equações de Maxwell são resolvidas através da formulação por potenciais desenvolvida na Seção 3.4.2.

Os elementos C_{mn} são dados por:

– Para $m = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

onde $i = 1, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$.

1. Para $n = i + (j-1)N_\rho + (k-3)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k-1/2)}\rangle}\right) \tag{C-1}$$

2. Para $n = i + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\right) \tag{C-2}$$

3. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\rho_{i-1/2}\Delta\varphi\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \tag{C-3}$$

4. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j+1/2,k)}\rangle}+\frac{1}{\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\right) \\ & + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k-1/2)}\rangle}+\frac{1}{\langle\mu_{(i+1/2,j,k+1/2)}\rangle}\right) \\ & + \frac{(\rho_{i+1/2})^2\Delta\varphi\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}+\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \\ & - \rho_{i+1/2}\Delta\varphi\Delta z\left[\langle\sigma_{(i+1/2,j,k)}\rangle-(i\omega)\langle\epsilon_{(i+1/2,j,k)}\rangle\right] \end{aligned} \tag{C-4}$$

5. Para $n = (i+1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+3/2}\rho_{i+1/2}\Delta\varphi\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}\right) \quad \text{(C-5)}$$

6. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\right) \quad \text{(C-6)}$$

7. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{(i+1/2,j,k+1/2)}\rangle}\right) \quad \text{(C-7)}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & -\frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \quad \text{(C-8)}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-2)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & +\frac{\rho_{i+1/2}(\rho_{i+3/2}-\rho_{i+1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}\right) \end{aligned} \quad \text{(C-9)}$$

10. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & -\frac{\Delta z\rho_i}{(i\omega)\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & +\frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \quad \text{(C-10)}$$

11. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & -\frac{\rho_{i+1/2}(\rho_{i+3/2}-\rho_{i+1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}\right) \end{aligned} \quad \text{(C-11)}$$

12. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)} \quad \text{(C-12)}$$

13. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)} \quad \text{(C-13)}$$

14. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)} \quad \text{(C-14)}$$

15. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\mu_{(i,j,k)}(\rho_{i+1}-\rho_i)} \quad \text{(C-15)}$$

16. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi\Delta z}{(\rho_{i+1}-\rho_i)}\left(\langle\sigma_{(i+1/2,j,k)}\rangle - (i\omega)\langle\epsilon_{(i+1/2,j,k)}\rangle\right) \quad \text{(C-16)}$$

17. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{(\rho_{i+1}-\rho_i)}\left(\langle\sigma_{(i+1/2,j,k)}\rangle - (i\omega)\langle\epsilon_{(i+1/2,j,k)}\rangle\right) \quad \text{(C-17)}$$

– Para $m = (N_\rho N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$ onde $i = 2,\cdots,N_\rho; j = 1,\cdots,N_\varphi; k = 2,\cdots,N_z$

1. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_i - \rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) - \frac{\rho_{i-1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \quad \text{(C-18)}$$

2. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) + \frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \quad \text{(C-19)}$$

3. Para $n = (i-1) + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_i - \rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) + \frac{\rho_{i-1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \quad \text{(C-20)}$$

4. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\langle\mu_{(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \quad \text{(C-21)}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{(i,j+1/2,k-1/2)}\rangle} \quad \text{(C-22)}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i(\Delta\varphi)^2\mu_{(i,j,k)}}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \quad \text{(C-23)}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i-1}}{(i\omega)\langle\mu_{(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \quad \text{(C-24)}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$\begin{aligned} C_{mn} = & \frac{\Delta z \rho_i}{(i\omega)\langle \mu_{(i-1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2} \right) \\ & + \frac{\Delta z \rho_i}{(i\omega)\langle \mu_{(i+1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2} \right) \\ & + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k-1/2)} \rangle} \\ & + \frac{2(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i (\Delta\varphi)^2 \mu_{(i,j,k)}} \left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2} \right) \\ & - (\rho_{i+1/2} - \rho_{i-1/2})\Delta z (\langle \sigma_{(i+1/2,j,k+1/2)} \rangle - (i\omega)\langle \epsilon_{(i+1/2,j,k+1/2)} \rangle) \end{aligned} \tag{C-25}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + I + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta z \rho_{i+1}}{(i\omega)\langle \mu_{(i+1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2} \right) \tag{C-26}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i (\Delta\varphi)^2 \mu_{(i,j,k)}} \left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2} \right) \tag{C-27}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{C-28}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k-1/2)} \rangle \rho_i \Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{(i,j,k)} \rho_i \Delta\varphi} \tag{C-29}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k-1/2)} \rangle \rho_i \Delta\varphi} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{(i,j,k)} \rho_i \Delta\varphi} \tag{C-30}$$

14. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k+1/2)} \rangle \rho_i \Delta\varphi} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{(i,j,k)} \rho_i \Delta\varphi} \tag{C-31}$$

15. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{(i,j+1/2,k+1/2)} \rangle \rho_i \Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{(i,j,k)} \rho_i \Delta\varphi} \quad \text{(C-32)}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \sigma_{\varphi\varphi(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{\varphi\varphi(i,j+1/2,k)} \rangle) \quad \text{(C-33)}$$

17. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \sigma_{(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{(i,j+1/2,k)} \rangle) \quad \text{(C-34)}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 1, \cdots, N_z$

1. Para $n = (i-1) + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{(i-1/2,j,k+1/2)} \rangle} - \frac{\rho_{i-1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{(i,j,k)}} \quad \text{(C-35)}$$

2. Para $n = i + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{(i+1/2,j,k+1/2)} \rangle} + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{(i,j,k)}} \quad \text{(C-36)}$$

3. Para $n = (i-1) + (j-1) N_\rho + (k-1) N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{(i-1/2,j,k+1/2)} \rangle} + \frac{\rho_{i-1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{(i,j,k)}} \quad \text{(C-37)}$$

4. Para $n = i + (j-1) N_\rho + (k-1) N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{(i+1/2,j,k+1/2)} \rangle} - \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{(i,j,k)}} \quad \text{(C-38)}$$

5. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \mu_{(i,j,k)}} \qquad \text{(C-39)}$$

6. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \mu_{(i,j,k)}} \qquad \text{(C-40)}$$

7. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \mu_{(i,j,k)}} \qquad \text{(C-41)}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{(i,j+1/2,k+1/2)} \rangle} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \mu_{(i,j,k)}} \qquad \text{(C-42)}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{(i,j,k)}} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \qquad \text{(C-43)}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j-1/2,k+1/2)} \rangle} \qquad \text{(C-44)}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-2) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{(i-1/2,j,k+1/2)} \rangle} \qquad \text{(C-45)}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j-1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j+1/2,k+1/2)} \rangle}$$
$$+ \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i)\langle \mu_{(i+1/2,j,k+1/2)} \rangle} + \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1})\langle \mu_{(i-1/2,j,k+1/2)} \rangle}$$
$$+ \frac{2\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{(i,j,k)}} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right)$$
$$- \Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle \right) \tag{C-46}$$

13. Para $(N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i)\langle \mu_{(i+1/2,j,k+1/2)} \rangle} \tag{C-47}$$

14. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{(i,j+1/2,k+1/2)} \rangle} \tag{C-48}$$

15. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + k(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{(i,j,k)}} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \tag{C-49}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \cdot \frac{1}{\Delta z} (\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle) \tag{C-50}$$

17. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \cdot \frac{1}{\Delta z} (\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle) \tag{C-51}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$

1. Para $n = (i-1) + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = \rho_{i+1/2} \Delta\varphi \Delta z (\langle \sigma_{(i-1/2,j,k)} \rangle - (i\omega) \langle \epsilon_{(i-1/2,j,k)} \rangle) \quad \text{(C-52)}$$

2. Para $n = i + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = -\rho_{i+1/2} \Delta\varphi \Delta z (\langle \sigma_{(i+1/2,j,k)} \rangle - (i\omega) \langle \epsilon_{(i+1/2,j,k)} \rangle) \quad \text{(C-53)}$$

3. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = (\rho_{i+1/2} - \rho_{i-1/2}) \Delta z (\langle \sigma_{(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{(i,j+1/2,k)} \rangle) \quad \text{(C-54)}$$

4. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -(\rho_{i+1/2} - \rho_{i-1/2}) \Delta z (\langle \sigma_{(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{(i,j+1/2,k)} \rangle) \quad \text{(C-55)}$$

5. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) (\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle) \quad \text{(C-56)}$$

6. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) (\langle \sigma_{(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k+1/2)} \rangle) \quad \text{(C-57)}$$

7. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-3)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{\Delta z} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) (\langle \sigma_{(i,j,k-1/2)} \rangle - (i\omega) \langle \epsilon_{(i,j,k-1/2)} \rangle) \quad \text{(C-58)}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \sigma_{(i,j-1/2,k)} \rangle - (i\omega) \langle \epsilon_{(i,j-1/2,k)} \rangle) \tag{C-59}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-2) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\rho_{i-1/2} \Delta\varphi \Delta z}{(\rho_i - \rho_{i-1})} (\langle \sigma_{(i-1/2,j,k)} \rangle - (i\omega) \langle \epsilon_{(i-1/2,j,k)} \rangle) \tag{C-60}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$\begin{aligned} C_{mn} = & -\Delta\varphi \Delta z \left(\frac{\rho_{i+1/2}}{(\rho_{i+1} - \rho_i)} \langle \sigma_{(i+1/2,j,k)} \rangle + \frac{\rho_{i-1/2}}{(\rho_i - \rho_{i-1})} \langle \sigma_{(i-1/2,j,k)} \rangle \right) \\ & + \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \sigma_{(i,j+1/2,k)} \rangle + \langle \sigma_{(i,j-1/2,k)} \rangle) \\ & + \frac{\Delta\varphi}{\Delta z} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \sigma_{(i,j,k+1/2)} \rangle + \langle \sigma_{(i,j,k-1/2)} \rangle \right) \\ & - (i\omega) \Delta\varphi \Delta z \left(\frac{\rho_{i+1/2}}{(\rho_{i+1} - \rho_i)} \langle \epsilon_{(i+1/2,j,k)} \rangle + \frac{\rho_{i-1/2}}{(\rho_i - \rho_{i-1})} \langle \epsilon_{(i-1/2,j,k)} \rangle \right) \\ & - (i\omega) \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \epsilon_{(i,j+1/2,k)} \rangle + \langle \epsilon_{(i,j-1/2,k)} \rangle) \\ & - (i\omega) \frac{\Delta\varphi}{\Delta z} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \epsilon_{(i,j,k+1/2)} \rangle + \langle \epsilon_{(i,j,k-1/2)} \rangle \right) \end{aligned} \tag{C-61}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + i + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2} \Delta\varphi \Delta z}{(\rho_{i+1} - \rho_i)} (\langle \sigma_{(i+1/2,j,k)} \rangle - (i\omega) \langle \epsilon_{(i+1/2,j,k)} \rangle) \tag{C-62}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} (\langle \sigma_{(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{(i,j+1/2,k)} \rangle) \tag{C-63}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) +$

$$(i-1)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$$

$$C_{mn}=-\frac{\Delta\varphi}{\Delta z}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{(i,j,k+1/2)}\rangle-(i\omega)\langle\epsilon_{(i,j,k+1/2)}\rangle\right) \tag{C-64}$$

Os elementos B_n são dados por:

$$B_n=\begin{cases} I_0, \text{ se } n=(N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1) \\ \qquad\qquad +(k-2)(N_\rho-1)N_\varphi; \\ 0, \text{ caso contrário.} \end{cases}$$

onde I_0 é a amplitude da corrente de excitação. $i=2,\cdots,N_\rho; j=1,\cdots,N_\varphi; k=2,\cdots,N_z$

D
Expressões dos elementos da matriz do sistema - Modelo anisotrópico - Formulação por campos

Neste apêndice são apresentadas as expressões dos elementos não nulos da matriz do sistema [C][X]=[B], decorrente da aplicação do método dos volumes finitos em meios anisotrópicos. Neste modelo as equações de Maxwell são resolvidas através da formulação por campos desenvolvida na Seção 4.3.1.

Os elementos C_{mn} são dados por:

– Para $m = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

onde $i = 1, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$.

1. Para $n = i + (j-1)N_\rho + (k-3)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle}\right) \tag{D-1}$$

2. Para $n = i + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\left[(\rho_{i+1})^2 - (\rho_i)^2\right]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\right) \tag{D-2}$$

3. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{2\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\left[(\rho_{i+1})^2 - (\rho_i)^2\right]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle} + \frac{1}{\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\right) \\ & + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle} + \frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle}\right) \\ & - \rho_{i+1/2}\Delta\varphi\Delta z\left[\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle - (i\omega)\langle\epsilon_{\rho\rho(i+1/2,j,k)}\rangle\right] \end{aligned} \tag{D-3}$$

4. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\left[(\rho_{i+1})^2 - (\rho_i)^2\right]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\right) \tag{D-4}$$

5. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle}\right) \tag{D-5}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{D-6}$$

7. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-2)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{D-7}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{D-8}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{D-9}$$

10. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{1}{4}\rho_{i+1/2}\Delta\varphi\Delta z\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{D-10}$$

11. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{1}{4}\rho_{i+1/2}\Delta\varphi\Delta z\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \quad \text{(D-11)}$$

12. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{1}{4}\rho_{i+1/2}\Delta\varphi\Delta z\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \quad \text{(D-12)}$$

13. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle(\rho_{i+1}-\rho_i)} - \frac{1}{4}\rho_{i+1/2}\Delta\varphi\Delta z\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \quad \text{(D-13)}$$

– Para $m = (N_\rho N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$ onde $i = 2,\cdots,N_\rho; j = 1,\cdots,N_\varphi; k = 2,\cdots,N_z$

1. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) - \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \quad \text{(D-14)}$$

2. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) - \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \quad \text{(D-15)}$$

3. Para $n = (i-1) + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) - \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \quad \text{(D-16)}$$

4. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{D-17}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle} \tag{D-18}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i-1}}{(i\omega)\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2}\right) \tag{D-19}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2}\right) \\ & + \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) \\ & + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle} \\ & - (\rho_{i+1/2} - \rho_{i-1/2})\Delta z(\langle\sigma_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle) \end{aligned} \tag{D-20}$$

8. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) \tag{D-21}$$

9. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} \tag{D-22}$$

10. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \tag{D-23}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle \rho_i \Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle \sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(D-24)}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle \rho_i \Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle \sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(D-25)}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle \mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle \rho_i \Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle \sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(D-26)}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 1, \cdots, N_z$

1. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle \sigma_{z\rho(i,j,k+1/2)}\rangle \quad \text{(D-27)}$$

2. Para $i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle \sigma_{z\rho(i,j,k+1/2)}\rangle \quad \text{(D-28)}$$

3. Para $n = (i-1) + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi\rho_{i-1/2}}{(i\omega)\Delta z\langle\mu_{\varphi\varphi(i-1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \quad \text{(D-29)}$$

4. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi\rho_{i+1/2}}{(i\omega)\Delta z\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \quad \text{(D-30)}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \quad \text{(D-31)}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \quad \text{(D-32)}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j-1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \quad \text{(D-33)}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \quad \text{(D-34)}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + (j-$

$2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j-1/2,k+1/2)} \rangle} \tag{D-35}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-2) + (j - 1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)} \rangle} \tag{D-36}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j - 1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j-1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j+1/2,k+1/2)} \rangle} \\ & + \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle} + \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)} \rangle} \\ & - \Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \sigma_{zz(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{zz(i,j,k+1/2)} \rangle \right) \end{aligned} \tag{D-37}$$

12. Para $(N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle} \tag{D-38}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j+1/2,k+1/2)} \rangle} \tag{D-39}$$

Os elementos B_n são dados por:

$$B_n = \begin{cases} I_0, \text{ se } n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) \\ \qquad\qquad + (k-2)(N_\rho - 1)N_\varphi; \\ 0, \text{ caso contrário.} \end{cases}$$

onde I_0 é a amplitude da corrente de excitação. $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$

E
Expressões dos elementos da matriz do sistema - Modelo anisotrópico - Formulação por potenciais

Neste apêndice são apresentadas as expressões dos elementos não nulos da matriz do sistema [C][X]=[B], decorrente da aplicação do método dos volumes finitos em meios anisotrópicos. Neste modelo as equações de Maxwell são resolvidas através da formulação por potenciais desenvolvida na Seção 4.4.1.

Os elementos C_{mn} são dados por:

– Para $m = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

onde $i = 1, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$.

1. Para $n = i + (j-1)N_\rho + (k-3)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle}\right) \tag{E-1}$$

2. Para $n = i + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\right) \tag{E-2}$$

3. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\rho_{i-1/2}\Delta\varphi\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \tag{E-3}$$

4. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}+\frac{1}{\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\right) \\ & +\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k-1/2)}\rangle}+\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle}\right) \\ & +\frac{(\rho_{i+1/2})^2\Delta\varphi\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}+\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \\ & -\rho_{i+1/2}\Delta\varphi\Delta z\left[\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle-(i\omega)\langle\epsilon_{\rho\rho(i+1/2,j,k)}\rangle\right] \end{aligned} \tag{E-4}$$

5. Para $n = (i+1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+3/2}\rho_{i+1/2}\Delta\varphi\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}\right) \quad \text{(E-5)}$$

6. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{2\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\,[(\rho_{i+1})^2-(\rho_i)^2]}\left(\frac{1}{\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\right) \quad \text{(E-6)}$$

7. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z}\left(\frac{1}{\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle}\right) \quad \text{(E-7)}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & -\frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \\ & -\frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \quad \text{(E-8)}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-2)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{zz(i+1/2,j-1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & -\frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \\ & +\frac{\rho_{i+1/2}(\rho_{i+3/2}-\rho_{i+1/2})\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+3/2})^2-(\rho_{i+1/2})^2}\right) \end{aligned} \quad \text{(E-9)}$$

10. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & -\frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ & -\frac{1}{4}\rho_i\Delta\varphi\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \\ & +\frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1}-\rho_i)}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \quad \text{(E-10)}$$

11. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z \rho_{i+1}}{(i\omega)\langle \mu_{zz(i+1/2,j+1/2,k)} \rangle} \left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2} \right) - \frac{1}{4} \rho_i \Delta\varphi \Delta z \langle \sigma_{\rho\varphi(i+1/2,j,k)} \rangle - \frac{\rho_{i+1/2}(\rho_{i+3/2} - \rho_{i+1/2})\Delta z}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1} - \rho_i)} \left(\frac{2}{(\rho_{i+3/2})^2 - (\rho_{i+1/2})^2} \right) \quad \text{(E-11)}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i - 1) + (j - 1)(N_\rho - 1) + (k - 2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\langle \mu_{\varphi\varphi(i+1/2,j,k-1/2)} \rangle (\rho_{i+1} - \rho_i)} - \frac{1}{4} \rho_{i+1/2} \Delta\varphi \Delta z \langle \sigma_{\rho z(i+1/2,j,k)} \rangle - \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1} - \rho_i)} \quad \text{(E-12)}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + i + (j - 1)(N_\rho - 1) + (k - 2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\langle \mu_{\varphi\varphi(i+1/2,j,k-1/2)} \rangle (\rho_{i+1} - \rho_i)} - \frac{1}{4} \rho_{i+1/2} \Delta\varphi \Delta z \langle \sigma_{\rho z(i+1/2,j,k)} \rangle + \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1} - \rho_i)} \quad \text{(E-13)}$$

14. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i - 1) + (j - 1)(N_\rho - 1) + (k - 1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle (\rho_{i+1} - \rho_i)} - \frac{1}{4} \rho_{i+1/2} \Delta\varphi \Delta z \langle \sigma_{\rho z(i+1/2,j,k)} \rangle + \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1} - \rho_i)} \quad \text{(E-14)}$$

15. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + i + (j - 1)(N_\rho - 1) + (k - 1)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle (\rho_{i+1} - \rho_i)} - \frac{1}{4} \rho_{i+1/2} \Delta\varphi \Delta z \langle \sigma_{\rho z(i+1/2,j,k)} \rangle - \frac{\rho_{i+1/2} \Delta\varphi}{(i\omega)\mu_{\rho\rho(i,j,k)}(\rho_{i+1} - \rho_i)} \quad \text{(E-15)}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i - 1) + (j - 1)(N_\rho - 1) + (k - 3)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{1}{4}\rho_{i+1/2}\Delta\varphi\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-16}$$

17. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ i+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{1}{4}\rho_{i+1/2}\Delta\varphi\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-17}$$

18. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ (i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{1}{4\rho_i}\rho_{i+1/2}\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-18}$$

19. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ i+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{1}{4\rho_{i+1}}\rho_{i+1/2}\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-19}$$

20. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ (i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi\Delta z}{(\rho_{i+1}-\rho_i)}\left(\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle - (i\omega)\epsilon_{\rho\rho(i+1/2,j,k)}\right) \tag{E-20}$$

21. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ i+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{(\rho_{i+1}-\rho_i)}\left(\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle - (i\omega)\epsilon_{\rho\rho(i+1/2,j,k)}\right) \tag{E-21}$$

22. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ (i-1)+j(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{1}{4\rho_i}\rho_{i+1/2}\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-22}$$

23. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ i+j(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{1}{4\rho_{i+1}}\rho_{i+1/2}\Delta z\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-23}$$

24. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+$

$(i-1)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{1}{4}\rho_{i+1/2}\Delta\varphi\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-24}$$

25. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+ i+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{1}{4}\rho_{i+1/2}\Delta\varphi\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-25}$$

– Para $m=(N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$ onde $i=2,\cdots,N_\rho; j=1,\cdots,N_\varphi; k=2,\cdots,N_z$

1. Para $n=(i-1)+(j-1)N_\rho+(k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = &\frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \\ &-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \\ &-\frac{\rho_{i-1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \tag{E-26}$$

2. Para $n=i+(j-1)N_\rho+(k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\Delta z\,(\rho_{i+1}-\rho_i)}{(i\omega)\Delta\varphi\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2-(\rho_i)^2}\right) \\ &-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \\ &+\frac{\rho_{i+1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \tag{E-27}$$

3. Para $n=(i-1)+jN_\rho+(k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\Delta z\,(\rho_i-\rho_{i-1})}{(i\omega)\Delta\varphi\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2-(\rho_{i-1})^2}\right) \\ &-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \\ &+\frac{\rho_{i-1/2}(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}\right) \end{aligned} \tag{E-28}$$

4. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta z\,(\rho_{i+1} - \rho_i)}{(i\omega)\Delta\varphi\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle - \frac{\rho_{i+1/2}(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi}\left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}\right) \quad \text{(E-29)}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle} \quad \text{(E-30)}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i(\Delta\varphi)^2\mu_{\varphi\varphi(i,j,k)}}\left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}\right) \quad \text{(E-31)}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i-1}}{(i\omega)\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2}\right) \quad \text{(E-32)}$$

8. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i-1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_i)^2 - (\rho_{i-1})^2}\right) + \frac{\Delta z\rho_i}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle} + \frac{2(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i(\Delta\varphi)^2\mu_{\varphi\varphi(i,j,k)}}\left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}\right) - (\rho_{i+1/2} - \rho_{i-1/2})\Delta z(\langle\sigma_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle) \quad \text{(E-33)}$$

9. Para $n = (N_\rho N_\varphi(N_z-1)) + I + (j-1)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\Delta z\rho_{i+1}}{(i\omega)\langle\mu_{zz(i+1/2,j+1/2,k)}\rangle}\left(\frac{2}{(\rho_{i+1})^2 - (\rho_i)^2}\right) \quad \text{(E-34)}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{(i\omega)\rho_i(\Delta\varphi)^2\mu_{\varphi\varphi(i,j,k)}}\left(\frac{2}{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}\right) \quad \text{(E-35)}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} \quad \text{(E-36)}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(E-37)}$$

13. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle\mu_{\rho\rho(i,j+1/2,k-1/2)}\rangle\rho_i\Delta\varphi} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(E-38)}$$

14. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle\rho_i\Delta\varphi} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(E-39)}$$

15. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\mu_{\varphi\varphi(i,j,k)}\rho_i\Delta\varphi} - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \quad \text{(E-40)}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-3)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \tag{E-41}$$

17. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+j(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \tag{E-42}$$

18. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_i - \rho_{i-1})}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{E-43}$$

19. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i\Delta\varphi}(\langle\sigma_{\varphi\varphi(i,j+1/2,k)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i,j+1/2,k)}\rangle) - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle\left(\frac{1}{(\rho_i - \rho_{i-1})} - \frac{1}{(\rho_{i+1} - \rho_i)}\right) \tag{E-44}$$

20. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+i+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_{i+1} - \rho_i)}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{E-45}$$

21. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-2)+j(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_i - \rho_{i-1})}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{E-46}$$

22. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+j(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i\Delta\varphi}(\langle\sigma_{\varphi\varphi(i,j+1/2,k)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i,j+1/2,k)}\rangle) + \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle\left(\frac{1}{(\rho_i - \rho_{i-1})} - \frac{1}{(\rho_{i+1} - \rho_i)}\right) \tag{E-47}$$

23. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + i + j(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_{i+1} - \rho_i)} \langle \sigma_{\varphi\rho(i,j+1/2,k)} \rangle \tag{E-48}$$

24. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4} \langle \sigma_{\varphi z(i,j+1/2,k)} \rangle \tag{E-49}$$

25. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4} \langle \sigma_{\varphi z(i,j+1/2,k)} \rangle \tag{E-50}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 1, \cdots, N_z$

1. Para $n = (i-1) + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)\Delta z \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)} \rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle \sigma_{z\rho(i,j,k+1/2)} \rangle - \frac{\rho_{i-1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{zz(i,j,k)}} \tag{E-51}$$

2. Para $n = i + (j-1) N_\rho + (k-2) N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)\Delta z \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle \sigma_{z\rho(i,j,k+1/2)} \rangle + \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z \mu_{zz(i,j,k)}} \tag{E-52}$$

3. Para $n = (i-1) + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi\rho_{i-1/2}}{(i\omega)\Delta z\langle\mu_{\varphi\varphi(i-1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle + \frac{\rho_{i-1/2}\Delta\varphi}{(i\omega)\Delta z\mu_{zz(i,j,k)}} \tag{E-53}$$

4. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi\rho_{i+1/2}}{(i\omega)\Delta z\langle\mu_{\varphi\varphi(i+1/2,j,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle - \frac{\rho_{i+1/2}\Delta\varphi}{(i\omega)\Delta z\mu_{zz(i,j,k)}} \tag{E-54}$$

5. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\mu_{zz(i,j,k)}} \tag{E-55}$$

6. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\mu_{zz(i,j,k)}} \tag{E-56}$$

7. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\langle\mu_{\rho\rho(i,j+1/2,k+1/2)}\rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z\mu_{zz(i,j,k)}} \tag{E-57}$$

8. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \langle \mu_{\rho\rho(i,j+1/2,k+1/2)} \rangle} - \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle \sigma_{z\varphi(i,j,k+1/2)} \rangle - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\Delta z \mu_{zz(i,j,k)}} \tag{E-58}$$

9. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{zz(i,j,k)}}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right) \tag{E-59}$$

10. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j-1/2,k+1/2)} \rangle} \tag{E-60}$$

11. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-2) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)} \rangle} \tag{E-61}$$

12. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j-1/2,k+1/2)} \rangle} + \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j+1/2,k+1/2)} \rangle} + \frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle} + \frac{\Delta\varphi \rho_{i-1/2}}{(i\omega)(\rho_i + \rho_{i-1}) \langle \mu_{\varphi\varphi(i-1/2,j,k+1/2)} \rangle} + \frac{2\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{zz(i,j,k)}}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right) - \Delta\varphi\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\left(\langle \sigma_{zz(i,j,k+1/2)} \rangle - (i\omega)\langle \epsilon_{zz(i,j,k+1/2)} \rangle\right) \tag{E-62}$$

13. Para $(N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi \rho_{i+1/2}}{(i\omega)(\rho_{i+1} + \rho_i) \langle \mu_{\varphi\varphi(i+1/2,j,k+1/2)} \rangle} \tag{E-63}$$

14. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{(i\omega)\rho_i \Delta\varphi \langle \mu_{\rho\rho(i,j+1/2,k+1/2)} \rangle} \tag{E-64}$$

15. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + k(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{(i\omega)(\Delta z)^2 \mu_{zz(i,j,k)}} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \tag{E-65}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\varphi(i,j,k+1/2)} \rangle \tag{E-66}$$

17. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-2) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4(\rho_i - \rho_{i-1})} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k+1/2)} \rangle \tag{E-67}$$

18. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$\begin{aligned} C_{mn} = -\Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \cdot \\ [\langle \sigma_{z\rho(i,j,k+1/2)} \rangle \frac{1}{4} \left(\frac{1}{\rho_i - \rho_{i-1}} - \frac{1}{\rho_{i+1} - \rho_i} \right) \\ - \frac{1}{\Delta z} (\langle \sigma_{zz(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{zz(i,j,k+1/2)} \rangle)] \end{aligned} \tag{E-68}$$

19. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + I + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4(\rho_{i+1} - \rho_i)} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k+1/2)} \rangle \tag{E-69}$$

20. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\varphi(i,j,k+1/2)} \rangle \tag{E-70}$$

21. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\varphi(i,j,k+1/2)} \rangle \qquad \text{(E-71)}$$

22. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-2) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4(\rho_i - \rho_{i-1})} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k+1/2)} \rangle \qquad \text{(E-72)}$$

23. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\Delta\varphi \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \cdot \left[\langle \sigma_{z\rho(i,j,k+1/2)} \rangle \frac{1}{4} \left(\frac{1}{\rho_i - \rho_{i-1}} - \frac{1}{\rho_{i+1} - \rho_i} \right) + \frac{1}{\Delta z} (\langle \sigma_{zz(i,j,k+1/2)} \rangle - (i\omega) \langle \epsilon_{zz(i,j,k+1/2)} \rangle) \right] \qquad \text{(E-73)}$$

24. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4(\rho_{i+1} - \rho_i)} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k+1/2)} \rangle \qquad \text{(E-74)}$$

25. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$C_{mn} = -\frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\varphi(i,j,k+1/2)} \rangle \qquad \text{(E-75)}$$

– Para $m = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1) N_\varphi$

onde $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$

1. Para $n = (i-1) + (j-1) N_\rho + (k-3) N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k-1/2)} \rangle \qquad \text{(E-76)}$$

2. Para $n = i + (j-1)N_\rho + (k-3)N_\rho N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle \tag{E-77}$$

3. Para $n = (i-1) + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \tag{E-78}$$

4. Para $n = i + (j-2)N_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \tag{E-79}$$

5. Para $n = (i-1) + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \\ &+ \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \\ &- \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \\ &+ \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle \\ &+ \rho_{i+1/2}\Delta\varphi\Delta z(\langle\sigma_{\rho\rho(i-1/2,j,k)}\rangle - (i\omega)\langle\epsilon_{\rho\rho(i-1/2,j,k)}\rangle) \end{aligned} \tag{E-80}$$

6. Para $n = i + (j-1)N_\rho + (k-2)N_\rho N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \\ &+ \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \\ &- \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \\ &+ \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle \\ &- \rho_{i+1/2}\Delta\varphi\Delta z(\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle - (i\omega)\langle\epsilon_{\rho\rho(i+1/2,j,k)}\rangle) \end{aligned} \tag{E-81}$$

7. Para $n = (i-1) + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{E-82}$$

8. Para $n = i + jN_\rho + (k-2)N_\rho N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle \tag{E-83}$$

9. Para $n = (i-1) + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \tag{E-84}$$

10. Para $n = i + (j-1)N_\rho + (k-1)N_\rho N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \tag{E-85}$$

11. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \tag{E-86}$$

12. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \tag{E-87}$$

13. Para $n = (N_\rho N_\varphi(N_z-1))+(i-2)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle \tag{E-88}$$

14. Para $n = (N_\rho N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle + \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle \\ &- \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \\ &+ \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \\ &+ (\rho_{i+1/2} - \rho_{i-1/2})\Delta z(\langle\sigma_{\varphi\varphi(i,j+1/2,k)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i,j+1/2,k)}\rangle) \end{aligned} \tag{E-89}$$

15. Para $n = (N_\rho N_\varphi(N_z-1)) + i + (j-2)(N_\rho-1) + (k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-90}$$

16. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-2) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle \tag{E-91}$$

17. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle + \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle \\ &- \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \\ &+ \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \\ &- (\rho_{i+1/2} - \rho_{i-1/2})\Delta z(\langle\sigma_{\varphi\varphi(i,j+1/2,k)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i,j+1/2,k)}\rangle) \end{aligned} \tag{E-92}$$

18. Para $n = (N_\rho N_\varphi (N_z - 1)) + i + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle \tag{E-93}$$

19. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \tag{E-94}$$

20. Para $n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \tag{E-95}$$

21. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-2)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j-1/2,k)}\rangle \tag{E-96}$$

22. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-2) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle \tag{E-97}$$

23. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1)N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle + \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle$$
$$-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle+\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j-1/2,k)}\rangle$$
$$+\Delta\varphi\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{zz(i,j,k+1/2)}\rangle-(i\omega)\langle\epsilon_{zz(i,j,k+1/2)}\rangle\right) \tag{E-98}$$

24. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+i+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-99}$$

25. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+(i-1)+j(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \tag{E-100}$$

26. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+(i-1)+(j-2)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j-1/2,k)}\rangle \tag{E-101}$$

27. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+(i-2)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle \tag{E-102}$$

28. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+(i-1)+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle + \frac{\rho_{i-1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle$$
$$-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle+\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j-1/2,k)}\rangle$$
$$-\Delta\varphi\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{zz(i,j,k+1/2)}\rangle-(i\omega)\langle\epsilon_{zz(i,j,k+1/2)}\rangle\right) \tag{E-103}$$

29. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+i+(j-1)(N_\rho-1)+(k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \tag{E-104}$$

30. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \tag{E-105}$$

31. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + (i-1) + (j-2)(N_\rho-1) + (k-3)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\langle\sigma_{\varphi z(i,j-1/2,k)}\rangle \\ &- \frac{1}{4\rho_i}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \end{aligned} \tag{E-106}$$

32. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + (i-2) + (j-1)(N_\rho-1) + (k-3)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\rho_{i-1/2}\Delta\varphi}{4}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle \\ &- \frac{\Delta\varphi}{4(\rho_i - \rho_{i-1})}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle \end{aligned} \tag{E-107}$$

33. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + (i-1) + (j-1)(N_\rho-1) + (k-3)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = &\frac{\Delta\varphi}{4}\left(\rho_{i+1/2}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle - \rho_{i-1/2}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle\right) \\ &+ \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\left(\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle - \langle\sigma_{\varphi z(i,j-1/2,k)}\rangle\right) \\ &-\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle\left(\frac{1}{(\rho_{i+1} - \rho_i)} - \frac{1}{(\rho_i - \rho_{i-1})}\right) \\ &-\frac{\Delta\varphi}{\Delta z}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{zz(i,j,k-1/2)}\rangle - (i\omega)\langle\epsilon_{zz(i,j,k-1/2)}\rangle\right) \end{aligned} \tag{E-108}$$

34. Para $n = (N_\rho N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi(N_z-1)) + ((N_\rho-1)N_\varphi N_z) + i + (j-1)(N_\rho-1) + (k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta\varphi}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle + \frac{\Delta\varphi}{4(\rho_{i+1}-\rho_i)}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle \quad \text{(E-109)}$$

35. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+j(N_\rho-1)+(k-3)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2}-\rho_{i-1/2})}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle + \frac{1}{4\rho_i}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k-1/2)}\rangle \quad \text{(E-110)}$$

36. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-2)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i-1/2}\Delta z}{4\rho_{i-1}}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle - \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4(\rho_i-\rho_{i-1})}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \quad \text{(E-111)}$$

37. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$\begin{aligned} C_{mn} = &-\frac{\Delta z}{4\rho_i}\left(\rho_{i-1/2}\langle\sigma_{\rho\varphi(i-1/2,j,k)}\rangle - \rho_{i+1/2}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle\right) \\ &- \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle\left(\frac{1}{(\rho_{i+1}-\rho_i)}-\frac{1}{(\rho_i-\rho_{i-1})}\right) \\ &+ \frac{1}{4\rho_i}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle - \langle\sigma_{z\varphi(i,j,k-1/2)}\rangle\right) \\ &- \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{\rho_i\Delta\varphi}\left(\langle\sigma_{\varphi\varphi(i,j-1/2,k)}\rangle - (i\omega)\langle\epsilon_{\varphi\varphi(i,j-1/2,k)}\rangle\right) \end{aligned} \quad \text{(E-112)}$$

38. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+i+(j-2)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn} = \frac{\rho_{i+1/2}\Delta z}{4\rho_{i+1}}\langle\sigma_{\rho\varphi(i+1/2,j,k)}\rangle + \frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4(\rho_{i+1}-\rho_i)}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle \quad \text{(E-113)}$$

39. Para $n = (N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+$

$$(i-2)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$$

$$C_{mn}=\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4(\rho_i-\rho_{i-1})}(\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle-\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle)$$
$$+\frac{\Delta\varphi}{4(\rho_i-\rho_{i-1})}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)(\langle\sigma_{z\rho(i,j,k+1/2)}\rangle-\langle\sigma_{z\rho(i,j,k-1/2)}\rangle)$$
$$-\frac{\rho_{i-1/2}\Delta\varphi\Delta z}{(\rho_i-\rho_{i-1})}(\langle\sigma_{\rho\rho(i-1/2,j,k)}\rangle-(i\omega)\langle\epsilon_{\rho\rho(i-1/2,j,k)}\rangle) \quad \text{(E-114)}$$

40. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+(i-1)+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn}=-\Delta\varphi\Delta z\left(\frac{\rho_{i+1/2}}{(\rho_{i+1}-\rho_i)}\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle+\frac{\rho_{i-1/2}}{(\rho_i-\rho_{i-1})}\langle\sigma_{\rho\rho(i-1/2,j,k)}\rangle\right)$$
$$+\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle\left(\frac{1}{(\rho_{i+1}-\rho_i)}-\frac{1}{(\rho_i-\rho_{i-1})}\right)$$
$$+\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4}\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle\left(\frac{1}{(\rho_i-\rho_{i-1})}-\frac{1}{(\rho_{i+1}-\rho_i)}\right)$$
$$+\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{\rho_i\Delta\varphi}(\langle\sigma_{\varphi\varphi(i,j+1/2,k)}\rangle+\langle\sigma_{\varphi\varphi(i,j-1/2,k)}\rangle)$$
$$+\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle\left(\frac{1}{(\rho_{i+1}-\rho_i)}-\frac{1}{(\rho_i-\rho_{i-1})}\right)$$
$$+\frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k-1/2)}\rangle\left(\frac{1}{(\rho_i-\rho_{i-1})}-\frac{1}{(\rho_{i+1}-\rho_i)}\right)$$
$$+\frac{\Delta\varphi}{\Delta z}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{zz(i,j,k+1/2)}\rangle+\langle\sigma_{zz(i,j,k-1/2)}\rangle\right)$$
$$-(i\omega)\Delta\varphi\Delta z\left(\frac{\rho_{i+1/2}}{(\rho_{i+1}-\rho_i)}\langle\epsilon_{\rho\rho(i+1/2,j,k)}\rangle+\frac{\rho_{i-1/2}}{(\rho_i-\rho_{i-1})}\langle\epsilon_{\rho\rho(i-1/2,j,k)}\rangle\right)$$
$$-(i\omega)\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{\rho_i\Delta\varphi}(\langle\epsilon_{\varphi\varphi(i,j+1/2,k)}\rangle+\langle\epsilon_{\varphi\varphi(i,j-1/2,k)}\rangle)$$
$$-(i\omega)\frac{\Delta\varphi}{\Delta z}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)\left(\langle\epsilon_{zz(i,j,k+1/2)}\rangle+\langle\epsilon_{zz(i,j,k-1/2)}\rangle\right) \quad \text{(E-115)}$$

41. Para $n=(N_\rho N_\varphi(N_z-1))+((N_\rho-1)N_\varphi(N_z-1))+((N_\rho-1)N_\varphi N_z)+i+(j-1)(N_\rho-1)+(k-2)(N_\rho-1)N_\varphi$

$$C_{mn}=-\frac{(\rho_{i+1/2}-\rho_{i-1/2})\Delta z}{4(\rho_{i+1}-\rho_i)}(\langle\sigma_{\varphi\rho(i,j+1/2,k)}\rangle-\langle\sigma_{\varphi\rho(i,j-1/2,k)}\rangle)$$
$$-\frac{\Delta\varphi}{4(\rho_{i+1}-\rho_i)}\left(\frac{(\rho_{i+1/2})^2-(\rho_{i-1/2})^2}{2}\right)(\langle\sigma_{z\rho(i,j,k+1/2)}\rangle-\langle\sigma_{\varphi\rho(i,j,k-1/2)}\rangle)$$
$$-\frac{\rho_{i+1/2}\Delta\varphi\Delta z}{(\rho_{i+1}-\rho_i)}(\langle\sigma_{\rho\rho(i+1/2,j,k)}\rangle-(i\omega)\langle\epsilon_{\rho\rho(i+1/2,j,k)}\rangle) \quad \text{(E-116)}$$

42. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-2) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta z}{4\rho_{i-1}} \langle \sigma_{\rho\varphi(i-1/2,j,k)} \rangle + \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_i - \rho_{i-1})} \langle \sigma_{\varphi\rho(i,j+1/2,k)} \rangle \quad \text{(E-117)}$$

43. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$\begin{aligned} C_{mn} = {} & \frac{\Delta z}{4\rho_i} \left(\rho_{i-1/2} \langle \sigma_{\rho\varphi(i-1/2,j,k)} \rangle - \rho_{i+1/2} \langle \sigma_{\rho\varphi(i+1/2,j,k)} \rangle \right) \\ & + \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4} \langle \sigma_{\varphi\rho(i,j+1/2,k)} \rangle \left(\frac{1}{(\rho_{i+1} - \rho_i)} - \frac{1}{(\rho_i - \rho_{i-1})} \right) \\ & - \frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \left(\langle \sigma_{z\varphi(i,j,k+1/2)} \rangle - \langle \sigma_{z\varphi(i,j,k-1/2)} \rangle \right) \\ & - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{\rho_i \Delta\varphi} \left(\langle \sigma_{\varphi\varphi(i,j+1/2,k)} \rangle - (i\omega) \langle \epsilon_{\varphi\varphi(i,j+1/2,k)} \rangle \right) \end{aligned} \quad \text{(E-118)}$$

44. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + I + j(N_\rho - 1) + (k-2)(N_\rho - 1)N_\varphi$

$$C_{mn} = -\frac{\rho_{i+1/2}\Delta z}{4\rho_{i+1}} \langle \sigma_{\rho\varphi(i+1/2,j,k)} \rangle - \frac{(\rho_{i+1/2} - \rho_{i-1/2})\Delta z}{4(\rho_{i+1} - \rho_i)} \langle \sigma_{\varphi\rho(i,j+1/2,k)} \rangle \quad \text{(E-119)}$$

45. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-2)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4} \langle \sigma_{\varphi z(i,j-1/2,k)} \rangle + \frac{1}{4\rho_i} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\varphi(i,j,k+1/2)} \rangle \quad \text{(E-120)}$$

46. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-2) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1)N_\varphi$

$$C_{mn} = \frac{\rho_{i-1/2}\Delta\varphi}{4} \langle \sigma_{\rho z(i-1/2,j,k)} \rangle + \frac{\Delta\varphi}{4(\rho_i - \rho_{i-1})} \left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2} \right) \langle \sigma_{z\rho(i,j,k+1/2)} \rangle \quad \text{(E-121)}$$

47. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$\begin{aligned} C_{mn} = & -\frac{\Delta\varphi}{4}\left(\rho_{i+1/2}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle - \rho_{i-1/2}\langle\sigma_{\rho z(i-1/2,j,k)}\rangle\right) \\ & - \frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\left(\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle - \langle\sigma_{\varphi z(i,j-1/2,k)}\rangle\right) \\ & + \frac{\Delta\varphi}{4}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle\left(\frac{1}{(\rho_{i+1} - \rho_i)} - \frac{1}{(\rho_i - \rho_{i-1})}\right) \\ & - \frac{\Delta\varphi}{\Delta z}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\left(\langle\sigma_{zz(i,j,k+1/2)}\rangle - (i\omega)\langle\epsilon_{zz(i,j,k+1/2)}\rangle\right) \end{aligned} \tag{E-122}$$

48. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + i + (j-1)(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$\begin{aligned} C_{mn} = & -\frac{\rho_{i+1/2}\Delta\varphi}{4}\langle\sigma_{\rho z(i+1/2,j,k)}\rangle \\ & - \frac{\Delta\varphi}{4(\rho_{i+1} - \rho_i)}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\rho(i,j,k+1/2)}\rangle \end{aligned} \tag{E-123}$$

49. Para $n = (N_\rho N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi (N_z - 1)) + ((N_\rho - 1) N_\varphi N_z) + (i-1) + j(N_\rho - 1) + (k-1)(N_\rho - 1) N_\varphi$

$$\begin{aligned} C_{mn} = & -\frac{(\rho_{i+1/2} - \rho_{i-1/2})}{4}\langle\sigma_{\varphi z(i,j+1/2,k)}\rangle \\ & - \frac{1}{4\rho_i}\left(\frac{(\rho_{i+1/2})^2 - (\rho_{i-1/2})^2}{2}\right)\langle\sigma_{z\varphi(i,j,k+1/2)}\rangle \end{aligned} \tag{E-124}$$

Os elementos B_n são dados por:

$$B_n = \begin{cases} I_0, \text{ se } n = (N_\rho N_\varphi (N_z - 1)) + (i-1) + (j-1)(N_\rho - 1) \\ \qquad\qquad + (k-2)(N_\rho - 1) N_\varphi; \\ 0, \text{ caso contrário.} \end{cases}$$

onde I_0 é a amplitude da corrente de excitação. $i = 2, \cdots, N_\rho; j = 1, \cdots, N_\varphi; k = 2, \cdots, N_z$