Pontifícia Universidade Católica
do Rio de Janeiro

Bernardo Silva de Carvalho Ribeiro

Does structural change lead to inequality change? A macroeconomic approach

Dissertação de Mestrado

Thesis presented to the Programa de Pós–graduação em Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia .

Advisor : Prof. Eduardo Zilberman
Co-advisor: Prof. Tiago Couto Berriel

Rio de Janeiro
April 2018

Pontifícia Universidade Católica do Rio de Janeiro

Bernardo Silva de Carvalho Ribeiro

Does structural change lead to inequality change? A macroeconomic approach

Thesis presented to the Programa de Pós–graduação em Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia . Approved by the undersigned Examination Committee.

Prof. Eduardo Zilberman
Advisor
Departamento de Economia – PUC-Rio

Prof. Tiago Couto Berriel
Co-advisor
Departamento de Economia – PUC-Rio

Prof. Carlos Viana de Carvalho
Departamento de Economia – PUC-Rio

Prof. Pedro Cavalcanti Ferreira
Escola de Pós-Graduação em Economia – EPGE-FGV

Prof. Augusto Cesar Pinheiro da Silva
Vice Dean of the Centro de Ciências Sociais – PUC-Rio

Rio de Janeiro, April the 9th, 2018

Bernardo Silva de Carvalho Ribeiro

B.A., Economics, Universidade Federal de Minas Gerais (Belo Horizonte, Brazil), 2015.

Bibliographic data

Ribeiro, Bernardo Silva de Carvalho

Does structural change lead to inequality change? A macroeconomic approach / Bernardo Silva de Carvalho Ribeiro; advisor: Eduardo Zilberman; co-advisor: Tiago Couto Berriel. – Rio de janeiro: PUC-Rio , Departamento de Economia, 2018.

v., 75 f: il. color. ; 30 cm

Dissertação (mestrado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia.

Inclui bibliografia

1. Economia – Teses. 2. Macroeconomia – Teses. 3. Transformação estrutural;. 4. Desigualdade;. 5. Preço relativo entre consumo e investimento;. 6. Consumo de subsistência;. 7. Mobilidade de trabalho;.
I. Zilberman, Eduardo. II. Berriel, Tiago. III. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. IV. Título.

CDD: 330

Acknowledgments

I thank God for showing me the way, even in the darkest times. I thank my father Rodrigo, my mother Adriana and my sister Luísa for becoming day after day part of my way. I thank Alice, who give me the strength to not fear the way, becoming the best partner in the journey. And I thank my grandmother Dilma, who taught me how to start everything.

I am grateful to Eduardo Zilberman and Tiago Berriel for their guidance and, most of all, for the excitement with which they supported this work. Nothing would be possible without their skills and understanding. I am indebted to Pedro Cavalcanti, to Carlos Viana and to Andre Brunelli for their valuable suggestions. Also, my colleagues Rodolfo Rigato, Guilherme Neves, Alvaro Cox and Alex Carrasco were always present with ideas and tips for this work.

I thank Márcio Garcia and Diogo Guillén for believing in my projects. They were crucial for my learning and growing process throughout the last two years. The same I can say in regard to my supportive and inspiring class.

To conclude, I thank CNPq and PUC-Rio for all the financial and administrative support.

Abstract

Ribeiro, Bernardo Silva de Carvalho; Zilberman, Eduardo (Advisor); Berriel, Tiago (Co-Advisor). **Does structural change lead to inequality change? A macroeconomic approach**. Rio de Janeiro, 2018. 75p. Dissertação de mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

While the structural change literature has been mainly focused on explaining the Kuznets Facts - a set of regularities concerning sectoral dynamics throughout economic growth - important issues were left apart. Inequality was one of them: Simon Kuznets, the father of this literature, when making some of the first documentation of structural change patterns, repetitively expressed his concern that inequality and sector reallocation were linked. In this regard, we seek to extend the benchmark model of structural change to introduce wealth and income distribution. We allow idiosyncratic risk and incomplete markets in a two-sector environment of growth. In a quantitative exercise, a secular transition from a poor and good's producer economy to a richer and service-based one is conducted. The model can account for an inverse U-shaped path for inequality as growth takes place. Our contribution is to suggest how a time-varying relative price of consumption and investment - yielded by the model's multi-sector structure - plays a role in the inequality behavior. We also show that the subsistence consumption requirement, typical of structural change setups, can influence distributional variables. The model is extended with a restriction over workers' capacity to move across sectors - and we show that it amplifies the inverse U-shaped path followed by the income and wealth Gini. Finally, the model is calibrated for the US economy (1950-2000), yielding qualitative and quantitative evidence of the effect on inequality from both mechanisms presented above (relative prices and subsistence consumption). Quantitative strength, however, is in some cases limited.

Keywords

Structural change; Inequality; Relative price of consumption and investment; Subsistence consumption; Labor mobility costs;

Resumo

Ribeiro, Bernardo Silva de Carvalho; Zilberman, Eduardo; Berriel, Tiago. **Transformação estrutural impacta a desigualdade? Uma abordagem macroeconômica**. Rio de Janeiro, 2018. 75p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

À medida que a literatura de transformação estrutural esteve focada em explicar os *Kuznets facts* - um conjunto de regularidades empíricas apresentado pela dinâmica dos setores de uma economia - questões importantes ficaram à margem. A desigualde foi uma delas: Simon Kuznets, o pai dessa literatura, repetitivamente defendeu que desigualdade e dinâmica setorial eram relacionadas. Nesse sentido, nosso objetivo é extender o modelo canônico de trasformação estrutual de forma a introduzir distribuição de renda e riqueza entre indivíduos. Permitimos que haja risco idiossincrático e mercados incompletos em um ambiente de crescimento com dois setores. Em um exercício quantitativo, é conduzida uma transição secular entre uma economia pobre baseada em bens para uma economia rica intensiva em serviços - ao longo da qual o modelo gera uma curva em U invertido para a trajetória da desigualdade. Nossa contribuição é sugerir como o preço relativo entre consumo e investimento (que varia no tempo e decorre da estrutura multi-setorial do modelo) tem um papel importante no comportamento das medidas distributivas. Também mostramos que o consumo de subsistência, típico de ambientes de transformação estrutural, pode influenciar a desigualdade. Na sequência, o modelo é extendido com uma restrição sobre a capacidade dos trabalhadores de se moverem entre os setores - e mostramos como essa fricção pode amplificar o o formato em U invertido seguido pelo Gini de renda e riqueza. Por fim, nossa economia é calibrada para os Estados Unidos (1950-2000), gerando evidências qualitativas e quantitativas do efeito sobre a desigualdade de ambos os efeitos acima (preço relativo e consumo de subsistência). A evidência quantitativa, porém, é em certos casos limitada.

Palavras-chave

Transformação estrutural; Desigualdade; Preço relativo entre consumo e investimento; Consumo de subsistência; Mobilidade de trabalho;

Table of contents

List of figures

List of tables

1
Introduction

Structural transformation has been an important issue in the growth literature over the last decades. While research and effort were allocated to understand the patterns pointed out by Kaldor (20) regarding the stability, over time, of variables such as the output-capital ratio and the growth rate, another concern also gained weight in the profession: how to explain the unbalanced and not stable configuration of sectoral shares in an economy throughout the growth process.

According to (19), structural transformation refers to this reallocation of economic activity across the broad sectors of agriculture, manufacture, and services, during the process of modern economic growth. It is also possible to generalize the definition to an environment of a k-sector economy, depending on the goals one has in mind. For instance, a two-sector economy (goods and services, agriculture and non-agriculture goods) is a rather common structure as well.

In a worldwide panorama, evidence presented by the literature suggests the existence of stylized facts regarding this sectoral dynamics: as the income level grows, agriculture share on GDP declines monotonically, services go in the opposite direction, increasing considerably, and manufacture, although tending to loose space in the highest levels of income, may grow before, showing a hump-shaped pattern. If the economy is instead divided into goods and services (or into agriculture and non-agriculture goods), a clear pattern emerges, showing how the growth in the income level leads to a monotonic rise in the latter sector and to a decrease in the former, in terms of GDP shares.

Given the regularity and the status of stylized facts achieved by the empirical observations above, the literature - starting with (22) - uses the term *Kuznets facts* to label them. This is because, in earlier times, Kuznets ((25), (26), (27)) directly and indirectly documented these patterns, putting the issue in the spotlight. Since then, and mostly in the last two decades, as growth researchers started to increasingly explain not only the *Kaldor facts*, but also the *Kuznets facts*, the structural change literature thrived.

When looking into Kuznets papers ((24), (25), (26)) for early documentation of structural change, one would probably notice, as a repetitive point made by the author, the link between sectoral reallocation (and growth) with inequality dynamics. In what he called the industrial structure of the income distribution, the labor force movement from agriculture to non-agriculture activities was an important cause underlying the rise in inequality during the process of growth acceleration.

With a growing share of the population becoming settled in the manufacture and good's production, which were activities characterized by higher income dispersion, it was a question of arithmetic to comprehend how inequality would soar, as advocated by Kuznets (25). Once the growth process moved forward, however, and higher levels of income were achieved, the initial rise in inequality would be substituted by a downward trend. In short, an inverse U-shaped curve would be displayed ((24), (25)).

Indeed, (30), in an empirical analysis, found that the structural change hypothesis proposed by Kuznets is supported by US data between 1919 and 2002. As the employment in manufacture and agriculture falls, inequality rises in the long run, in accordance with his estimations.

Even though the goal here is not to consider in detail Kuznets' thesis themselves, we must underscore that, in important papers of this seminal author, just attached to the descriptions of the later called *Kuznets facts*, there were often inequality issues. Therefore, exactly where the growth literature found the motivation to insert a multi-sector structure into a growth model, we find the motivation to introduce inequality measures in a model of growth with distinct sectors. We ask: does structural change matter for inequality? Is there another long run trend (now concerning inequality) that may also be clarified with the help of a multi-sector growth model?

To answer the questions, in this paper, we bring two frameworks together: a structural change environment and a heterogeneous agents model with incomplete markets. We do it by borrowing from benchmark setups: on the one hand, we build over (19)'s model of sectoral reallocation and, on the other hand, over Aiyagari's seminal economy (3).

Starting from benchmark models (i.e, from workhorse setups) allows us to answer questions based on a general context (and not on a specific environment restricted by a set of particular hypotheses). Just as the structural change literature seeks to introduce the *Kuznets facts* in a standard and simple model of growth, we want to analyze inequality in an environment as close as possible to the workhorse multi-sector growth model.

We present a continuous time model inhabited by a set of agents with fixed unitary mass. Agents are subject to an idiosyncratic Poisson process affecting their productivity level - which allows us to gain the tractability and numerical efficiency as in (2). They inelastically supply labor and derive utility from the consumption of two imperfectly substitute goods, which we will label as *goods* and *services*[1]. There is a minimum subsistence level below which goods consumption cannot fall, and a natural endowment of services. Firms in each sector produce according to constant returns to scale using labor and capital hired from households. Capital good - the only asset in which agents can save on - is produced with a combination of goods and services.

We then undertake, as a quantitative exercise with a standard calibration, a secular transition from a poor economy, in which good's production prevail, to a richer one, with a dominant service sector. Throughout the transition, we allow two channels of structural change to act: income effects and relative price effects. Following the literature, we use nonhomothetic Stone-Geary preferences, which favor the demand for services as the income level grows. In addition, we allow for distinct technical progress across sectors - leading to relative price changes, affecting the spending shares of each good.

Results indicate a linkage between the paths of structural change and of inequality. While it is widely known that the presence of different sectors and distinct technical growth rates among them can yield the stylized facts of sectoral reallocation, the main contribution of this paper is to indicate how these same factors can play a role in the inequality long-run dynamics.

At the heart of this mechanism, we have the existence of a time-varying relative price of composite consumption and capital good. Since consumption and capital are both made from goods and services, but not necessarily in the same proportions (given that preferences and technology - as always - are independent), consumption and capital are not exchanged in a one for one basis. And since each sector (goods and services) has its own growth rate, not only are prices of capital and consumption different, but they also change differently along time. Therefore, the trade-off between savings and spendings is time-varying, affecting the optimal path of consumption and capital accumulation in the transition (and hence the interest rate path as well).

Our baseline calibration, consistently with data, yields a falling relative price of capital in comparison to consumption. The farther we go in time, the cheaper becomes the investment, and the more expensive (relatively speaking) it is to consume. It creates an additional desire among agents to anticipate consumption - contributing to a smaller capital accumulation, and to a rise in interest rates. The higher interest rates become, the more the richest save in

[1]The model can be easily extended to a k-sector economy, although no insight or a new result would be achieved for our purposes in this paper.

proportion to the poorest households - and hence, the more inequality soars.

Saving policy functions, which were negatively sloped during the transition beginning (in the assets *vs* savings space), display a positive inclination when interest rates peak. With the subsequent rise in capital accumulation, interest rates fall, saving functions become negatively sloped again, and capital accumulation follows ahead with a decreasing trend in inequality. After all, the transition displays a Kuznets curve.

Not only can structural change's driving forces influence this process by means of the relative price of capital and consumption, but also by means of the subsistence requirement of goods. For instance, a fall in this requirement in the transition beginning can augment utility without an extra need of spendings, lowering household's willingness to deccumulate assets for consumption smooth purposes. It yields a lower rise in interest rates and in inequality measures.

We also expand the model with a labor market friction which aims to capture mobility costs across sectors. Workers cannot more switch freely from one sector to the other, despite any wage differential that may exist. Opportunities to do so follow a Poisson process.

Results show that the friction becomes active after a growth shock which creates pressure for significant sectoral reallocation. The instantaneous initial transfer of labor between sectors (and hence, the instantaneous initial structural change) is spread out in the following years - or even decades. The first outcome is a gap in wages, which can lead income inequality to become significantly higher in comparison to the frictionless model. Also, as the final steady state stills the same but the economy gets initially worse due to the friction, the desire to smooth consumption becomes stronger. Interest rates soar abruptly, triggering a rise in wealth inequality.

We finally calibrate the model for the US economy and observe that there is qualitative and quantitative evidence for the main mechanisms presented above. Quantitative strength, however, is in some cases limited. We provide evidence that if the subsistence requirement of goods was lowered, in 1950, to its real value of 2000 (and remained fixed), the top 10 wealth share would be lower throughout the $1950-2000$ period. If the relative price of goods and services were constant, inequality would be lower in the 1960's and higher after the 1970s.

The paper is organized as follows. In the subsection below, we present the related literature. In Chapter 2 the main model is built and quantitative exercises are conducted. Chapter 3 extends our environment with the labor market friction and Chapter 4 presents the calibrated model. The final chapter concludes.

1.1 Related Literature

Other authors have studied inequality issues in an environment of structural change. Their focus has been mostly placed on wage inequality, and their models, generally speaking, greatly depart from the core models of this literature[2], in order to gain complexity. We claim, however, that the standard multi-sector growth model already allows one to draw conclusions concerning the inequality path through time, without the need for further hypotheses. It turns out, also, that the dynamics of savings has a first-order importance for these results, which cannot be considered by models focused on wage inequality only.

In this regard, (28) consider wage inequality by meticulously modeling labor markets in a two-sector economy. The authors do not include savings behavior and wealth accumulation, which rules out any kind of wealth inequality measure. (7), as well, develop a model where high-skilled specialized workers have a comparative advantage in the production of more complex goods, which are associated with the service sector. When technical progress shifts demand towards services, the model accounts not only for structural change but also for the rise in the skill premium and in wage inequality. Given the lack of capital and savings into the model, the paper incorporates neither wealth dispersion, nor capital income inequality, though.

In addition to that, (7)'s model is also an example of how our analysis differs from the literature by its use of a benchmark model of structural change. In (7), wage dispersion emerges in a specific framework, with different technologies for each sector, with satiable preferences, with assumptions regarding the comparative advantages of each kind of worker in the continuum of goods, and with other hypotheses not found in the benchmark models of neoclassical growth and structural change.

The same can be said of models whose environments are described as *a dual economy*, where one sector, normally labeled as the modern one, faces higher returns to scale, spillover effects, complementarities with high-skilled labor, or any other advantages in comparison to the traditional one. Income inequality arises naturally between workers from both sectors. (29) and (8) are examples in this regard.

The traditional literature of the Kuznets curve is also an example of the dual economy approach. In the seminal paper (25), Kuznets, as mentioned

[2]By core (or benchmark) models of structural transformation, we refer to the widely used setups consisting of a neoclassical growth framework extended with different sectors whose production functions differs only in the levels of TFP. Non-homothetic preferences may appear too. For some seminal examples, see (19), (12), (22) and (31).

before, linked inequality and sectoral dynamics by means of an environment where a fast-growing sector, with wages both higher and more volatile than the traditional sector, expands its shares of employment, creating a gap between workers within it - and between its workers and the ones in the traditional sector. Although in Chapter 3 we insert a labor market friction and observe our model behavior in this context, our main mechanism to link inequality and structural change comes from the behaviour of savings and interests.

Moving on, a recent influential paper that features a distribution of agents in a structural change model is (6). The paper aims to conciliate Kaldor and Kuznets facts and also to explain another empirical regularity: poor household spent a larger fraction of their wealth in the consumption of goods. In his model, agents are heterogeneous in the initial distribution of assets, there are complete markets and the growth of consumption expenditures is always equal for every agent. What is important to highlight is that the interaction of inequality and structural change is only approached to show that poor households consume proportionally more goods: there is not any consideration in the other direction, i.e, if structural change can influence inequality, which is the question of this work.

It is important to consider the literature of growth and inequality since our structural change setup is a multi-sector extension of this setup. Chatterjee has important works to which our paper is connected. In (9) and (10), the authors studied a neoclassical growth model with an unequal distribution of assets. They found that the structure of preferences and the growth rate can influence the inequality path along a transition to the balanced growth steady state. The relationship between savings propensity and the number of assets held by individuals determines if inequality is soaring or decreasing throughout growth.

An important result in (9) is that the existence of a minimum consumption necessity makes this relationship upward monotonic: the intertemporal elasticity of substitution (IES) from the poorer individuals becomes lower than the rich ones, and hence the former group saves relatively less, yielding a rising trend in inequality. In a similar setup and using the same intuition, (11) showed how a growth model with subsistence requirements can explain, qualitatively and quantitatively in US data, an inverse U-shaped movement in inequality. In the transition beginning, the IES of the poorer households is lower due to the minimum consumption requirement - and inequality soars. While the real value of the subsistence depreciates, the IES of the poor rises and inequality trend turns down.

We use the core intuitions of (9), (10) and (11) to analyze the inequality dynamics in our model, i.e, the use of savings behavior in the cross-section of households. While papers of structural change generally include inequality by means of the labor market, these papers are closer to ours in the sense that trends in wealth and income disparities are essentially connected to savings and interests dynamics. However, our underlying mechanism is not the same. The subsistence requirement does not show up in the intertemporal part of the model, and it hence does not play a role here as in (9), (10) and (11). We explore how consumption smooth, idiosyncratic risk and sectoral dynamics can influence savings' and interests' behavior, producing long-run effects over inequality.

Finally, as the relative price between consumption and capital plays an important role in our environment, we must mention the seminal paper from (16), in which a great quantitative role in the post-war US growth was explained by technical progress specific to the investment sector (which, therefore, has changed continuously the relative prices of consumption and capital). Although this work and the related models which followed didn't introduce inequality[3], they give quantitative and theoretical weight to the main mechanism underlying our conclusions.

[3]An exception was (23), where the authors make use of the falling relative price of capital to explain capital stock expansion and the subsequent effects on skill premium. Once more, however, we have a model in which inequality only appears in the labor market within a technological bias environment.

2
The model

The continuous time economy which follows is made of two sectors, goods and services, and a continuum of households potentially heterogeneous in their units of efficient labor supplied. The structural change part of the model lies within the benchmark framework detailed by (19). Heterogeneity and incomplete markets components borrow from the workhorse model of (3) and from some of the efficient tools presented by (2).

2.1
Households

The economic environment consists of a continuum of households with fixed unitary mass. Households consume, inelastically supply labor and save on a risk-free asset (a_t). For each household j, its labor efficiency units, z_j, follows a Poisson process with two possible levels $z_j \in \{z_1, z_2\}$. We have that $z_2 > z_1$ and that z_j jumps from z_1 to z_2 with intensity λ_1 (and vice versa with intensity λ_2).

Each household solves the following problem (we omit the subscript j from now on as always as possible):

$$\begin{aligned}
&\max_{\{C_{g,t},C_{s,t},a_t\}_{t=0}^{\infty}} u(C_t) = \int_0^{\infty} e^{-\rho t}\frac{C_t^{1-\sigma}}{1-\sigma}dt \\
&\text{s.t. } \dot{a}_t = w_t z_t + a_t r_t - p_{s,t}C_{s,t} - p_{m,t}C_{m,t} \qquad (2\text{-}1)\\
&\text{and } a_t \geq \underline{\text{a}}
\end{aligned}$$

where $C_{g,t}$ and $C_{s,t}$ are goods and services consumption - and C_t is their aggregation:

$$C_t \equiv [\omega_g^{\frac{1}{\epsilon}}(C_{g,t} - \bar{C}_g)^{\frac{\epsilon-1}{\epsilon}} + \omega_s^{\frac{1}{\epsilon}}(C_{s,t} + \bar{C}_s)^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}}$$

The non-homothetic *positive* terms $\bar{C}_g$ and $\bar{C}_s$ are responsible for a higher income elasticity in the service sector, which gives structural change a channel to happen. One can interpret $\bar{C}_g$ as a subsistence level above which $C_{g,t}$ must always be, and $\bar{C}_s$ as an endowment of services individuals naturally possess. Another channel for structural change can also be seen: the elasticity of substitution, governed by ϵ, which may lead to sectoral budget shares reallocation due to relative prices dynamics.

As usual, w_t, r_t, $p_{g,t}$ and $p_{s,t}$ denote labor and capital payments and prices, respectively. There is no aggregate uncertainty. Markets are incomplete in the sense that agents face an idiosyncratic risk but can only save (or borrow up to certain limit $\underline{a}$) on the risk free asset a_t.

2.2 Firms and technology

Each sector $i \in \{g, s\}$ is operated by a representative firm that hires capital, K_i, and a fraction of the aggregate labor $N \equiv \int_0^1 z_j dj$. Therefore, we are assuming that firms do only care about the homogeneous mass of efficiency units they hire, independently of the composition of this mass in terms of the two different types of workers.

Firms have labor augmenting technology (which is sector-specific) with constant returns to scale. They pay wages, w_t, interest rates R_t, and sell at price p_i. In short, for every time period, their problem is:

$$\max_{N_i, K_i} \ p_i(A_i N_i)^{1-\alpha} K_i^{\alpha} - R_t K_i - w_t N_i \tag{2-2}$$

To remain, at least in a first moment, the closest as possible of the benchmark model of structural change, equal capital shares in both sectors are assumed, i.e, the α above is not sector specific. In Chapter 3 we show how to extend this setup with distinct capital shares[1].

As in (36), we consider that the investment good, x_t, requires value added from both sectors, with a fraction ν_t coming from goods, and with $1 - \nu_t$ coming from services[2]. These fractions are part of the technology set - and not endogenously determined. A competitive retailer buys goods and services to deliver units of investment. Such units, when produced, immediately become part of the capital stock, which depreciates at a rate δ. The price charged by the competitive retailer is given by:

$$p_{x,t} = \nu_t p_{g,t} + (1 - \nu_t) p_{s,t}$$

[1]The simplification of equal capital shares in benchmark models of structural change, like (19) and (12), was dropped by (1), who concluded that the quantitative effect of this extension is smaller, not closer to the traditional income and relative prices effects.

[2]Although it is common practice in the literature to assume $\nu_t = 1$, we allow the investment good to depend on both goods and services motivated by (18). The authors showed that, while in the 1950s services indeed account for no more than 30% in value added of investment's spending, this share raised to around 50% in the 2000s (see Figure B.1).

From now on, consider the investment good as the numeraire: $p_{x,t} = 1$ for all t. Also, assume that technology variables are time dependent:

$$A_{i,t} = A_{i,0} \exp\{\gamma_{i,t} t\} \;\; i = g, s$$

$$\nu_t = \nu_0 \exp\{\gamma_{\nu,t} t\}$$

2.3 Model's properties

Since factors are freely mobile, firms first order conditions yield two main conclusions: capital labor ratio is the same in both sectors (being equal to the aggregate capital labor ratio), and relative prices are only function of technology:

$$\frac{K_i}{N_i} = \frac{\alpha}{1-\alpha}\frac{w}{R} = \frac{K}{N} \tag{2-3}$$

$$\frac{p_g}{p_s} = \left(\frac{A_{s,t}}{A_{g,t}}\right)^{1-\alpha} \tag{2-4}$$

Notice, as well, that the model aggregates in the production side. Defining aggregate production in units of the numeraire, $Y_t \equiv Y_{g,t} p_{g,t} + Y_{s,t} p_{s,t}$, and using firms first order conditions, it is possible to arrive at:

$$Y_t = p_{g,t}(A_{g,t} N_t)^{1-\alpha} K_t^{\alpha} = p_{s,t}(A_{s,t} N_t)^{1-\alpha} K_t^{\alpha} \tag{2-5}$$

It means that we can work with a single production function which uses aggregate capital and labor. Factor prices can hence be obtained as the marginal returns of this aggregate function. The productive side of the model, therefore, becomes embedded in an only equation.

Another result common to structural change literature that also enhances tractability is that households' problem can be broken into two independent stages: *dynamic* and *static*. To see this, first, consider the price level of the aggregate consumption - derived as usual from the consumer optimization.

$$P_t \equiv (\omega_g p_{g,t}^{1-\epsilon} + \omega_s p_{s,t}^{1-\epsilon})^{\frac{1}{1-\epsilon}}$$

With this definition, one can rewrite the household budget constraint, shown in 2-1, yielding:

$$\dot{a}_t = w_t z_t + a_t r_t - P_t C_t - p_{g,t}\bar{C}_g + p_{s,t}\bar{C}_s \tag{2-6}$$

It turns out that households' problem consists first of choosing a path for $\{C_t\}$, in accordance to 2-1, but now subject to the budget constraint given by 2-6. Services and goods' consumption no longer appear directly into the problem - and the non-homothetic terms do not distort any margin since they are taken as given. This is the *dynamic* part of the optimization. The *static* one happens later, when households choose, at every period of time and given sectoral prices, the optimal amount of goods and services to yield the previously chosen quantity of C_t.

2.4 Equilibrium

As the first step to characterize equilibrium, the model is rewritten in terms of detrended variables. For now, consider it as if we were restating the same model with transformed variables. In Subsection 2.4.1, we will point out the conditions under which this detrended version admits a steady state.

Consider $A_{lr,t} \equiv \exp\{\gamma_{lr} t\}$ a new variable, growing at the rate γ_{lr}, named the long run growth rate. In this regard, keep in mind the notation according to which, for a given variable x_t, its detrended value is denoted by $\hat{x}_t \equiv \frac{x_t}{A_{lr,t}}$, and also define a new discount rate $\tilde{\rho}$ such that $\tilde{\rho} \equiv \rho - \gamma_{lr}(1-\sigma)$.

The household's problem 2-1, with the budget constraint in terms of composite consumption 2-6, becomes:

$$\begin{aligned}
&\max_{\{\hat{c}_t;\hat{a}_t\}} \int_0^\infty e^{-\tilde{\rho}t} \frac{\hat{c}_t^{1-\sigma}}{1-\sigma} dt \\
&\text{s.t } \dot{\hat{a}}_t = \hat{w}_t z_t + \hat{a}_t(r_t - \gamma_{lp}) - P_t \hat{c}_t - p_{g,t}\hat{\bar{c}}_{g,t} + p_{s,t}\hat{\bar{c}}_{s,t} \qquad (2\text{-}7) \\
&\hat{a}_t \geq \hat{\underline{a}}_t
\end{aligned}$$

The productive side of the economy, which were embedded in the aggregate production function 2-5, is also rewritten bellow in its stationary version. Remember that wages (now in the detrended form) and interests are once more obtained from its marginal returns.

$$\hat{Y}_t = p_{g,t}(\hat{A}_{g,t}N_t)^{1-\alpha}\hat{K}_t^\alpha = p_{s,t}(\hat{A}_{s,t}N_t)^{1-\alpha}\hat{K}_t^\alpha \qquad (2\text{-}8)$$

We must now state equilibrium conditions for the static and dynamic parts of the model - as well as the market clearing restrictions.

Dynamic equilibrium conditions For the households in our economy, an optimal path for assets and composite consumption must satisfy two differential partial equations. The first, is the Hamilton-Jacobi-Bellman (HJB), a continuous time analog of the Bellman equation. The second is the Kolmogorov Forward (KF) equation, which determines how evolves wealth and idiosyncratic productive joint distribution across the continuum of individuals. Formally, we have:

$$
\begin{aligned}
\tilde{\rho} v(\hat{a}, z, t) &= \max_{\hat{c}} \left\{ \frac{\hat{c}_t^{1-\sigma}}{1-\sigma} + \partial_a v(\hat{a}, z, t) s(\hat{a}, z, t) + \partial_t v(\hat{a}, z, t) \right. \\
&\qquad \left. + \lambda_z (v(\hat{a}, z', t) - v(\hat{a}, z, t)) \right\} \\
s(\hat{a}, z, t) &= \hat{w}_t z_t + \hat{a}_t (r_t - \gamma_{lp}) - P_t \hat{c}_t - p_{g,t} \hat{\hat{c}}_{g,t} + p_{s,t} \hat{\hat{c}}_{s,t} \qquad \text{(2-9)} \\
\frac{dg_t(\hat{a}, z)}{dt} &= -\partial_{\hat{a}} [g_t(\hat{a}, z) s(\hat{a}, z)] - \lambda_z g_t(\hat{a}, z) + \lambda_{z'} g_t(\hat{a}, z')
\end{aligned}
$$

where $v(\cdot)$ is the value function defined over the state space (containing all possible levels of assets, idiosyncratic productivity, and all aggregate variables), $s(\cdot)$ is the saving function - and where $g_t(\cdot)$ is the joint probability distribution of wealth and productivity.

Static Equilibrium conditions The static problem of households concerns choices of services and goods' consumption, at each period of time, taking prices and total consumption expenditures as given. For individuals, an optimal path of sectoral consumption must satisfy the demands below. They are yielded by the minimization of total costs needed to achieve a given level of composite consumption.

$$
\hat{c}_{s,t} = \frac{P_t \hat{c}_t}{p_{s,t}} \left[1 + \frac{\omega_g}{\omega_s} \left(\frac{p_{g,t}}{p_{s,t}} \right)^{1-\epsilon} \right]^{-1} - \hat{\hat{c}}_{s,t} \qquad \text{(2-10)}
$$

$$
\hat{c}_{g,t} = (\hat{c}_{s,t} + \hat{\hat{c}}_{s,t}) \left(\frac{\omega_g}{\omega_s} \right) \left(\frac{p_{s,t}}{p_{g,t}} \right)^{\epsilon} + \hat{\hat{c}}_{m,t} \qquad \text{(2-11)}
$$

We need to make an assumption at this point in order to have a well defined problem, in which services consumption is not negative, and the goods consumption is at least $\hat{\hat{c}}_g$, for *every household*. It means that the nonhomothetic terms may not be unbounded - or, putting it differently, the time varying endowment in the dynamic problem $(p_{s,t}\hat{\hat{c}}_{s,t} - p_{g,t}\hat{\hat{c}}_{g,t})$ must not exceed, in absolute value, a certain proportion of the household's income. That is what Assumption 2.1 guarantees (the derivation is at Appendix A).

Assumption 2.1 *The nonhomothetic terms $\hat{\hat{c}}_{s,t}$ and $\hat{\hat{c}}_{g,t}$ satisfy for every time:*

$$
(1 - \zeta_t)(p_{s,t}\hat{\hat{c}}_{s,t}) + \zeta_t(p_{g,t}\hat{\hat{c}}_{g,t}) \leq \zeta_t(\hat{w}_t z_1 + \underline{\hat{a}}_t(r_t - \gamma_{lr}))
$$

where $\zeta_t \equiv \left(1 + \frac{\omega_g}{\omega_s}\left(\frac{p_{g,t}}{p_{s,t}}\right)^{1-\epsilon}\right)^{-1} \in (0,1)$.

By integrating demands 2-10 and 2-11, aggregate amounts of consumption in each market are obtained. Also, at each instant of time, total investment expenditures by households are given by the aggregation of savings and the depreciation cost of effective units (γ_{lr}):

$$\hat{C}_{i,t} = \int_{\underline{\hat{a}}}^{\infty} \hat{c}_{i,t}(a, z_1) g_t(\hat{a}, z_1) d\hat{a} + \int_{\underline{\hat{a}}}^{\infty} \hat{c}_{i,t}(a, z_2) g_t(\hat{a}, z_2) d\hat{a}, \quad i \in \{g, s\}$$
$$\hat{X}_t = \int_{\underline{\hat{a}}}^{\infty} (s_t(\hat{a}, z_1) + \gamma_{lr}\hat{a}_t) g_t(\hat{a}, z_1) d\hat{a} + \int_{\underline{\hat{a}}}^{\infty} (s_t(\hat{a}, z_2) + \gamma_{lr}\hat{a}_t) g_t(\hat{a}, z_2) d\hat{a}$$

Market clearing Market clearing conditions requires that all capital used in the production must be supplied by households' savings (2-12 and 2-13), with a similar condition holding for labor as well (2-14 and 2-15). Condition 2-16 is an outcome of 2-13: since in equilibrium savings must equal the economy capital stock, returns must also be the same. Finally, production in each market must satisfy consumption and investment needs (2-17 and 2-18).

$$\hat{K}_t = \hat{K}_{g,t} + \hat{K}_{s,t} \tag{2-12}$$
$$\hat{K}_t = \int_{\underline{\hat{a}}}^{\infty} \hat{a} g_t(\hat{a}, z_1) d\hat{a} + \int_{\underline{\hat{a}}}^{\infty} \hat{a} g_t(\hat{a}, z_2) d\hat{a} \tag{2-13}$$
$$N_t = N_{g,t} + N_{s,t} \tag{2-14}$$
$$N_t = \int_0^1 z(j) dj = \frac{z_1\lambda_2 + z_2\lambda_1}{\lambda_1 + \lambda_2} \tag{2-15}$$
$$r_t = \partial_{\hat{K}} \hat{Y}_t - \delta \tag{2-16}$$
$$\hat{Y}_{g,t} = \hat{C}_{g,t} + \nu_t \hat{X}_t \tag{2-17}$$
$$\hat{Y}_{s,t} = \hat{C}_{s,t} + (1 - \nu_t)\hat{X}_t \tag{2-18}$$

We are now in position to define equilibrium:

Definition 2.1 *Given an initial wealth and labor efficiency distribution,* $g(\hat{a}, z, 0)$*, and a technology path* $\{A_{g,t}; A_{s,t}; \nu_t\}_{t=0}^{\infty}$*, the* ***dynamic equilibrium*** *is characterized by sequences of prices* $\{P_t; p_{g,t}; p_{s,t}; r_t; \hat{w}_t\}_{t=0}^{\infty}$*, policy functions* $\{s_t(\hat{a}, z), c_t(\hat{a}, z)\}_{t=0}^{\infty}$*, value functions* $\{v_t(\hat{a}, z)\}_{t=0}^{\infty}$*, and of distributions* $\{g(\hat{a}, z, t)\}_{t=0}$ *such that:*

1. *Policy and value functions solve households' intertemporal problem taken prices as given, i.e, the HJB in 2-9 is attended, as well as the borrowing constraint.*

2. *The distributions' path obey the KF equation in 2-9.*

3. *Aggregate labor and capital are consistent with firms optimal choices. It means that 2-8 is attended and wages and interest rates correspond to its first order conditions.*

4. *Market clearing conditions 2-12 to 2-18 are verified, as well as Assumption 2.1.*

Definition 2.2 *Given the objects of the dynamic equilibrium, the* ***static equilibrium****, for each instant t, is characterized by policy functions $\{\hat{c}_{g,t}, \hat{c}_{s,t}\}$ in accordance to 2-10 and 2-11 - and by labor and capital allocations among sectors $\{N_{g,t}, N_{s,t}, \hat{K}_{g,t}, \hat{K}_{s,t}\}$ such that market clearing conditions 2-12,2-14,2-17 and 2-18 are attended.*

When conditions for Definitions 2.1 *and* 2.2 are met, the model has an equilibrium path.

2.4.1
Necessity and existence of a stationary system

For numerical solutions and computations, a stationary system (from which we will solve backward) is needed, meaning that a terminal condition for the value function $v_t(\cdot)$ and for all policy functions in the Dynamic equilibrium 2.1 must be obtained. Theoretically, this terminal condition could be in $t \to \infty$, but in practice, it is necessary that it happens in a finite period of time, as argued by (2). In this steady state, the dynamics given by the HJB and KF equation boils down to the system 2-19 below.

$$
\begin{aligned}
\tilde{\rho} v(\hat{a}, z) = \max_{\hat{c}} \Big\{ & u(\hat{c}) + \frac{\partial v(a,z)}{\partial a} \big[\hat{w} z + \hat{a}_t (r - \gamma_{lp}) - P\hat{c} \\
& - p_g \hat{\bar{c}}_g + p_s \hat{\bar{c}}_s \big] + \lambda_z (v(\hat{a}, z') - v(\hat{a}, z)) \Big\} \\
0 = {} & -\partial_{\hat{a}} [g(\hat{a}, z) s(\hat{a}, z)] - \lambda_z g(\hat{a}, z) + \lambda_{z'} g(\hat{a}, z') \\
\hat{a}_t \geq {} & \underline{\hat{a}}
\end{aligned}
\tag{2-19}
$$

Once we need the system to assume the above form in finite time, we must make assumptions to guarantee it:

Assumption 2.2 *Consider the technology set $\{A_{g,t}, A_{s,t}, \nu_t\}$, the nonhomothetic terms $\{\bar{c}_g, \bar{c}_s\}$, two time periods arbitrarily chosen, τ and T, such that $0 \leq \tau \leq T$, and the debt limit $\underline{a}_t$. We assume that the following conditions hold:*

1. *Growth rates $\gamma_{g,t}$, $\gamma_{s,t}$, $\gamma_{\nu,t}$ can assume any value for $t \in [0, \tau]$.*

2. *For $t \geq T$, $\gamma_{g,t} = \gamma_{s,t} = \gamma_{lr}$, and $\gamma_{\nu,t} = 0$.*

3. *For $t \in (\tau, T)$, growth rates converge smoothly from their respective values in $t = \tau$ to the new level in $t = T$.*

4. *For $t \geq T$, $\bar{c}_g$ and $\bar{c}_s$ are both set to zero or start to grow at rate γ_{lr}. It means that $\hat{\bar{c}}_{g,t}$ and $\hat{\bar{c}}_{s,t}$ become constant for $t \geq T$.*

5. *$\underline{a}_t = \underline{a}_0 \exp\{\gamma_{lr} t\}$, and hence $\hat{\underline{a}}_t$ is constant.*

With Assumption 2.2 we bring to a finite (but *arbitrarily* large) period of time, what we would observe asymptotically in a model where growth rates could always be set freely in accordance with the stylized facts of structural change[3]. Placing τ and T sufficiently distant from $t = 0$ makes it possible to obtain a steady state system with a service share arbitrarily closer to 1, and with nonhomothetic terms with real value $\hat{\bar{c}}_{i,t} \sim 0$.

Given these assumptions, a stationary equilibrium as in 2-19 indeed exists in the model. We can extend the results of existence presented in the literature, like in (2), to the setup of this paper. That is what states Proposition 2.1, whose discussion and proof can be seen in Appendix A.

Proposition 2.1 *Given a technology set which respects Assumption 2.2, given that Assumption 2.1 does hold and considering that $\hat{\underline{a}}_t$ is always smaller (in absolute value) than the natural debt limit for all agents, then there exists a steady state equilibrium for this model.*

As the model converges to the steady state, interest rates, wages, and all aggregate variables are constant in the detrended model, and Kaldor facts trivially hold. As we will see in the following exercises, however, it is in the first decades of growth, when a poor and goods producer economy begins the process of catching up to become richer and service intensive, that the association between structural change elements with the inequality behavior is stronger, with long-lasting effects.

[3](8) uses a similar assumption in order to solve numerically a two-sector model by obtaining the set of stationary policy functions.

2.5
Quantitative Exercises

In Chapter 4, we will bring the model to the data and analyze some of the main results. Before that, however, we can gain intuition and explore the underlying mechanisms by running some simple quantitative exercises. That is what we do in this Section: using a standard calibration we observe the model's performance, focusing on the interaction between structural change driving mechanisms and the inequality path.

The section is organized as follows. Firstly, we discuss the general set up and the parameters' values chosen. Next, we run the model with a benchmark calibration (Simulation 1), in which both mechanisms of structural change (income and relative prices effects) are turned on. We identify some important intuitions and then run a set of Simulations (2 to 4) in order to perform a sort of counterfactual analysis (like turning off relative prices changes or the nonhomothetic terms) and robustness checks.

2.5.1
General idea and calibration

In all simulations performed in this Chapter, we will have the same panorama: a secular transition from an economy with low levels of income per capita, in which goods' production is predominant, to a richer economy, in which services account for the most part of GDP. We go, therefore, from a steady state to another, through a transition that takes place in a long run window of time. The driver mechanism to propel the economy to this change is the acceleration of technical growth rates from both sectors.

Technical growth rates accelerate, but not necessarily homogeneously. We allow a distinction in sectoral rates to take place for an arbitrary period of time until they start to converge for the common (long run) rate, as detailed in the model's description. This step is done in consistency to the empirical literature in structural change, which estimates higher growth rates in the goods' sector[4].

The long-run technical progress is equal in both economies. The difference lies in the secular period during which the economy speeds up its growth rate and accumulates efficient units of capital.

Calibration Our baseline calibration can be seen in Table 2.1. It borrows mostly from standard macro literature values and from (18)'s regression estimates of a demand system close to the one in this paper. We calibrate the Poisson Process to make individuals, on average, switch from one productivity

[4]See (19) for some estimations and for a review of estimations on the literature.

state to the other once in every 20 years. It makes our idiosyncratic shocks closer to a career shock than to a frequent change in wages. In order to have a Wealth Gini of 60% in the transition beginning, a number not distant to general levels of this index, we determined the values of the productive levels z_1 and z_2.

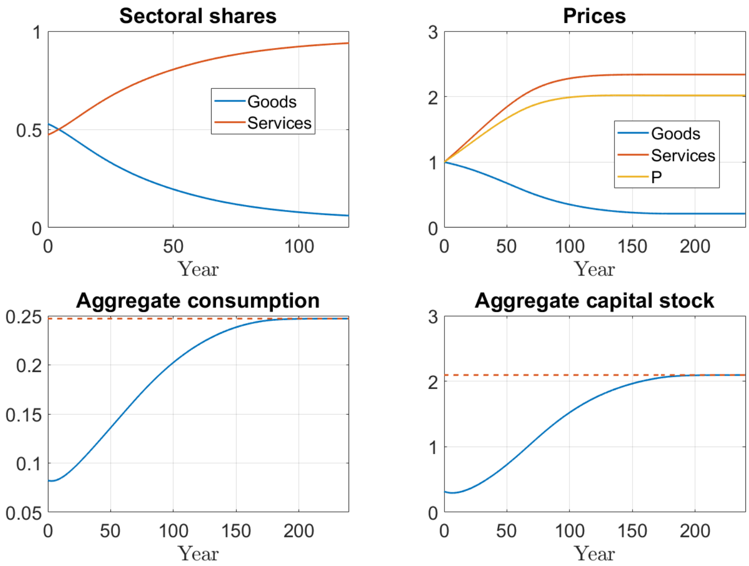
Targeting a secular transition, we set $\tau = 60$ and $T = 180$. Our long-run growth rate is of 1.5%, following (11)' estimations. We also adjusted the path for $\{\nu_t\}$ aiming to qualitatively reproduce estimations from (18) - according to which the share of goods in the production of final investment units decreased from 70% to 50%, in the course of 60 years (see Figure B.1). Therefore, we set an initial value for $\nu_t = 80\%$ and a final steady state level of 53%, setting up a smooth path between these extremes. Non-homothetic terms were chosen to make the fraction of goods in our beginning economy greater than 50%.

What we must underscore, however, is that the main results we will present do not depend on a specific calibration strategy, being robust to a wide range of alternative calibrations concerning, for instance, the Poisson process, the duration of the transition and the path of $\{\nu_t\}$.

Table 2.1: Calibration for numerical simulations

Parameter	Value	Source/Target
σ	2.00	Standard value
ρ	.035	Standard value
α	1/3	Standard value
δ	0.05	Standard value
ω_g	0.15	Estimated by (18)
ω_s	0.85	Estimated by (18)
ϵ	.002	Estimated by (18)
λ_1	1/20	Career shocks
λ_2	1/20	Career shocks
z_1	0.05	Model scale
z_2	.125	$\mathrm{WealthGini}_{t=0} \sim 0.6$
γ_{lr}	.015	Estimated by (11)
$\tau; T$	60;180	Secular transition
$\{\bar{c}_{g,t}\}_{t=0}^{\tau}$	0.055	$N_{g,t0}/N \sim 75\%$
$\{\bar{c}_{g,t}\}_{t\geq T}$	$.004\mathrm{e}^{\gamma_{lr}t}$	Avoids discontinuity at T
$\{\bar{c}_{s,t}\}_{t=0}^{\tau}$	0.055	$N_{g,t0}/N \sim 75\%$
$\{\bar{c}_{s,t}\}_{t\geq T}$	$.004\mathrm{e}^{\gamma_{lr}t}$	Avoids discontinuity at T
$\{\underline{\mathrm{a}}_t\}_{t=0}^{\infty}$	$0.1\mathrm{e}^{\gamma_{lr}t}$	$\sim$ 2 months of wages in $t = 0$
$\{\nu_t\}_{t=0}^{\tau}$	$.8\mathrm{e}^{-.002t}$	Matches Figure B.1
$\{\nu_t\}_{t\geq T}$	0.53	Matches Figure B.1

Figure 2.1:



2.5.2 Simulation 1

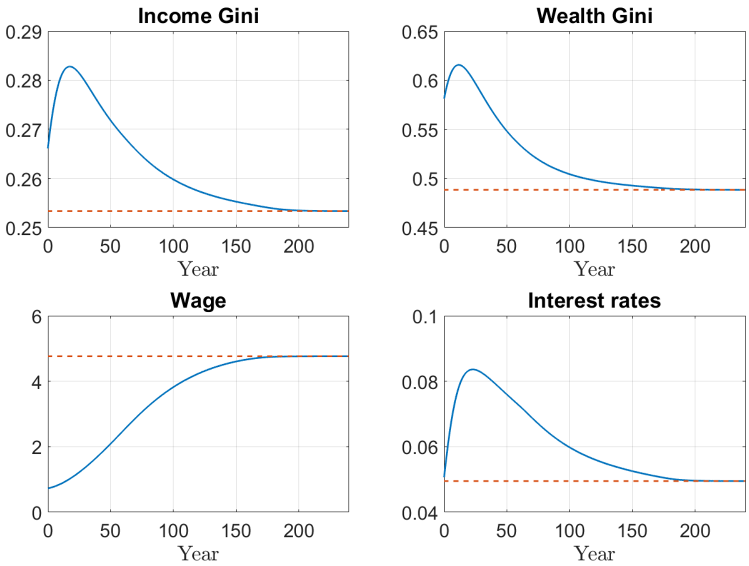
In this simulation (our benchmark), we set the rate of technical growth in goods' sector, for $0 \leq t \leq \tau$, at $\gamma_g = 0.05$ and the growth rate of services at $\gamma_s = 0.02$. As mentioned, a calibration such that $\gamma_g > \gamma_s$ is consistent with evidence from the empirical literature. In addition to that, our main points presented bellow, qualitatively, do not depend on how large is this inequality, provided that it holds. We chose these specific values for γ_g and γ_s so as to yield, at the end of the transition, a service share of about 90%.

Figure 2.1 and Figure 2.2 display the path followed by our economy throughout the transition. Notice how the structural transformation takes place in accordance to the stylized facts[5] - and consumption, capital, and wages soar as expected. Inequality (both wealth and income Gini) peaks in the first thirty years, and then decreases steadily to a level closer to the initial steady state, following an inverse U-shaped path and resembling the Kuznets curve.

To understand the mechanisms behind this inverse U-shaped pattern - and how structural change matters in this process, we need to go through three questions: Why does inequality peak in the first decades, why savings are relatively small (and interest rates higher) in the transition beginning and how does sectoral dynamics play a role in this process.

[5]Keep in mind that we have both mechanisms for structural change turned on (income effects and relative prices effects).

Figure 2.2:



Why does inequality peak in the first decades? We find that those periods in which capital accumulation is speeding, interest rates are decreasing - and so is inequality. For instance, notice in Figures 2.1 and 2.2 how, in the first decade, variations in the capital stock are almost null and inequality soars abruptly. In the following decades, as capital becomes a steeper function of time, inequality measures start to come back to lower levels. Another way to comprehend it is to look at Table 2.2: we see how the correlation between percentage variations in capital stock (interest rates) and in the Gini index is almost perfectly negative (positive).

This behavior is exactly what the seminal paper from (9) pointed out: inequality and capital accumulation in a neoclassical growth model move together - if savings' propensity is decreasing (increasing) in the wealth of households, the correlation is negative (positive). The more capital accumulation speeds, the more equal (unequal) becomes the distribution of resources.

Indeed, in our model, looking into the savings' policy function of households, it becomes clear why the Gini index and the capital accumulation are negatively correlated. Figure 2.3 shows how this policy generally looks in our model at the beginning of the transition and always that interest rates are not following a significant upward trend.

The group which mostly saves, both in absolute and in relative terms, is the one with higher productive (z_2) *and* with the lower level of assets. The self-insurance behavior, common to incomplete market environments like our model, lead the ones with the high wages - but not yet protected against an eventual negative shock - to have the greatest willingness to save. Therefore, while capital accumulation takes place mostly driven by this group with lower asset stock, inequality (in terms of assets and of capital income) decreases significantly[6].

Periods of relatively low capital accumulation and savings are accompanied by a rising trend in the interest rates (equation 2-16 and Table 2.2 convey this negative correlation). When interest rates become sufficiently high, savings become closer to a normal good: the policy function becomes upward sloping in an interval of time around the peak in interest rates. The wealthier start to save more (or borrow less, at least) and is in that moment that wealth inequality peaks. As the supply of savings (and hence capital accumulation) start to soar, interests go down, savings policy functions return to their natural shape (downward sloping), and capital accumulation keeps going on - driven mostly by the high income but poor asset holders individuals (as highlighted in Figure 2.3), which leads to a falling trend in inequality.

In Figure B.4 we try to illustrate the above-described changes in the savings policy functions throughout the transition path. One can observe, for the current simulation, how their shapes start and end in a fashion closer to Figure 2.3, but become upward slopping while interest rates soar. Observe that, for each saving policy function across time, we write its respective year t in the transition. We also write the interest rate level observed at each of these periods.

Table 2.2: Correlations for years 0-120

Simulation	corr($\Delta\%K, \Delta\%Gini$)		corr($\Delta\%r, \Delta\%Gini$)		corr($\Delta\%K, \Delta\%r$)
	Wealth	Income	Wealth	Income	
I	-0.94	-0.80	0.89	0.98	- 0.69
II	-0.83	-0.63	0.91	0.99	- 0.54
III	-0.96	-0.97	0.86	0.99	-0.93
IV	-0.93	-0.68	0.71	0.98	-0.54

Why do interest rates soar in the transition beginning? Now that we have already discussed the reason why inequality is negatively correlated to savings and capital accumulation, the remaining question is why does the willingness to accumulate assets is relatively small (i.e, why interest rates rise abruptly) during the first decades of growth acceleration (leading to a peak in the Gini index). One important reason, widely understood, is consumption smoothing. When the transition begins, given the absence of aggregate uncertainty, individuals know that the economy will grow - and

[6]See Figure B.2, the policy for savings as function of the subsistence requirement, and notice that the same conclusions regarding the groups which mostly save can be easily made.

they will become richer. They want to bring part of this higher consumption to the present, deccumulating assets, and interest rates must rise to partially dissuade their plans since the economy is not rich yet.

For future clarification, we will refer to the above mechanism (and intuition) as the consumption smooth desire due to a wealth gap between final and terminal states.

Why sectoral dynamics matters? Since consumption smooth and savings' dynamics are important forces behind the increase in inequality during the first decades, sectoral dynamics can make a difference when it affects these forces. In our model, this happens by means of the relative prices' path and by the nonhomothetic terms: both are drivers of structural change and can influence savings and consumption smooth as well.

Relative prices across sectors interact with the consumption smooth mechanism making it more or less intense in the transition beginning. Generally speaking, sectoral prices, by means of the consumption bundle's price, P_t, influence the trade-off between savings and consumption for a given period, since P_t is the relative price between these two options, or, in the perspective of the aggregate output, Y_t, it is a marginal rate of transformation between consumption and capital[7].

Just intuitively, if P_t will rise tomorrow, the agents have incentives to bring part of tomorrow's consumption to the present, while consumption stills cheaper - and to accumulate proportionally more assets tomorrow, when these assets in turn become cheaper. It can add some strength, therefore, to the smooth consumption desire, i.e, the desire to bring consumption from tomorrow to today. The Euler equation can also help us to grasp the intuition. For each individual, an optimal path of consumption and savings must satisfy:

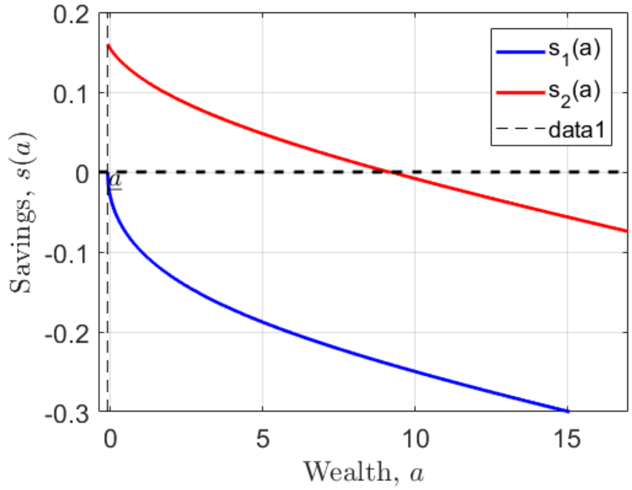
$$\frac{E_t\{du'(c(a,z,t))\}}{u'(c(a,z,t))} = \Big[\tilde{\rho} + \gamma_{lr} + \frac{\dot{P}_t}{P_t} - r_t\Big]dt \tag{2-20}$$

$$\dot{\hat{a}}_t - \hat{a}_t(r_t - \gamma_{lr}) = \hat{w}_t z_t - P_t \hat{c}_t - p_{g,t}\hat{\bar{c}}_{g,t} + p_{s,t}\hat{\bar{c}}_{s,t} \tag{2-21}$$

Equation 2-20 is an *Euler equation* and 2-21 is just the savings' dynamics already presented. The derivation of 2-20 can be seen in the Appendix A. Notice here that a rising relative price of consumption and capital, everything else constant in 2-20, would contribute to an expected increase in the marginal

[7]Remember that output (in units of the investment good) is written as $Y_t \equiv Y_{g,t}p_{g,t} + Y_{s,t}p_{s,t}$. Using firms first other conditions, and definitions of price level and aggregate consumption, we can write $Y_t = P_tC_t + X_t - p_{a,t}\bar{C}_{a,t} + p_{m,t}\bar{C}_{m,t}$. With this formulation, it becomes clear, as stressed by (19), how the relative price of consumption and capital, P_t, acts as marginal rate of transformation between investment and consumption.

Figure 2.3: Savings policy functions



utility, and hence, to less consumption. As P_t becomes higher and higher, agents have less willingness to consume[8].

In this environment with a fast growing P_t, and everything else constant, we would observe that consumption in t is higher than in $t+\Delta$. This can be an extra channel fostering the desire to anticipate consumption (bringing it from $t+\Delta$ to t). In Simulations 2-3 we explore these intuitions.

The nonhomothetic terms, although not so directly, can also affect the savings' dynamics in the transition beginning. A relaxation in the subsistence requirements, for instance, even without changes in the production side, could represent for households an utility gain with consumption, offsetting the desire to smooth consumption by deccumulation of assets, during the beginning of the transition. That is what we explore in Simulation 4.

2.5.3 Simulation 2

We maintain the same initial and final steady-state levels of Simulation 1 (i.e, the same beginning and terminal values for all variables). The only difference is that we speed the change of the consumption relative price in the first decades even faster here, so that it achieves the final level ahead on time, as shown by Figure 2.4. The sequences of TFP growth rates for both

[8]It turns out that in our calibration the capital good indeed becomes cheaper throughout the simulations - which is by far consistent with time series for the American economy along the last century, as shown by (15) and (16). As the growth in goods' sector is higher than in services, the relative price of goods decreases. Since the weight of services in the consumption bundle (preferences) is higher than the weight of services in the production of capital (technology), the relative price between consumption and capital ends up increasing along time.

sectors are calibrated to yield this price sequence - but respecting always the restriction $\gamma_{g,t} \geq \gamma_{s,t}\ \forall t$.

Figure 2.4: Prices - comparing simulations 1 and 2

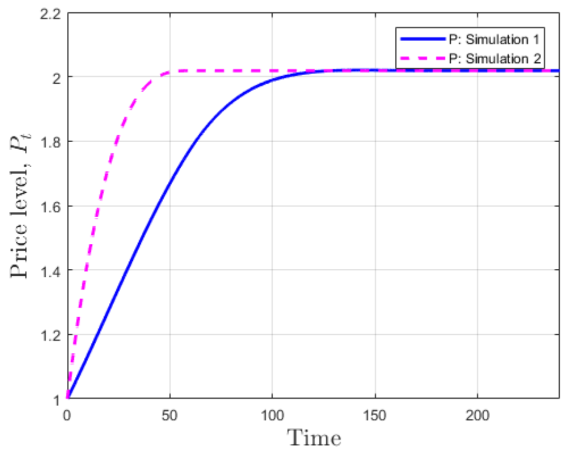
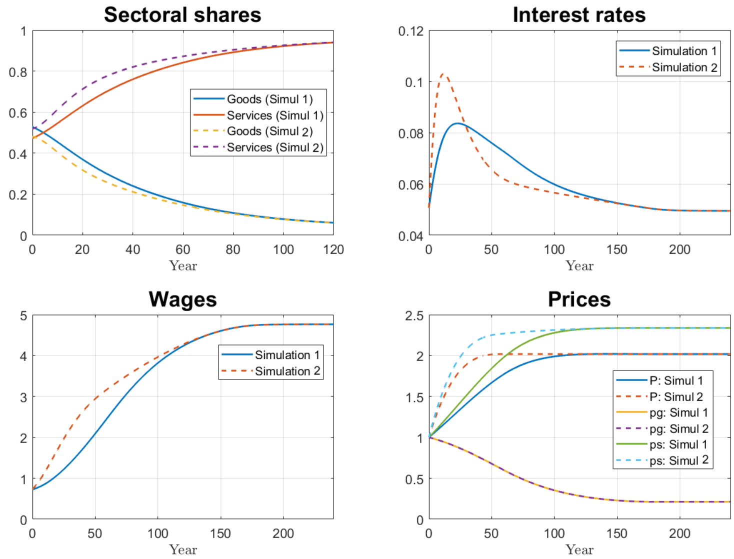


Figure 2.5:

As the final levels of consumption and output are the same of Simulation 1, the consumption smoothing channel motivated by the wealth gap between the final and the beginning steady state does not have reasons to be bigger. However, as P_t achieves higher values even faster, agents may have greater incentives, by means of this channel, to bring more consumption to the very beginning of the transition.

Indeed, notice in Figure 2.5 and Figure 2.6, how the consumption in $t = 0$ and in the transition beginning is greater here than in Simulation 1. Then, as the price starts to grow faster, consumption in this simulation assumes a flatter time trend - and gradually converges to the path of Simulation 1. Conversely,

Figure 2.6:



during the same time, capital is faster accumulated here, in accordance with the predictions of the Euler equation.

With more pressure for consumption and fewer savings in the transition very beginning, it is not a surprise, as previously discussed, that interest rates (and inequality) will have a larger peak, and also that this peak will happen ahead in time in comparison to Simulation 1. This last observation is a result of a greater consumption already in $t = 0$, the transition very beginning, when prices haven't soared yet. At the same, as P_t is one of the driver force of structural change, we observe a faster sectoral reallocation in the first decades for Simulation 2.

2.5.4 Simulation 3

Keeping the same calibration of Table 2.1, our economy will now undertake modern growth with equal rates of technical progress in both sectors. This means that the relative price of consumption and capital will not change. Therefore we will turn off the relative price channel of the peak in inequality. In addition to that, we set equal growth rates at $\gamma_g = \gamma_s \sim 0.03$, so as to have the same levels of capital and output we had in Simulation 1 (concerning the final steady state). With this, we want the channel of consumption smoothing due to the wealth gap between initial and final states to remain as active as possible.

Indeed, here we will have a higher level of consumption in the final steady state in comparison to Simulation 1. This happens because the amounts of

Figure 2.7:

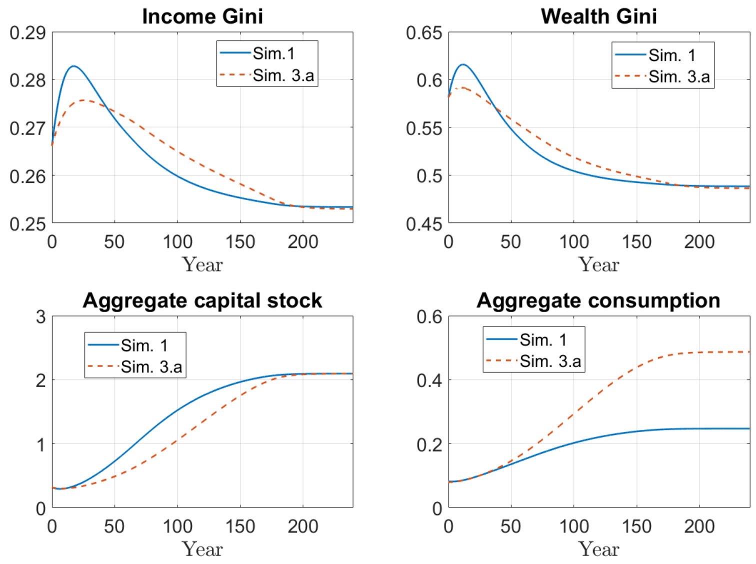
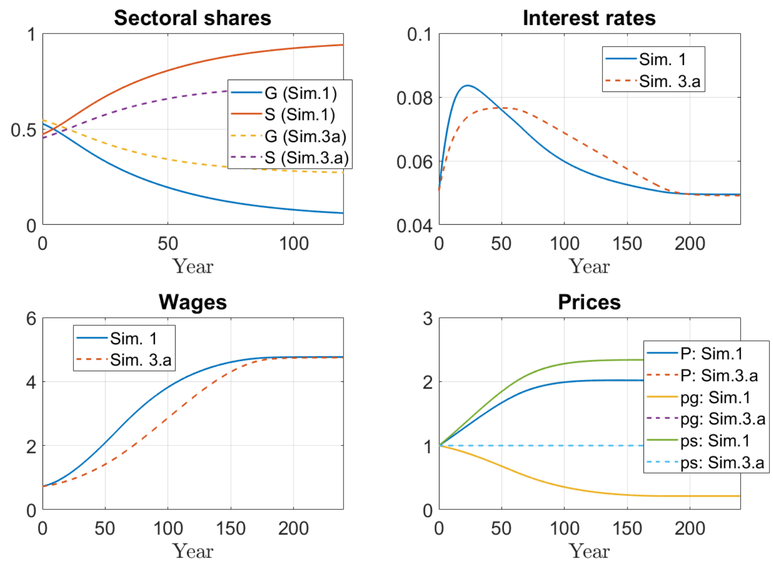


Figure 2.8:

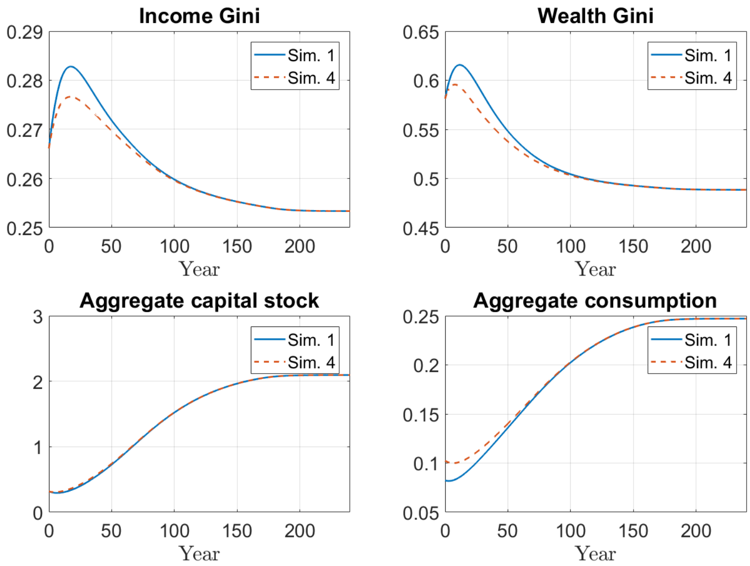


capital and output (measured in capital units) are the same in both cases - and the economy transforms capital in consumption at a higher rate in this Simulation[9]. Hence, there is no reason for the consumption smoothing channel due to the wealth gap to be lower.

Comparative results in Figures 2.7 and 2.8 show how the same patterns

[9] If consumption is cheaper, agents will choose a higher fraction c/k, i.e, the proportion of consumption and capital in the steady state. To have the same level of capital as before with a lower P, the consumption needs to be higher. Another way to see the intuition is remembering the expression $Y_t = P_tC_t + X_t - p_{a,t}\bar{c}_{a,t} + p_{m,t}\bar{c}_{m,t}$: once, in a steady state, we keep the same levels of output and capital (and hence of investment), but cut down on P_t, the level of consumption will be pressured to rise.

Figure 2.9:



underscored in Simulation 1 emerge here as well. However, the peak of inequality and interest rates loses a significant strength, achieving a lower maximum. While capital accumulation in Simulation 1 becomes a steeper function of time during the first decades, it is much flatter here, again in accordance with the Euler equation.

If in Simulation 2 we had a faster increase of P_t leading to a greater peak in inequality and more intense structural change in comparison to Simulation 1, here we have the opposite. A constant path for P_t leads to a lower level achieved by Gini indexes during the transition - and to a lower reallocation of activity across sectors, even in the final steady state.

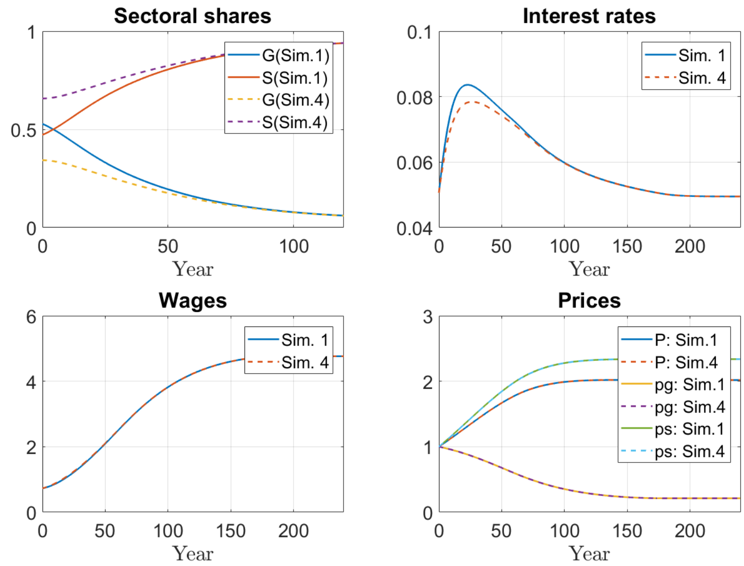
The reader can also see Figures B.5-B.6 (Simulation 3.b) in Appendix B, in which we do the same exercise of here, targeting, however, a steady state with the same level of consumption. Since consumption is the same - and not higher - in the final steady state, all results pointed out above are more pronounced in this Simulation.

2.5.5 Simulation 4

In this simulation we take the same environment of Simulation 1 - even the growth rates calibration is the same. The difference here is that we make the following experiment: in $t = 0$ the subsistence requirement level $\hat{\bar{c}}_{g,t}$ drops immediately to its steady-state level, which is more than ten times lower than its value in $t = 0$.

We interpret this experiment as a way to give more of the consumption

Figure 2.10:



bundle C_t for the agent in the transition beginning - without the need for more expenses. Looking from a different perspective, it is as if we augment the time-varying endowment in the dynamic problem 2-7, allowing agents to consume more. But notice that we are not changing anything in the steady state, it is just a *temporary* gain of resources.

We have the same wealth and consumption gap between steady states, but now agents can smooth consumption not only by saving less, but also using the resources freed by the drop in the subsistence requirements. As a result, one would expect the downward pressure over savings to be smaller in the first decades as well as the rise in the interest rates, leading to a smaller peak in inequality. That is exactly what happens in Figures 2.9 and 2.10: consumption is higher, interest rates and inequality lower during the first decades.

Finally, notice that the shock of $t = 0$ in the subsistence requirement immediately yielded a reallocation from goods production to services. Throughout the transition, however, this reallocation becomes less intense, since the income effect channel for structural change, given by the nonhomothetic terms, is over after $t = 0$.

3
A Calvo framework for labor mobility

The model is augmented with the following friction: in our continuum of workers, individuals cannot freely switch sectors. Opportunities to do so follow a Poisson process, with arrival rate θ. Hence, only when selected by this random device, can they move across sectors. From now on, this friction will be referred as our Calvo framework in the labor market.

The goal here is twofold. Firstly, the dynamic and intertemporal stages of household's problem become not independent anymore, which allows us to explore in a deeper sense the association between structural change and inequality. Secondly, as noticed in Section 1.1, most of the existing papers in the literature, even the Kuznets' ones (25), have in the labor market the driving mechanism to link inequality movements with sectoral dynamics. Therefore, we introduce here a channel to observe this issue and to compare and contrast it with our previous results.

Also, we bring into the analysis the empirical evidence. (28) estimated for the period 1968-2000 that labor mobility costs across sectors are significant, reaching almost 75% percent of the annual average wage - while an occupational change within one sector is much less costly. As noticed by (19), the majority of jobs creation and destruction occurs within rather than across narrow industrial classifications - which has also been highlighted by (21) for Korea's industrialization. The Calvo device is a reduced form to capture these costs and rigidities.

Another reason for the importance of the current exercise comes from last chapter's results. Since our simulations showed an inverse U-shaped curve for the Gini index, it is worth to mention that, historically, the literature on the Kuznets curve, i.e, on this inverse U-shaped behavior for inequality, has placed great importance on labor mobility across sectors.

Beginning with the seminal paper of Kuznets (25) and continuing in a range of related papers like (32) and (4), the idea of a two-sector environment, one of them (the growing one) with higher average and variance of wages, have been widely explored. Even with higher wages in the growing sector, a flux of workers capable of offsetting this gap cannot take place due to labor market frictions, contributing to income disparity. At the same time, the higher variance of income within the growing sector makes the aggregate inequality higher as this sector expands. More recently, as mentioned in Chapter 1, (30) revisited the Kuznets hypothesis and empirically found that for the 1919-2002 period, the structural transformation explanation for movements in inequality is supported by data.

With a labor market friction and the possibility of distinct wages across sectors, we can observe how some of the traditional explanations of the Kuznets curve behave in our set up - whether they amplify or not our findings of the previous chapter.

3.1 The extended model

Assume that the basic environment of Chapter 2 holds. It means that technology is the same, preferences are the same, as well as the notation. The following paragraphs, on the other hand, will specify the changes in equilibrium equations yielded by the introduction of Definition 3.1.

Definition 3.1 *For each agent, opportunities to choose in which sector to work follow a Poisson process with arrival rate θ. Without such opportunity, agents cannot change their current sector of employment despite any possible wage discrepancy that may exist. Given the law of large numbers, in aggregate terms, this friction imposes that, for every time period, at most a measure θ of the workers within sector i can move to sector j.*

Previously, free mobility of labor and capital allowed us to equate wages and capital rental rates in both sectors, which, when applied to firms first order conditions, yielded 2-3 and 2-4. Here we can no more guarantee equal wages, but only a unique interest rate in our economy. It turns out that capital-labor ratio will not necessarily be the same across sectors and prices may not be pinned down exclusively by technology. If the constraint is binding, wages may differ - and also, therefore, the capital-labor ratio in each sector (denoted here by φ_i, with $i \in \{g, s\}$).

While in Chapter 2 firms optimality conditions were represented by 2-3, 2-4 and 2-5, we now have:

$$\frac{\hat{w}_{g,t}}{\hat{w}_{s,t}} = \frac{\varphi_{g,t}}{\varphi_{s,t}} \tag{3-1}$$

$$\frac{p_{g,t}}{p_{s,t}} = \left(\frac{A_{s,t}\varphi_{g,t}}{A_{g,t}\varphi_{s,t}}\right)^{1-\alpha} \tag{3-2}$$

Turning to the households problem, consider $\Omega \in \{g, s\}$ a new state variable, denoting the current sector of employment - and $b_t \in \{0, 1\}$ a new control, indicating whether the individual will change ($b = 1$) or not ($b = 0$) across sectors if she has the opportunity to do so, at time t. When choosing, agents take into account the probability of remaining fixed in the new sector along time and also the wages path. Remember that, as in Chapter 2, there is no aggregate uncertainty, waiving us to use expectations in the optimal policy defined below:

$$b : \mathbb{R}_+ \text{ x } \{g, s\} \rightarrow \{0, 1\} \qquad \text{such that:}$$

$$b(t, \Omega) = \begin{cases} 1 & \int_t^\infty e^{-(\tilde{\rho}+\theta)\tau}(w_{\tau,\Omega'} - w_{\tau,\Omega})d\tau > 0 \\ 0 & \int_t^\infty e^{-(\tilde{\rho}+\theta)\tau}(w_{\tau,\Omega'} - w_{\tau,\Omega})d\tau < 0 \\ \in \{0, 1\} & \int_t^\infty e^{-(\tilde{\rho}+\theta)\tau}(w_{\tau,\Omega'} - w_{\tau,\Omega})d\tau = 0 \end{cases} \tag{3-3}$$

Notice that the policy function $b_t(\Omega)$ neither depends on the individual level of assets nor on her current productivity state z. Also, this policy is determined independently of consumption choices, which must be, as before, a solution to the control problem 2-7 - whose value function and optimal policies satisfy an HJB equation, now expressed in 3-4. The distribution of assets and productivity is synthesized by the KF expression in 3-6.

Differences in comparison to the frictionless model are intuitive: in the HJB 3-4, the continuation value takes into account not only the possibility of a productivity shock (in which z changes to z') but now also the possibility of sector switching (weighted by the rate θ). In the KF expression 3-6, the parameter θ (interacted with the policy b_t) governs the transition of mass between the two sectors, as expected.

$$\begin{aligned} \tilde{\rho} v_t(\hat{a}, z, \Omega) = \max_{\hat{c}} \Bigg\{ & u(\hat{c}_t) + \frac{\partial v_t(a, z, \Omega)}{\partial a} s(\hat{a}, z, \Omega) + \theta b_t(\Omega)(v_t(\hat{a}, z, \Omega') \\ & - v_t(\hat{a}, z, \Omega)) + \lambda_z(v_t(\hat{a}, z', \Omega) - v_t(\hat{a}, z, \Omega)) + \frac{\partial v_t(\hat{a}, z, \Omega)}{\partial t} \Bigg\} \end{aligned} \tag{3-4}$$

$$s(\hat{a}, z, \Omega) = \hat{w}_{t,\Omega} z_t + \hat{a}_t(r_t - \gamma_{lp}) - P_t \hat{c}_t - p_{g,t}\hat{\bar{c}}_{g,t} + p_{s,t}\hat{\bar{c}}_{s,t} \tag{3-5}$$

$$\begin{aligned} \frac{\partial g_t(\hat{a}, z, \Omega)}{\partial t} &= -\partial_{\hat{a}}[g_t(\hat{a}, z, \Omega)s(\hat{a}, z, \Omega)] - \lambda_z g_t(\hat{a}, z, \Omega) + \lambda_{z'} g_t(\hat{a}, z', \Omega) \\ &+ \theta(b_t(\Omega')g_t(\hat{a}, z, \Omega') - b_t(\Omega)g_t(\hat{a}, z, \Omega)) \end{aligned} \tag{3-6}$$

Finally, the demand system 2-10 and 2-11 for goods and services stills the same, as well as all market clearing conditions 2-12 to 2-18, with exception of 2-16. Assumption 2.1 is again needed for a well defined problem. For completeness, we restate it here for the case in which there is not a single wage rate:

Assumption 3.1 *The nonhomothetic terms $\hat{\bar{c}}_{s,t}$ and $\hat{\bar{c}}_{g,t}$ satisfy for every time:*

$$(1-\zeta_t)(p_{s,t}\hat{\bar{c}}_{s,t}) + \zeta_t(p_{g,t}\hat{\bar{c}}_{g,t}) \leq \zeta_t(\tilde{w}_t z_1 + \underline{\hat{a}}_t(r_t - \gamma_{lr}))$$

where $\zeta_t \equiv \left(1 + \frac{\omega_g}{\omega_s}\left(\frac{p_{g,t}}{p_{s,t}}\right)^{1-\epsilon}\right)^{-1} \in (0,1)$ *and* $\tilde{w}_t \equiv \min\{\hat{w}_{t,g}; \hat{w}_{t,s}\}$.

An equilibrium definition can be written as follows:

Definition 3.2 *Given a path for technology* $\{A_{g,t}, A_{s,t}, \nu_t\}_{t=0}^{\infty}$ *and an initial joint distribution of wealth and productivity,* $g(\hat{a}, z, 0)$, *an equilibrium consists of paths for prices, wages and rental rates,* $\{p_g(t), p_s(t), \hat{w}_g(t), \hat{w}_s(t), r(t)\}_{t=0}^{\infty}$, *as well as policy functions* $\{b(\Omega,t), c(\hat{a},z,t), s(\hat{a},z,t), c_g(\hat{a},z,t), c_s(\hat{a},z,t)\}_{t=0}^{\infty}$, *distributions* $\{g(a,z,t)\}_{t=0}^{\infty}$, *value functions* $\{v(\cdot)\}_{t=0}^{\infty}$, *allocations of aggregate capital and labor among sectors,* $\{\hat{K}_g(t), \hat{K}_s(t), L_g(t), L_s(t)\}$, *such that:*

1. *The policy and value functions solve the household's problem taken prices, wages and rental rates as given. In other words, 3-3, 3-4 and 3-5 are attended, as well as the borrowing constraint.*

2. *Sectoral allocations of labor and capital maximize firms problem 2-2, taken prices, wages and rental rates as given. This implies that 3-1 and 3-2 are observed.*

3. *Market clearing conditions 2-12 to 2-18 (with exception of 2-16) are observed.*

4. *The distributions' path obey the KF equation 3-6.*

5. *The restrictions imposed by Definition 3.1 and by Assumption 3.1 are respected for every instant of time.*

Notice that the separability between the dynamic and static problems has been broken, and now one cannot define and compute each equilibrium at a time. While the friction is biding in the labor market, prices, wages, and rental rates depend on the capital-labor ratio in each sector - which are pinned down by sectoral demands. These prices, then, affect the spending decisions, and hence the consumption demands of households and capital-labor ratios again, in a feedback loop.

Once this separability has been broken, some extensions become straightforward to be made. For instance, suppose that we want distinct capital shares in the productions functions: α_g in the goods sector and α_s in the services. The firms' problem becomes (for $i \in \{g,s\}$):

$$\max_{N_i, \hat{K}_i} \; p_i(\hat{A}_i N_i)^{1-\alpha_i}\hat{K}_i^{\alpha_i} - R_t\hat{K}_i - \hat{w}_t N_i \tag{3-7}$$

First order conditions lead to:

$$\frac{\hat{w}_{g,t}}{\hat{w}_{s,t}} = \frac{\alpha_s}{1-\alpha_s}\frac{1-\alpha_g}{\alpha_g}\frac{\varphi_{g,t}}{\varphi_{s,t}} \tag{3-8}$$

$$\frac{p_{g,t}}{p_{s,t}} = \frac{\alpha_s}{\alpha_g}\left(\frac{A_{s,t}}{\varphi_{s,t}}\right)^{1-\alpha_s}\left(\frac{\varphi_g}{A_{g,t}}\right)^{1-\alpha_{g,t}} \tag{3-9}$$

The new equilibrium, therefore, would be described in the same way as in Definition 3.2, except for a change in topic 2, where one would replace 2-2, 3-1 and 3-2 by 3-7, 3-8 and 3-9. Numerical computations wouldn't change also.

Stationary system The extended model in this chapter admits the same steady-state equilibrium of the benchmark setup of the last chapter. It may admit other solutions, but we can assure that the last Chapter's stationary equilibrium is also a stationary equilibrium here. Although it is not relevant for the general analysis, we may state it clearly, since for numerical solutions a steady equilibrium is needed as before. Given the Proposition bellow, we can depart backward for the same steady state found in the frictionless model.

Proposition 3.1 *Consider that conditions of Assumption 2.2 are respected, yielding a time invariant technology path, $\{\hat{A}_g, \hat{A}_s, \nu\}$. We have previously shown that a stationary solution for the frictionless model exists. For an equal stationary technology vector, the model in this Chapter admits a solution with the same allocation of capital and labor, the same policy functions and with sectoral distributions $g(a, z, \Omega)$ such that $g(\cdot, \Omega_G) + g(\cdot, \Omega_S) = g(\cdot)$, where $g(\cdot)$ is the equilibrium distribution of the frictionless model.*

See Appendix A for the proof.

3.2 Quantitative exercises

The goal here is the same as the one we had in Section 2.5. By going through a set of simple simulations, the reader can gain intuitions regarding the model's main mechanisms. Not only the goal, but also the general set up and the calibration is the same, with two exceptions (refer to Table 2.1 to remember the parameters' values chosen, and to Table 3.1 to see the two changes in the current exercise).

The first one, is the new parameter θ. Given our choice for career shocks (when calibrating $\lambda_{1,2}$), and given the empirical evidence according to which almost all job changes occur within sectors (19), and not across them, we set θ equal to $\lambda_{1,2}$, i.e, $\theta = 1/20$. On average, hence, workers will have a costless opportunity to change their sector of employment once in every 20 year, as if it was a career change that not commonly occurs.

The second change concerns the nonhomothetic terms. It turns out that Assumptions 2.1 and 3.1 are tighter for our model than for the Representative Agent one, since here, it must hold for the poorest household and not only for the average household. When the friction is binding it becomes even tighter, as services prices soar (the demand for services, not fully attended by the available labor force for this sector, contributes to a rise in p_s). In this regard, while in Chapter 2 we had both nonhomothetic terms with the same value, here we set $\bar{c}_s$ to 0 - and rise $\bar{c}_g$ in order to respect Assumption 3.1 and have an initial economy with approximately the same share of goods and services[1].

Table 3.1: Calibration for numerical simulations

Parameter	Value	Comment
θ	1/20	Same value of $\lambda_{1,2}$: career shocks
$\{\bar{c}_{g,t}\}_{t=0}^{\tau}$	0.06	$N_{g,t0}/N \sim 50\%$
$\{\bar{c}_{g,t}\}_{t\geq T}$	$.005e^{\gamma_{lr}t}$	Avoids discontinuity at T
$\{\bar{c}_{s,t}\}_{t=0}^{\tau}$	0.00	Respect Assumption 3.1
$\{\bar{c}_{s,t}\}_{t\geq T}$	0.00	Respect Assumption 3.1
Others	$\cdots$	Same as in Table 2.1

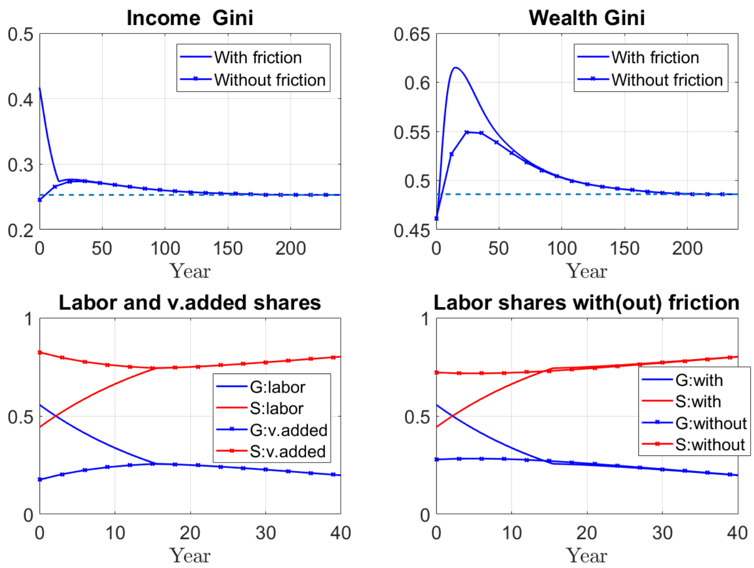
3.2.1
Model with and without labor market friction

Figures B.7 to B.11 present the results comparing the frictionless economy with the economy in which $\theta = 1/20$. Figures 3.1 to 3.3 have the same setup, but one modification: as in the last Chapter, we introduced a shock in $t = 0$ according to which the subsistence requirement is immediately set to its steady value.

In all simulations performed the last chapter, the greatest flux of workers across sectors occurred in $t = 0$, creating a gap between the share of labor in this period in comparison to $t = -1$, i.e, the steady state of the goods producer economy. After the initial boom, the reallocation of labor kept a smooth pace, with not more than a 2-percent-change per year in the goods sector labor force. It turns out, therefore, that our friction actually limits the initial massive flux in $t = 0$, spreading it along the following years.

[1]Although this point is a quantitative weakness of our model in comparison to the Representative Agent one, we must remember that papers in structural change usually use only one channel: income or relative price effects. Classical examples on the first case (using relative price effects) are (31) and (1) asymptotically, and in the second case are (22) and (14). Therefore, although we drop here the term $\bar{c}_s$, we still keep able to use both measures of structural change.

Figure 3.1:

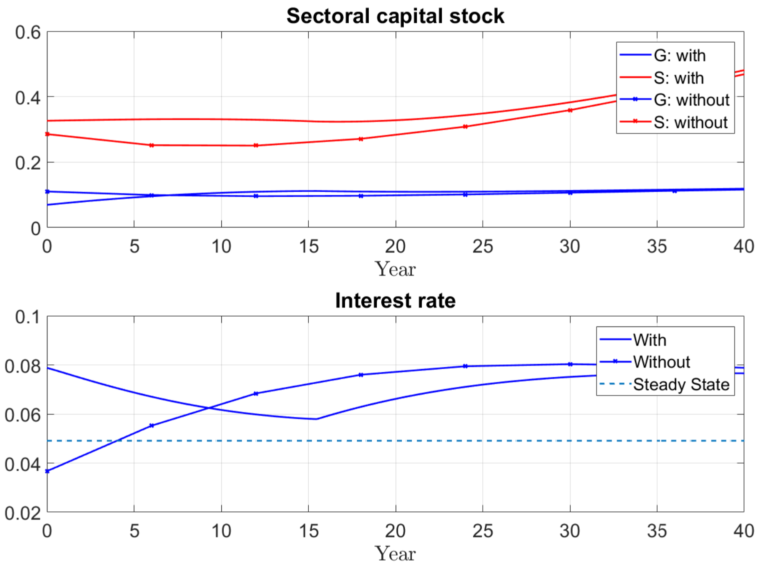


When we run the model in this Section, the result is that with approximately 4.5 years all of the initial flux of labor across sectors is over, and the friction is no more strong enough to bind. Although we can observe some interesting results (discussed below), the window of time is short. That is why we used the shock in the subsistence requirement: as noticed in Simulation 4, in the last Chapter, such shock makes the initial reallocation of resources much greater, giving us the opportunity to observe the effects of our labor market friction if it binds for a longer period of time. In the case of Figures 3.1 to 3.3, it is binding for $t \in [0, 15.5]$.

There are three main points to highlight. The first is related to income inequality, as shown in Figures 3.1 and B.7. The Gini index jumps in $t = 0$ driven by the gap in wages: since labor cannot move in enough amounts to the service sector, the following rise in prices induces capital reallocation out of goods production (see Figures 3.2 and B.10). Although in the case of Figure 3.1 the rise in the Gini index is more significant and lasting than in Figure B.7, we have the same pattern in both cases: there is an initial high peak that completely vanishes when the friction stops binding and wages equalize.

When it comes to wealth inequality there is not a same pattern in Figures 3.1 and B.8. If the friction binds for a small period of time, the wealth Gini index does not rise in comparison to the frictionless case and stay a little below in the three first decades. When the friction binds for a longer period, instead, there is a huge upward trend in the Gini - and it doesn't vanish when the friction stops bindings. Actually, the convergence of the Gini on that case to the frictionless economy path takes around 75 years in 3.1.

Figure 3.2:



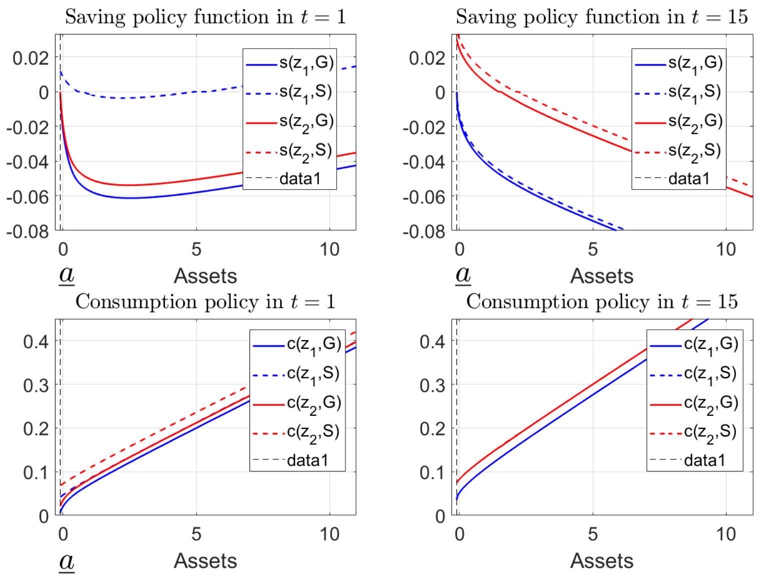
To understand this wealth inequality behavior, as always, we look to savings dynamics - and this is the second (and main) point to be highlighted. Notice in Figure 3.2 how there is a peak in interest rates in $t = 0$ in the model with friction. This huge peak induces an upward slope in saving policies already in the first moments of the transition: attracted by a huge (and *temporary*) rise in interest rates, savings become a normal good, with the richer saving more (Figure 3.3) - which, as discussed in the last chapter, triggers a peak in inequality. Given this initial jump in interest rates, inequality peaks much ahead on time (and in greater amounts) than in the frictionless economy. In $t = 15$, with the friction almost not binding anymore, with interest rates lower than the frictionless economy, notice that savings already returned to their natural behavior (Figure 3.3). In the case when the friction binds for a short period of time, the initial rise in interest rates is lower and even more temporary and doesn't induce an upward trend in savings, as shown in Figure B.11[2].

Finally, notice that we have here a difference between value-added shares and labor shares (which are always equal in the frictionless economy), as shown in Figures B.9 and 3.1. While labor shares don't jump in $t = 0$ as before, value added shares do. It is because the relative price of services and goods soars significantly in $t = 0$ (so as to discourage people to consume services). As this price slowly converges to the frictionless model's path, the value- added

[2]Agents prefer to perfectly smooth consumption. Notice in Figure B.11, that the shifty in the consumption policy functions occasioned by the sector of employment is almost null: individuals know the wage discrepancy will shortly pass and smooth consumption: those in the service (goods) sector save (borrow) more while the friction is binding, and in the overall there is not a great change in the wealth distribution path.

of services decrease a little, before start rising again.

Figure 3.3:



Kuznets curve traditional explanations In general terms and qualitatively speaking, it is interesting to notice how the model supports traditional explanations of the Kuznets curve, according to which a rising sector, with higher wages, yields a gap between their workers and the lower paid ones from traditional occupations[3], triggering a rise in inequality measures. Also, we see that the rise in the Gini coefficients, in this case, can be much more striking than in the case without the labor market friction, as shown in Figure 3.1.

Notice that the effect of the friction on wealth inequality is not disconnected from the mechanism pointed out in the last Chapter based on the rising interest rates during the transition beginning. Now, however, we know that structural change can impact this rise in interest rates (and hence in inequality) not only by means of a higher desire to bring consumption to the beginning of the transition, but also by means of a huge amount of labor reallocation across sectors that touch labor market restrictions and make consumption more limited. It means that the connection between sectoral dynamics and inequality gained here another element.

[3]See (25), (4) and (32), as previously discussed

4
Bringing the model to the data

4.1
USA: 1950-2000

The model from Chapter 2 is now calibrated for the US economy. The time span, 1950-2000, was determined by the availability of data for our purposes. We do not incorporate the labor market friction in this exercise for two reasons. Firstly, as discussed in the previous Chapter, the friction influences the economy just after the hit of a growth shock, ruling out a discontinuity in labor shares that otherwise would have happened. We believe that this environment does not apply to the US economy in 1950, already intensive in services and growing relatively steadily.

Secondly, (28) showed that the wage gap between goods and services sectors were remarkably stable and close to zero over time in the US economy. Therefore, given that our friction would be a channel to introduce a rising wage gap, it is not of quantitative relevance to meet the data in the US case.

It is important to highlight that we calibrated the model assuming that $c_{g,t}$ and $c_{s,t}$ are value-added amounts of goods and services to be consumed - which means that the production functions in our model are value added production functions. Notice that it would have been different had we assumed that goods and services refer to final consumption units[1]: data sources of labor and production would need to be changed.

The majority of parameters and exogenous variables were defined based on estimations, moments and standard values found in the literature. Two variables, however, were directly chosen so as to fit the model to the data: the endowment of services ($\bar{c}_{s,t}$) and the debt limit, ($\underline{a}_t$).

4.1.1
Data and calibration

Sectoral value added (at constant 2005 US prices) and employment come from the GGDC 10 sector database (35). We define the goods sector as the

[1]In the latter case, for instance, a car is a unit of the goods sector, while in the former a car includes both units of goods and services, since there are different stages in its production performed by different sectors.

aggregation of ISIC categories A to F[2], while services were set as the union of the remaining categories, G to P[3].

Aggregate capital stock (at constant 2011 US prices) comes from the Penn World Table (13), as well as the depreciation rate. Since the model in Chapter 2 features equal capital-labor ratios, we only need aggregate capital to determine TFPs paths, provided that sectoral employment is available.

The path for ν_t was taken from (18), who developed a technique to extract, from the production value added of each sector, how much is delivered for consumption and how much is used to deliver investment goods. Figure B.1 presents this series calculated by (18). From (18) we also obtained estimations to calibrate our preference parameters, ϵ, ω_g and ω_s - and some moments regarding the nonhomothetic terms (to be discussed below).

Standard values for ρ, σ and α were assumed. Also, we set $\lambda_{1,2} = 1/20$, exactly as in the quantitative exercises of Chapter 2. With this, we make the idiosyncratic shocks looks like more career shifts than short-term fluctuations since agents will spend on average 20 years in each productive state.

The productive states $z_{1,t}$ and $z_{2,t}$ were calibrated to reflect the college/high school educated wage ratio, presented by (7) (see Figure B.13). Therefore, we make $z_{2,t} = \gamma_t z_{1,t}$, where γ_t is this wage ratio[4]. To pin down the values of $z_{1,t}$ and $z_{2,t}$ we set the constant aggregate labor mass, N, chosen in accordance to the model scale. After the year 2000, the college/high school ratio is assumed to be constant in its 2000 level.

Inequality data used here can be found in (33) and in (5). From the former comes the wealth share of the top 10 percent, and from the later, the income Gini. We use the top 10 wealth share as the target for the calibration of $\{\underline{\hat{a}}_t, \hat{\hat{c}}_{s,t}\}$, the two variables used to shape the model to the data. Other inequality measures (income Gini, top 1 and top 5 wealth share) conveys how the model can fit the variables not target by the calibration. For the three of them, a similar panorama is observed: although our economy can replicate trends and qualitative patterns, it cannot match the *level* of data, staying always in lower ones (less inequality). Given this similarity, we only report in the results the comparison between the model and the income Gini index.

The detrended debt limit $\underline{\hat{a}}_t$ was calibrated as a constant for all time

[2]This aggregation, hence, includes: agriculture, hunting, forestry and fishing (A and B), mining and quarrying (C), manufacturing (D), electricity, gas and water supply (E) and construction (F).

[3]Services sector hence includes: wholesale and retail trade, hotels and restaurants (G to H), transport, storage, and communication (I), finance, insurance, real estate and business services (J to K), government services (L to N), community, social and personal services (O to P).

[4]Since (7) presents this statistic for each decade, and not yearly, we interpolated the data.

Table 4.1: Model Calibration

Parameter	Calibrated value	Comment
ρ	0.3	Standard value.
σ	2.0	Standard value.
γ_{lr}	0.015	Estimated by (11) for the US economy in the long run.
α	1/3	Standard value.
$\{\delta_t\}_{1950}^{2000}$	$\{\cdot\}$	Calculated by (13): Penn World Table.
ϵ	0.002	Estimated by (18) for the US economy.
$\{A_{g,t}; A_{s,t}\}_{1950}^{2000}$	$\{\cdot,\cdot\}$	Calculated with data and the model's production functions.
ω_g	0.15	Estimated by (18) for the US economy.
ω_s	0.85	Estimated by (18) for the US economy.
$\lambda_1; \lambda_2$	1/20	Career shocks.
$\{\underline{\hat{a}}_t\}_{1950}^{2000}$	$\tilde{d}_{1950}/2$	$\tilde{d}$ is the natural debt limit
$\{\nu_t\}_{1950}^{2000}$	$\{\cdot\}$	Estimated by (18). See Figure B.1
$\{z_1, z_2\}_{t=1950}^{2000}$	$\{\cdot,\cdot\}$	$z_{2,t} = \gamma_t z_{1,t}$. γ_t is college/high-school wage ratio by (7)

periods - and equal to almost half of the households natural debt limit in 1950. Given the rest of the calibration, it allowed us to obtain, in 1950 a top 10 wealth share of 56%, as in the data.

The endowment $\hat{\bar{c}}_{s,1950}$, in turn, was set to yield a service share of value-added equal to the data in 1950. In equilibrium, this value was approximately the number of market services consumed by the model's average household in 1950. A smooth path between $\hat{\bar{c}}_{s,1950}$ and and the steady state value $\hat{\bar{c}}_{s,T}$ was built. This steady state level was set to $0.15\hat{\bar{c}}_{s,1950}$, in order to minimize the distance between the top 10 wealth share path in the data and in the model.

The sequence for $\hat{\bar{c}}_{g,t}$ was calibrated to yield the moments estimated by (18)[5] and reported in Table 4.2. In (18)'s estimations, the moments were presented for the years 1947 and for 2010. While the latter year is part of our simulation (since our steady state is projected into the future, as discussed below), the first year is not. For simplicity, we target in the first year of our simulation, 1950, the same moments reported by (18) for 1947.

Table 4.2: Herrendorf et al (2013) *vs* model

	Herrendorf et al (2013)		Calibrated model	
	1947	2010	1950	2010
$p_g\hat{\bar{c}}_g/PC$	0.08	0.004	0.08	0.004

In order to guarantee a steady state for the model, Assumption 2.2 is assumed to hold here, where T is set to the year 2055, and τ is the final sample year, 2000. It means that growth rates will converge to the long run rate

[5]Calibration also guaranteed that this path was a smooth depreciation of the detrended values $\hat{\bar{c}}_{g,t}$, until a final steady state, in which this term is only a very small fraction of its initial values (see Table 4.3). Also, a remark should be done: in (18)'s model, the goods sector is divided into agriculture and manufacture, and all of the subsistence requirement comes from agriculture. It doesn't change the fact that the reported moment was the ratio between nominal expenditures with subsistence and aggregate consumption.

Table 4.3: How exogenous variables are projected into the future

Exogenous variable	Its projection
$\{\delta\}$	For $t \geq 2055$, δ_t is the predicted value for 2070 by a cubic interpolation to the data. $2000 < t < 2055$: smooth convergence (see Figure 4.1).
$\{\gamma_{g,t}\}$	For $t \geq 2055$, $\gamma_{g,t} = \gamma_{lr}$. For $2000 < t < 2055$, $\gamma_{g,t}$ converges smoothly to γ_{lr} (see Figure 4.1).
$\{\gamma_{s,t}\}$	For $t \geq 2055$, $\gamma_{s,t} = \gamma_{lr}$. For $2000 < t < 2055$, $\gamma_{g,t}$ converges smoothly to γ_{lr} (see Figure 4.1).
$\{z_{1,t}, z_{2,t}\}$	For $t > 2000$, $z_1 = \gamma z_2$, where γ is the college/high school wage ratio of 2000, from (7).
$\{\bar{c}_{g,t}\}$	For $t \geq 2055$, $\hat{\bar{c}}_{g,t}$ is $0.004\hat{\bar{c}}_{g,1950}$. For $1950 < t < 2055$, smooth convergence. The goal is to attend moments presented by (18). See Table 4.2.
$\{\bar{c}_{s,t}\}$	For $t \geq 2055$, $\hat{\bar{c}}_{g,t}$ is $0.15\hat{\bar{c}}_{s,t0}$. For $1950 < t < 2055$, smooth convergence. The goal is to attend moments presented by (18). See Table 4.2.
$\{\underline{a}_t\}$	For all periods, $\underline{a}_t$ grows at the long run growth rate. It means that $\hat{\underline{a}}_t$ is constant $\forall t$.
$\{\nu_t\}$	For $t \geq 2055$, ν_t is the predicted value for 2070 by a cubic interpolation to the data. For $2000 < t < 2055$: smooth convergence (see Figure 4.1).

smoothly between these years. Indeed, notice in Figure 4.1 how sectoral TFPs extracted from data for the sample years have a trend of natural convergence to an interval close to γ_{lr}, which makes the projection seems plausible.

To guarantee Assumption 2.2 we also need to make δ_t, ν_t, $\hat{\underline{a}}_t$ and the nonhomothetic terms constant after the year 2055. For the cases of ν_t and δ_t, a cubic polynomial is fit to the data - and the values until the year 2070 are extrapolated. This final point becomes the steady state for the respective variable, and a smooth path is constructed between data and the terminal levels in the year 2055, as shown in Figure 4.1.

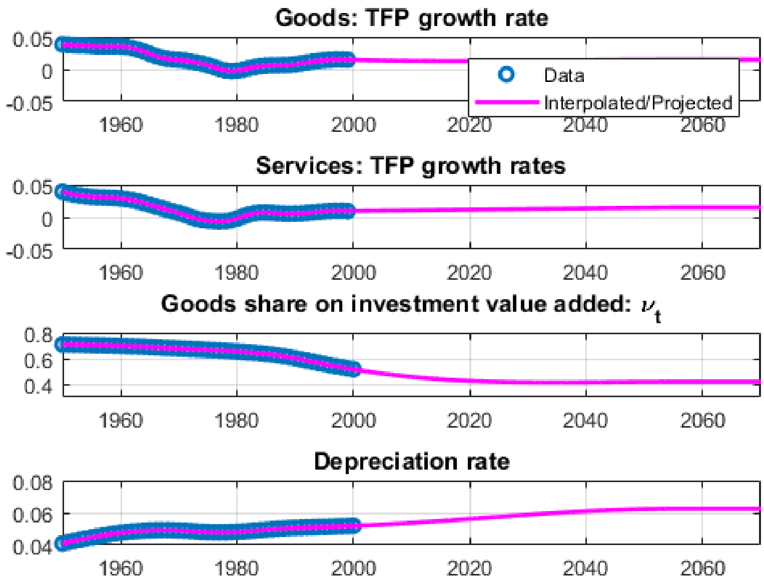
Nonhomothetic terms, as discussed above, had their paths adjusted to satisfy the moments and targets in our model. After 2055 their detrended values are a constant - and hence Assumption 2.2 is attended. The same conclusions hold trivially for the case of $\hat{\underline{a}}_t$ since it has been assumed that the detrended debt limit is constant for every t.

4.2 Results

In Figure 4.2, one can see prices and the TFPs extracted from data. Figures 4.3 and 4.4 present data and the model fit for sectoral shares and for the top 10 wealth share. By construction, the model has a reasonable performance in both cases, with the caveat that the last 15 years of the inequality path could not be properly followed by our simulation. For all simulations, we present the fit for the Gini index in the Appendix B. Generally speaking, we can follow the patterns in the Gini qualitatively, with a similar (but shifted downwards in the y-axis) shape.

In Chapter 2, we explored the intuition for the inequality path based on

Figure 4.1: Data and projected path for exogenous variables



the wealth gap between steady states and on the dynamics of the relative price of consumption and investment. For the US case (1950-2000), however, these mechanisms have relatively a much lower strength. While in Simulation 1 we observed that P_t more than doubled (remember Figure 2.1), here, in Figure 4.2, it rises by less than 10%. While in Simulation 1 consumption becomes around three times higher, in the calibrated model it achieves a small difference between 1950 and 2000 (in detrended units), with a fall after the 1970s.

These differences did not come as surprise. In our quantitative exercises, we studied a poor economy starting a fast and deeper growth process to reach, within some decades, a balanced growth path in a much richer position. Here, we start with the American economy in 1950, already wealthy and growing relatively steadily. At the same time that it points out possible limitations of our current exercise, it gives us a unique opportunity to test our mechanisms and intuitions in a much less favorable environment. That is what we look for in the next counter-factual exercises.

4.2.1
A lower subsistence requirement (Exercise 1)

What if American households were waived of subsistence requirements? To observe what would have happened in the current model, we do the same exercise of previous chapters: $\hat{\bar{c}}_{g,t}$ is made constant and equal to its steady level in the benchmark calibration of Table 4.3. Figures 4.5 and 4.6 compare the data, the benchmark calibrated model and the current counter-factual

Figure 4.2:

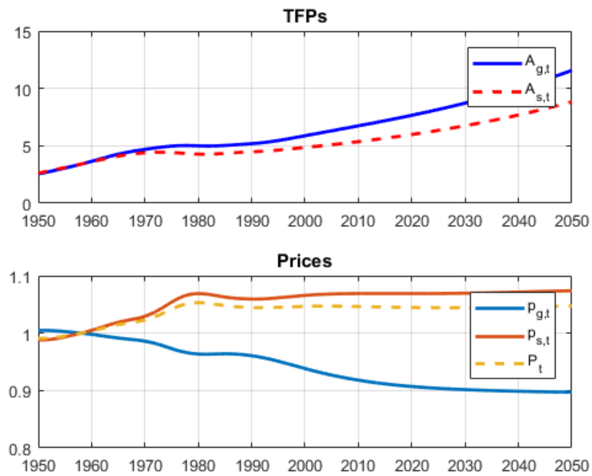
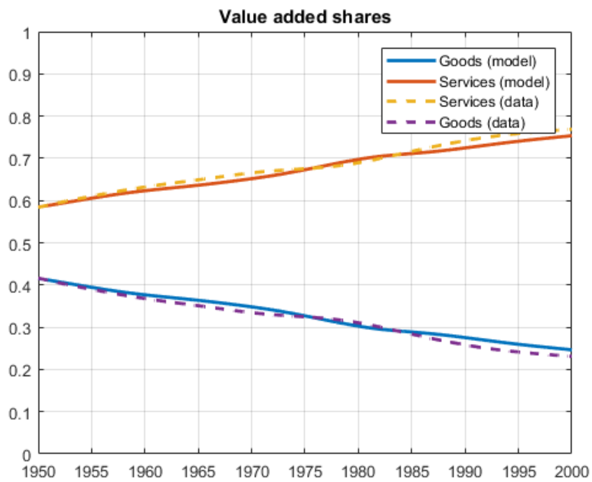


Figure 4.3:



simulation (Exercise 1)[6].

The same qualitative observations made in Simulation 4 of Chapter 2 can be found here: in comparison to the benchmark calibration, the inequality path has an equal shape but shifts downwards; there is an initial greater reallocation of value added to services followed by a less intense structural change dynamics; consumption initially rises more and interests soar less, yielding the lower peak in inequality.

As presented in Section 2.5, a small subsistence requirement augments households endowment in the dynamic problem, allowing them to consume more without a parallel rise in spendings. The down pressure over savings (for consumption smooth purposes) becomes smaller and interest rates soar less, making the effects on inequality less significant.

[6]Remember that data for the Gini index is available on the Appendix B, Figure B.14.

Figure 4.4:

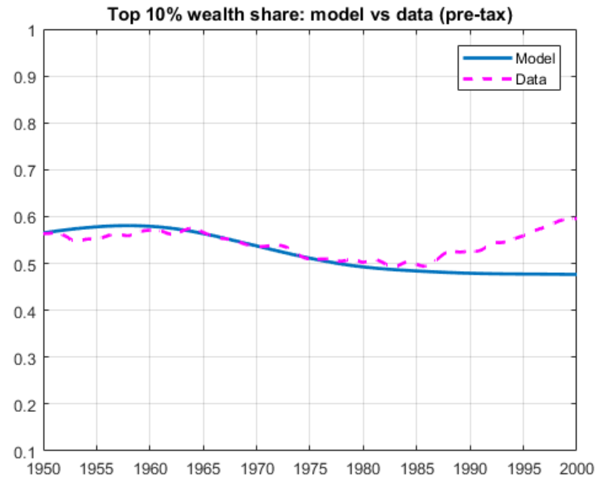


Figure 4.5:

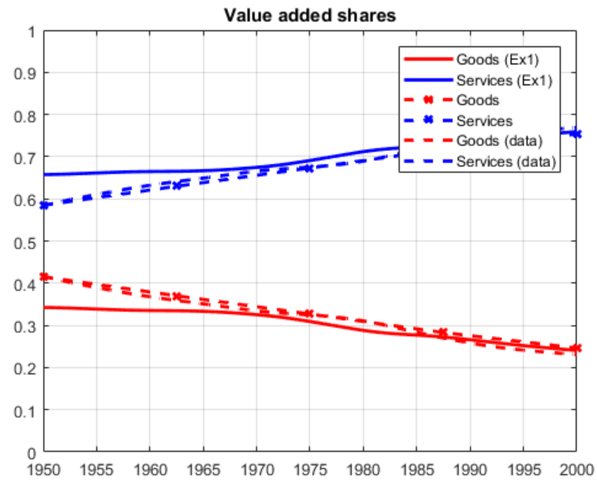


Figure 4.6:

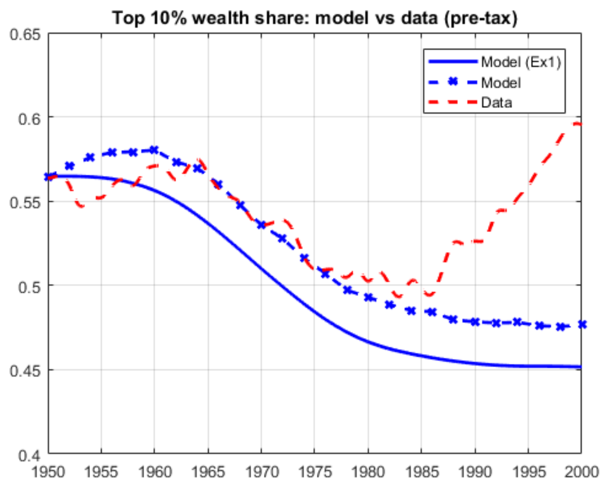
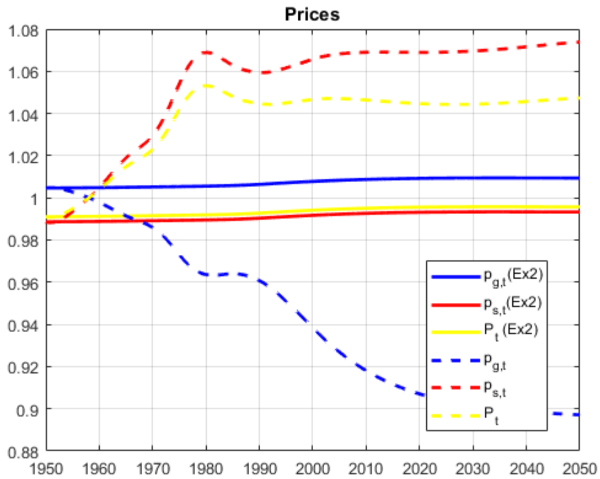


Figure 4.7:



Meanwhile, with the reduction of $\hat{\hat{c}}_{g,t}$, agents can also buy services with resources that would otherwise have been spent on goods. It makes the share of services jumps initially in comparison to the baseline calibration. Since $\hat{\hat{c}}_{g,t}$ will not decrease over time anymore, the income channel for structural change is cut down - and the process has a less intense progression in the following decades.

4.2.2
Uniform growth rates (Exercise 2)

The idea of Simulation 3 in Chapter 2 is the motivation for this exercise. Instead of using values extracted from data for γ_g and γ_s, we find here a common growth rate (approximately 0.013) capable of delivering a steady state economy with the same level of capital as the one in the benchmark calibration. For $t > 2000$ this common rate converges to γ_{lr} as in Table 4.3, always equal for both sectors.

It turns out, in Figure 4.7, that prices are almost unchanged through the transition (the only remaining mechanism affecting then is the path of ν_t). Remember, however, that the difference between prices in the benchmark calibration and prices in this exercise is much less significant than it was in Simulation 3 of Chapter 2. This explains why the change in sectoral allocations of value added in the initial period (see Figure 4.8) is not much significantly here. However, the same qualitative patterns emerge: the share of value added in goods production initially rise (given that goods prices don't decrease) and structural change follows a less intense dynamics with one of its main channels turned off.

With the top wealth share (Figure 4.9), we observe the exact pattern

Figure 4.8:

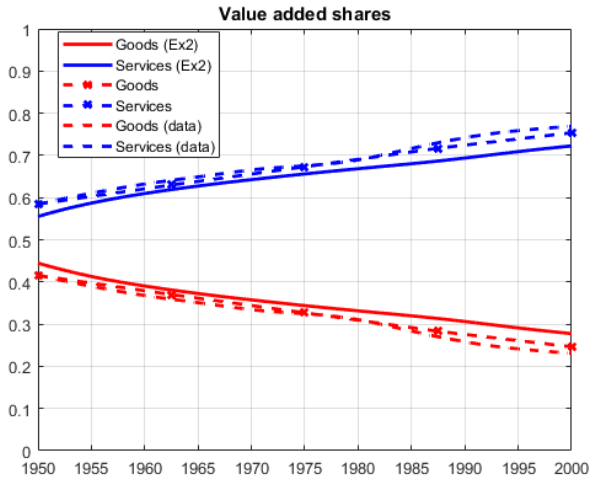
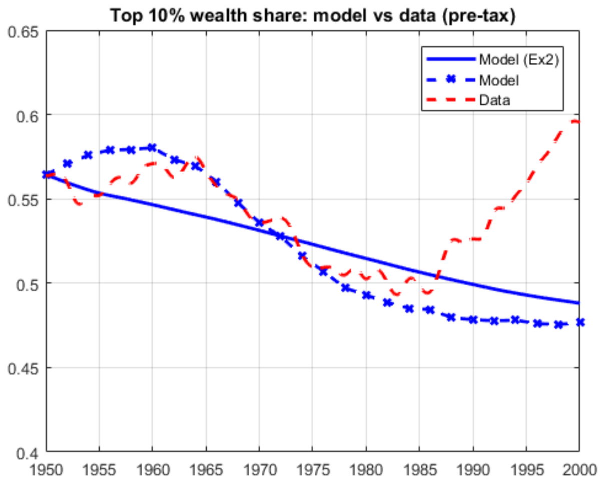


Figure 4.9:

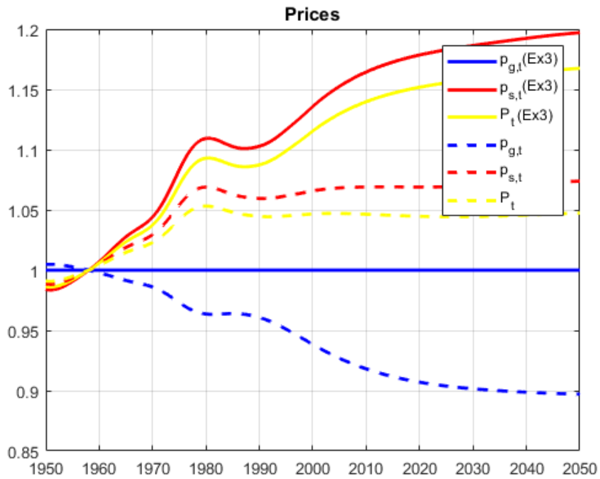


highlighted in Simulation 3: inequality initially achieves lower levels, but decreases more slowly than in the benchmark model[7] and ends up with higher levels after a couple of years.

A more stable relative price of consumption and investment limits the rise in interest rates during the first years of growth, as the pressure for consumption smooth is relieved (the opportunity cost of consumption in terms of investment will not increase in the first decades as before). This makes capital accumulation flatter in time and causes inequality to both rises less and to reduces more slowly, as observed in Section 2.5.

[7]The same is true for the Gini index as well (see Figure B.15).

Figure 4.10:



4.2.3
No services needed for the investment good (Exercise 3)

What if, from 1950 onward, the investment good was produced only by the goods sectors? Such exercise is interesting since assuming $\nu = 1$ is common practice in the literature. In Figures 4.10, 4.12 and in 4.11, one can see the outcomes of the model with the benchmark calibration and with an equal environment except for the permanent shock in 1950 that sets ν equal to one (Exercise 3).

Notice how the relative price of consumption and investment rises in the current exercise in comparison to the benchmark case. Investment becomes cheaper when its composition share of services (the more expensive sector) decreases. This relative price, therefore, achieves higher levels and higher growth rates in the first years.

Given the discussion in Section 2.5 we would expect a greater rise in inequality being occasioned by the fast-growing path for prices: as the consumption good is about to shortly become more expensive, there is a higher willingness to consume in 1950 and an initial greater pressure on interest rates and on inequality. In the opposite direction, however, we have a stimulus for lower consumption in 1950 yielded by the rise in P_t: as this price is about to become higher, the optimal balance between investment and consumption becomes more investment biased, and households proportionally cut down on consumption. Not surprisingly, given this balance of opposite forces, the outcome of the current exercise over interest rates and inequality is almost null, as conveyed by Figure 4.12.

The same cannot be said about sectoral shares, for which there is indeed a significant change occasioned by the exercise. With the same initial level of

Figure 4.11:

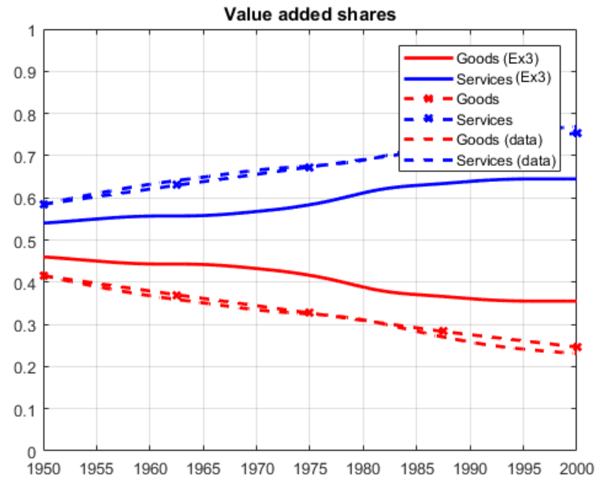
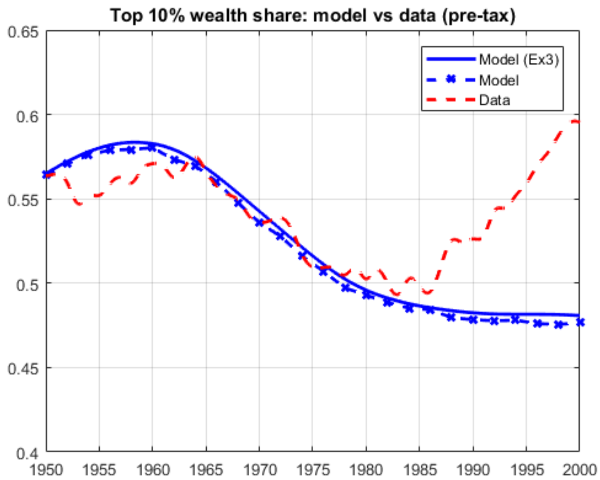


Figure 4.12:



capital stock and depreciation costs, there is a higher demand for goods for investment purposes here in comparison to the baseline calibration. There is, as a consequence, an initial rise in the share of goods. As the proportional level of consumption relative to assets decreases over time (in comparison to the baseline calibration), we have that a smaller amount of consumption lowers the income channel of structural change, making it less intense and the share of goods permanently higher.

4.2.4 A different path for P_t (Exercise 4)

For completeness, we do once more the exercise of Simulation 2, in Section 2.5. The same initial and final steady states of the benchmark calibration are maintained, but the relative price of consumption and investment achieves its steady level ahead on time, with higher initial growth rates. We present this exercise (Exercise 4) in Figures B.16 to B.19 at the Appendix.

Inequality peaks ahead on time and in greater amounts, while the share of goods in the value added decreases initially in comparison to the benchmark model (but these differences here are extremely low, given the small amplitude in the dynamics of P_t). Once more, however, the calibrated model yields the qualitative results previously discussed.

5
Conclusions

Without departing from the benchmark model of structural change, we built a direct linkage between sectoral dynamics and the path of inequality. We believe to have succeeded in providing evidence that this benchmark setup has lessons to tell about the behavior of inequality in the long run. The fact that savings and capital accumulation have played a major role in this model - while, on the other hand, related literature uses mainly labor market frictions to study inequality in multi-sector growth frameworks - gives extra weight to our approach.

Our main contribution has been to suggest that sectoral dynamics matters for the inequality path. The relative price of goods and services impacts the trade-off between investment and consumption and hence impacts savings decisions and the path of interest rates. Finally, inequality ends up being affected as a result of changes in these last two variables.

If capital is to become cheaper in terms of consumption as time goes by (which was driven by our structural change setup), it is optimal for agents to allocate more consumption into the earlier periods of time, since the intertemporal trade-off associated to consumption is then smaller. Such anticipation phenomenon leads to fewer savings and higher interests during the first decades. The higher interest rates become, the more the richest save in proportion to the poorest households - and hence, the more inequality soars.

Exogenous variations in the subsistence consumption of goods (an important source of structural change) during the transition beginning can also affect the behavior of interest rates in the first decades of growth - impacting inequality. A small subsistence requirement, for instance, allows households to augment their utility with consumption without a parallel rise in spendings. The down pressure over savings (for consumption smooth purposes) becomes smaller and interest rates soar less, making the rise on inequality less significant than it would otherwise be observed.

A labor mobility friction which limits the maximum mass of workers capable of switching sectors at each period creates a greater interplay between inequality and structural change. The friction becomes active after a shock that puts pressure over sectoral reallocation - and it spreads into a wider period of time the labor reallocation (and hence the structural change) which otherwise would instantaneously occur.

Also, the friction yields a miss-allocation of labor across sectors while it is active, making the economy momentously worse in comparison to the frictionless case. Since the friction does not bind in the final steady state, the consumption smooth desire becomes higher when it is binding. Interest rates soar, as a result, leading to great inequality in the first decades.

Finally, the calibrated version of the model (for the US economy, 1950-2000) founds qualitative and (in some cases limited) quantitative evidence of the main intuitions highlighted by the paper. In accordance to some counterfactual analysis, if the subsistence requirement of goods was lowered, in 1950, to its real value of 2000 (and remained fixed), the top 10 wealth share would be lower throughout the $1950-2000$ period. If the relative price of goods and services were constant, inequality would be lower in the 1960's and higher after the 1970s.

Bibliography

[1] ACEMOGLU, D.; AND GUERRIERI, V. **Capital deepening and non-balanced economic growth**. Journal of Political Economy, 116(3):467–498, 2008.

[2] ACHDOU, Y.; HAN, J.; LASRY, J. M.; LIONS, P. L.; MOLL, B. **Income and wealth distribution in macroeconomics: A continuous-time approach**. NBER Working Paper No. 23732, 2017.

[3] AIYAGARI, R. **Uninsured idiosyncratic risk and aggregate saving**. The Quarterly Journal of Economics, 109(3):584–616, 1994.

[4] ALDERSON, A. S.; NIELSEN, F. **The kuznets curve and the great u-turn: Income inequality in u.s. counties, 1970 to 1990**. American Sociological Review, 62:12–33, 1997.

[5] BERNADETTE P.; SEMEGA, J.; KOLLAR, M. **Income and poverty in the united states: 2015**. Current Population Reports 1,P60-256(RV), U.S. Bureau of the Census, 2016.

[6] BOPPART, T. **Structural change and the kaldor facts in a growth model with relative price effects and non-gorman preferences**. Econometrica, 82(6):2167–2196, 2014.

[7] BUERA, F.; KABOSKI, J. P. **The rise of the service economy**. The American Economic Review, 102(6):2540–69, 2012.

[8] CASELLI, F.; COLEMAN, W. **The u.s. structural transformation and regional convergence: A reinterpretation**. Journal of Political Economy, 109(3):584–616, 2001.

[9] CHATTERJEE, S. **Transitional dynamics and the distribution of wealth in a neoclassical growth model**. Journal of Public Economics, 54:97–119, 1994.

[10] CHATTERJEE, S; RAVIKUMAR, B. **Minimum consumption requirements: Theoretical and quantitative implications for growth and distribution**. Macroeconomic Dynamics, 3:482–505, 1999.

[11] DÍAZ, A.; ÁLVAREZ-PELÁEZ, M. **Minimum consumption and transitional dynamics in wealth distribution**. Journal of Monetary Economics, 52:633–667, 2005.

[12] DUARTE, M.; DIEGO R. **The role of the structural transformation in aggregate productivity**. Quarterly Journal of Economics, 125(1):129–173, 2010.

[13] FEENSTRA, R. ;INKLAAR, R.;TIMMER, M . **The next generation of the penn world table**. American Economic Review, 105(10):3150–3182, 2015.

[14] FOELLMI, R.; ZWEIMUELLER, J. **Structural change, engel's consumption cycles and kaldor's facts of economic growth**. Journal of Monetary Economics, 55(7):1317–1328, 2008.

[15] GORDON, R. **The measurement of durable goods prices**. The University of Chicago Press, Chicago, IL, 1997 edition edition, 1990.

[16] GREENWOOD, J., HERCOWITZ, Z., KRUSELL, P. **Long-run implications of investment-specific technological change**. The American Economic Review, 87(3):342–362, 1997.

[18] HERRENDORF, B., ROGERSON, R., VALENTINYI, Á. **Two perspectives on preferences and structural transformation**. The American Economic Review, 103(7):2752–2789, 2013.

[19] HERRENDORF, B.; ROGERSON, R.; VALENTINYI, Á. **Growth and structural transformation**. Handbook of Economic Growth, vol. 2B:855–941, 2014.

[20] KALDOR, N.. **Capital accumulation and economic growth**. In: CONFERENCE HELD BY THE INTERNATIONAL ECONOMICS ASSOCIATION, London, Macmillan, 1963.

[21] KIM, D.; TOPEL, R. H. **Differences and Changes in Wage Structures**, chapter Labor Markets and Economic Growth: Lessons from Korea's Industrialization 1970–1990. University of Chicago Press.

[22] KONGSAMUT, P.; REBELO, S.; AND XIE, D. **Beyond balanced growth**. Review of Economic Studies, 68:869–882, 2001.

[23] KRUSELL, P.; OHANIAN, L.; RÍOS-RULL, R-R.; AND VIOLANTE, G. **Capital-skill complementarity and inequality: A macroeconomic analysis**. Econometrica, 68(5):1029–1053, 2000.

[24] KUZNETS, S; JENKS, E. **Shares of Upper Income Groups in Income and Savings**, chapter Shares of Upper Income Groups in Savings. National Bureau of Economic Research.

[25] KUZNETZ, S. **Economic growth and income inequality**. The American Economic Review, 45:1–28, 1955.

[26] KUZNETS, S. **Quantitative aspects of the economic growth of nations: Ii, industrial distribution of national product and labour forcce**. Economic Development and Cultural Change, 5(4), Supplement:1–111, 1957.

[27] KUZNETS, S. **Modern economic growth: Findings and reflections**. The American Economic Review, 63(3), 1973.

[28] LEE, D.; WOLPIN, K. **Intersectoral labor mobility and the growth of the service sector**. Econometrica, vol. 74:1–46, 2006.

[29] MATSUYAMA, K. **Agricultural productivity, comparative advantage and economic growth**. Journal of Economic Theory, 58(2):317–334, 1992.

[30] MOLLICK, A. V. **Income inequality in the u.s.: The kuznets hypothesis revisited**. Economic Systems, 36:127–144, 2012.

[31] NGAI, R.; PISSARIDES. **Structural change in a multisector model of growth**. The American Economic Review, 97(1):429–433, 2007.

[32] NIELSEN, F. **Income inequality and industrial development: Dualism revisited**. American Sociological Review, 59:654–677, 1994.

[33] PIKETTY, T.; SAEZ, E.; ZUCMAN, G. **Distributional national accounts: Methods and estimates for the united states**. NBER Working Paper No. 22945, 2016.

[35] TIMMER, M.; DE VRIES, G; DE VRIES, K. **Routledge Handbook of Industry and Development**, chapter Patterns of Structural Change in Developing Countries.

[36] VELOSO, P. **Introducing capital in a three sector model of structural transformation: a cross-country analysis**. Dissertação de mestrado, Pontifícia Universidade Católica do Rio de Janeiro - PUC-Rio, Rio de Janeiro, 2015.

A
Proofs and derivations

A.1
Derivation of Assumption 2.1

The goal of Assumption 2.1 is to guarantee a well defined problem, in which services consumption is not negative, and the goods consumption is at least $\hat{\hat{c}}_g$, for *every household in our continuum of unitary mass* $j \in [0,1]$. Imposing it in the demand for services 2-10, we have:

$$\hat{\hat{c}}_{s,t} \leq \frac{P_t \min_{j\in[0,1]} \hat{c}_t(j)}{p_{s,t}} \left[1 + \frac{\omega_g}{\omega_s}\left(\frac{p_{g,t}}{p_{s,t}}\right)^{1-\epsilon}\right]^{-1} \tag{A-1}$$

As shown by (2) in a problem like ours, the following results are in place: $c_t(\hat{a}, z_2) \geq c_t(\hat{a}, z_1)$; $\partial_a c_t(\hat{a}, z) \geq 0$ and $s(\underline{\hat{a}}, z_1) = 0$. It means that the we can substitute $\min_{j\in[0,1]} \hat{c}_t(j)$ for $c_t(\underline{\hat{a}}, z_1)$.

Taking the budget constraint of the poorest household we write:

$$s_t(\underline{\hat{a}}, z_1) = 0 = \hat{w}_t z_1 + \underline{\hat{a}}_t(r_t - \gamma_{lp}) - P_t\hat{c}_t(\underline{\hat{a}}, z_1) - p_{g,t}\hat{\hat{c}}_{g,t} + p_{s,t}\hat{\hat{c}}_{s,t} \tag{A-2}$$

Use A-2 to substitute $P_t c_t$ out in A-1, and find the expression in Assumption 2.1. One could insert technology variables in place of prices in the final result. However, this does not bring additional intuition to the explanation and makes the algebra less clear.

A.2
Proof or Proposition 2.1

Imposing that $\underline{a}$ is smaller (in absolute value) than the natural debt limit makes the coefficient of absolute risk aversion bounded when a becomes closer to $\underline{a}$. Also, the coefficient of relative risk aversion is clearly bounded above for all c. If we weren't working with CRRA utility, this conditions would need to be met before one states Proposition 2.1.

Now, consider the asset supply function and the implicit relation between capital and interest rates in 2-16:

$$S(r) \equiv \int_{\underline{a}}^{\infty} ag(a, z_1)da + \int_{\underline{a}}^{\infty} ag(a, z_2)da \tag{A-3}$$

$$K(r) \equiv \{k \in \mathbb{R}_{\geq 0};\ \partial_k \hat{Y} = r + \delta\} \tag{A-4}$$

In the literature of Hugget and Ayagari models, a graphical approach is often used to prove existence of equilibrium. In this regard, here we must assure the continuity of functions $S(r)$ and $K(r)$ and also show the following limits: $\lim_{r \to -\infty} S(r) = \underline{a}$, $\lim_{r \to \tilde{\rho}} S(r) = \infty$, $\lim_{r \to -\delta} K(r) = \infty$ and $\lim_{r \to \infty} K(r) = 0$. Hence, there exists at least one value of r for which $S(r) = K(r)$.

The continuity and the above limits concerning $S(r)$ have been extensively proved by (2) in a one sector model. However, if one takes our stationary model in 2-19 and transforms it inserting $\tilde{r} \equiv r - \gamma_{lr}$, and $\tilde{y}_i \equiv \hat{w}z_i - p_g\hat{\bar{c}}_g + p_s\hat{\bar{c}}_s$, the same system and the same hypotheses of (2) emerge here (i.e, $\tilde{r} < \rho$, $y_2 > y_1$, coefficients of relative and absolute risk aversion bounded). The only exception in this equivalence is the presence of the constant P that multiplies $\hat{c}$ in the HJB. It can be directly verified, on the other hand, that the presence of P neither affects the proof of $S(\tilde{r})$ continuity, nor the proof of its limits when $\tilde{r} \to -\infty$ and $\tilde{r} \to \rho$, provided that $P \in [0, \infty)$. Further details of this verification are available under request.

Given our Cobb-Douglas aggregate production function 2-8, the continuity of $K(\tilde{r}) \equiv \{k \in \mathbb{R}_{\geq 0};\ \partial_{\hat{k}} \hat{Y} = \tilde{r} + \gamma_{lr} + \delta\}$ is straightforward, since $K(\tilde{r}) = \Gamma(\tilde{r} + \gamma_{lr} + \delta)^{-\frac{1}{1-\alpha}}$ where $\Gamma \equiv (\alpha p_s (A_s N)^{1-\alpha})^{\frac{1}{1-\alpha}} > 0$. It is also clear that, as $r \to -\delta$, i.e, $\tilde{r} \to -\gamma_{lr} - \delta$, $K(\tilde{r}) \to \infty$ - and that, as $\tilde{r} \to \infty$, $K(\tilde{r}) \to 0$.

Therefore, for at least one $\tilde{r}$, and hence, for at least one r, we have the equality $S = K$, clearing the assest/capital market. By the Walras' law the goods' market is also cleared. To see this, take agents' budget constrain 2-6 and integrate its both sides over the density $g(a, z)$. Use the equality $S(r) = K(r)$ and the Euler theorem to obtain $\hat{Y}_t = p_g\hat{Y}_g + p_s\hat{Y}_s = X + p_g\hat{c}_g + p_s\hat{c}_s$, which synthesizes the equilibrium in the goods' market.

A.3 Euler equation 2-20

To arrive in the Euler equation 2-20, we follow (2). Take the HJB expression in 2-9 and differentiate it in terms of the state $\hat{a}_t$. Apply the

Envelope theorem to obtain:

$$\tilde{\rho}\partial_a v(\hat{a}_t, z, t) = \partial^2_{aa} v(\hat{a}_t, z, t)\dot{\hat{a}}_{t,z} + \partial_a v(\hat{a}_t, z, t)(r_t - \gamma_{lr}) + \lambda_z[\partial_a v(\hat{a}_t, z', t) - \partial_a v(\hat{a}_t, z, t)] \\ + \partial^2_{a,t} v(\hat{a}_t, z, t)$$

Use that, optimally, agents choose $\hat{c}$ such that $u'(\hat{c}(a_t, z, t)) = \partial_a v(\hat{a}, z, t)P_t$. This relationship is yielded by the FOC implicit in the HJB. Substitute $\partial_a v(\cdot)$ out:

$$(\tilde{\rho} + \dot{P}_t/P_t + \gamma_{lr} - r_t)u'(\hat{c}(\hat{a}_t, z, t)) = u''(\hat{c}(\hat{a}_t, z, t))[\partial_a \hat{c}(\hat{a}_t, z, t)\dot{\hat{a}}_t + \dot{\hat{c}}(\hat{a}_t, z, t)] \\ + \lambda_z[u'(\hat{c}(\hat{a}_t, z', t)) - u'(\hat{c}(\hat{a}_t, z, t))]$$

The right hand side is the expected change in marginal utility of consumption for an individual, i.e, $E_t\{du'(\hat{c}(\cdot))\}/dt$. Using this, we have (2-20).

A.4
Proof of Proposition 3.1

We have the time invariant vector $\{\hat{A}_g, \hat{A}_s, \nu\}$, all the stationary value functions, a distribution $g(\cdot)$, policy functions and allocations of capital and labor satisfying Definitions 2.1 and 2.2. We must show that these same objects are an equilibrium for the extended model as well, with sectoral distributions $g(a, z, \Omega)$ being such that $g(\cdot, \Omega_G) + g(\cdot, \Omega_S) = g(\cdot)$.

To begin with, notice that the allocation of capital and labor features equal and constant capital-labor ratio. Plugging this allocation into 3-1 and 3-2 (and into firms problem first order conditions), we see that wages, prices and interest rates in the extended model will equal the ones we already have from the benchmark model (which are time-invariant).

Since sectoral wages are always equal, we have that $v(\cdot, G) = v(\cdot, S)$. Hence, the dynamic system given by the HJB and the KF equations becomes:

$$\tilde{\rho}v(\hat{a}, z, \Omega) = \max_{\hat{c}} \{u(\hat{c}) + \partial_a v(a, z, \Omega)s(\hat{a}, z, \Omega) + \lambda_z(v(\hat{a}, z', \Omega) - v(\hat{a}, z, \Omega))\}$$
$$s(\hat{a}, z_i, \Omega) = \hat{w}_\Omega z_i + \hat{a}(r - \gamma_{lp}) - P\hat{c} - p_g\hat{\bar{c}}_g + p_s\hat{\bar{c}}_s$$
$$0 = -\partial_{\hat{a}}[g(\hat{a}, z, \Omega)s(\hat{a}, z, \Omega)] - \lambda_z g(\hat{a}, z, \Omega) + \lambda_{z'} g(\hat{a}, z', \Omega) \\ + \theta(b_t(\Omega')g(\hat{a}, z, \Omega') - b(\Omega)g(\hat{a}, z, \Omega))$$

If $v^*(\cdot)$ is a value function from the equilibrium of Definition 2.1, it satisfies the HJB in 2-19 by definition, as well as the borrowing constraint. Hence, for Ω_G or Ω_S, the same $v^*(\cdot)$ solves the HJB just above, attending the borrowing constraint. Once $c(\cdot) = u'^{-1}(\partial_a v(\cdot))$, the same can be said of consumption and savings policy functions coming from equilibrium of Chapter 2.

The policy function $b(\Omega, t)$ is implicitly determined (trivially). Wages are equal for all periods - and, in accordance to 3-3, any path of zeros or ones satisfy an optimum. With this, we have finished showing that the first item of Definition 3.2 is satisfied.

Define $\tilde{g} = g(\Omega_G, .) + g(\Omega_S, .)$. Add the KF equation above in the case where $\Omega = G$ to the KF when $\Omega = S$. Remember that $v(\cdot, G) = v(\cdot, S)$, $c(\cdot, G) = c(\cdot, S)$ and hence $s(\cdot, G) = s(\cdot, S) = s(\cdot)$, where $s(\cdot)$ is the policy from equilibrium of Chapter 2. Obtain the following expression:

$$0 = -\partial_{\hat{a}}[\tilde{g}(\hat{a}, z)s(\hat{a}, z)] - \lambda_z \tilde{g}(\hat{a}, z) + \lambda_{z'} \tilde{g}(\hat{a}, z')$$

By construction, the same distribution $g(\cdot)$ that satisfies the equilibrium in Definition 2.1 satisfies the equation above in the place of $\tilde{g}(\cdot)$. Hence, any pdfs $g(\cdot, \Omega = G)$ and $g(\cdot, \Omega = S)$ such that $g(\cdot, \Omega = G) + g(\cdot, \Omega = S) = g(\cdot)$ are possible distributions path for the stationary equilibrium. We already have that $g(\cdot)$ satisfies the boundary conditions:

$$\int_{\underline{a}}^{\infty} g(a, z_i) da = \frac{\lambda_j}{\lambda_i + \lambda_j} \quad \text{for } (i, j) = \{(1, 2); (2, 1)\}$$

Here, to be consistent with the labor allocations, we impose, in addition to the boundary condition above, another one:

$$\int_{\underline{a}}^{\infty} g(a, z_1, \Omega_i) da + \int_{\underline{a}}^{\infty} g(a, z_2, \Omega_i) da = N_i/N \quad \text{for } i =\in \{g, s\}$$

Therefore two pdfs $g(\cdot, \Omega_G)$ and $g(\cdot, \Omega_S)$ such that $g(\cdot, \Omega_G) + g(\cdot, \Omega_S) = g(\cdot)$, where $g(\cdot)$ is the distribution of the stationary equilibrium from Chapter 2, and such that the boundary conditions hold, are possible distributions paths for the extended model. We have shown that condition 4 of the Definition 3.2 is satisfied.

Since firms problem is identical both in the extended model and in the benchmark model, and since our allocation of capital and labor comes from an equilibrium of the benchmark model, condition 2 is satisfied by definition. Also, the same prices and the same policy functions guarantee here that market clearing conditions are attended, which is condition 3 in the Definition 3.2.

Condition 5 is also verified. Since wages are always equal and constant in this equilibrium path, sectoral employment is stable and we can consider that no labor reallocation with positive mass occurs. Assumption 3.1 is exactly the same as 2.1 because wages are equal - and 2.1 is already attended by definition.

B
Additional figures

Figure B.1: Manufacture share on total value added used to deliver final investment consumption (US). Data from (18)'s decomposition.

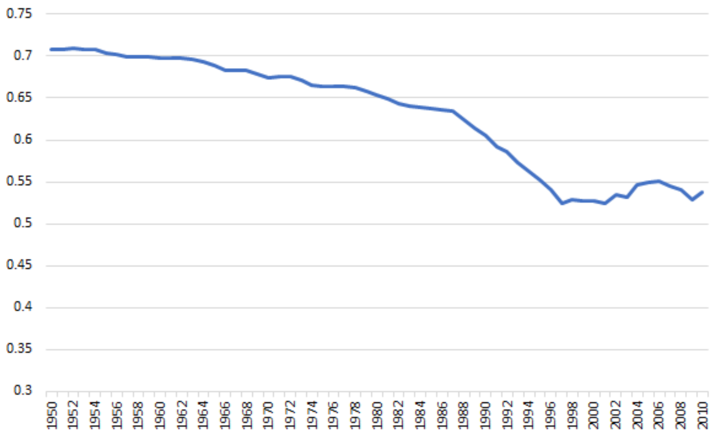


Figure B.2: Savings policy function (Simulation 1)

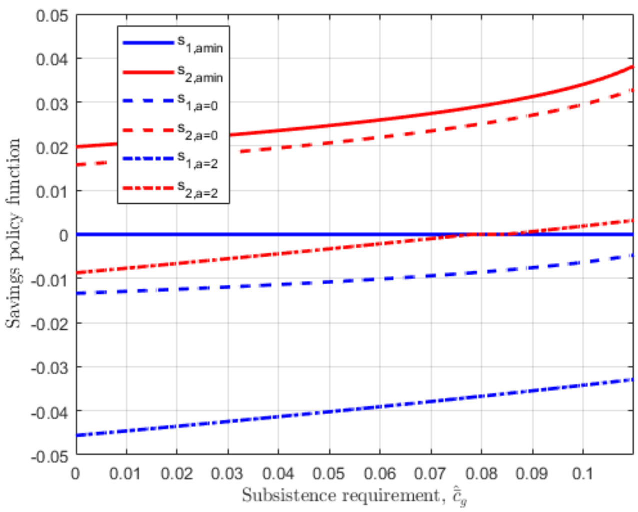


Figure B.3: Consumption policy function (Simulation 1)

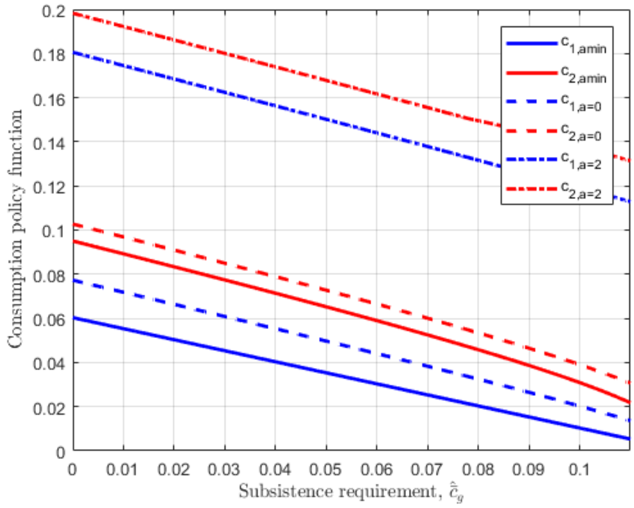


Figure B.4: Savings policy functions. Blue line: $s(a, z_1)$. Red line: $s(a, z_2)$.

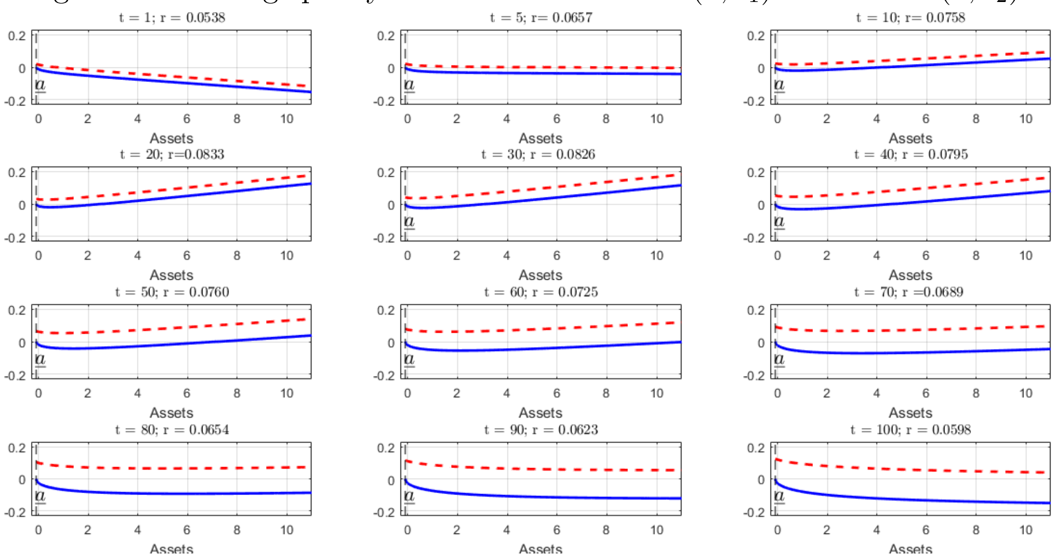


Figure B.5: Simulation 3.b

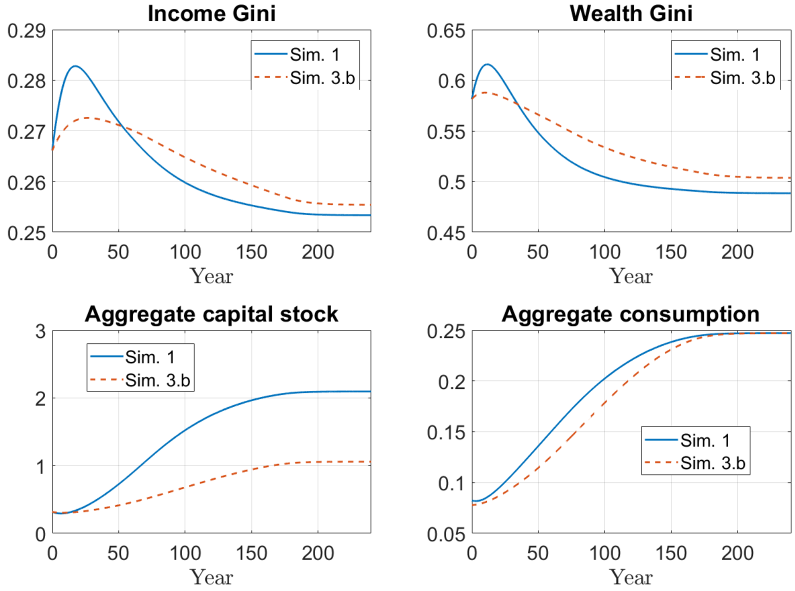


Figure B.6: Simulation 3.b

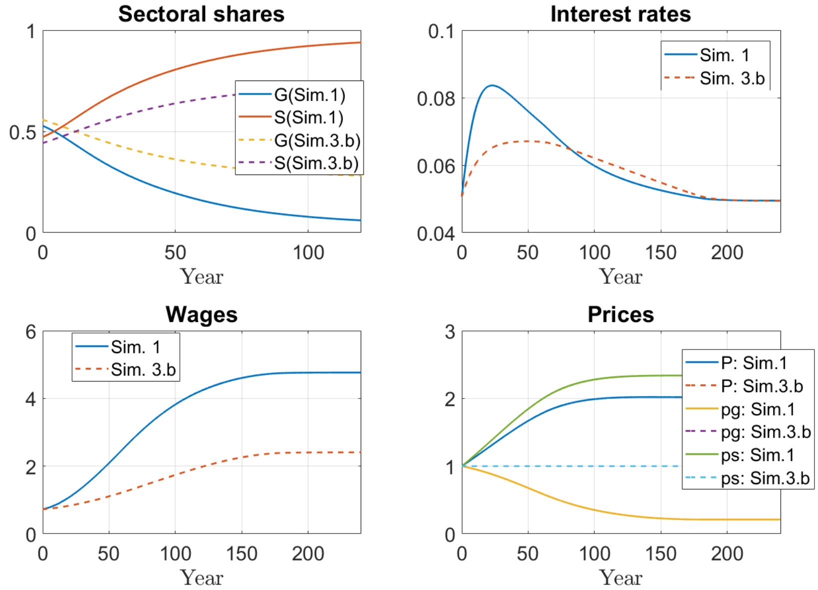


Figure B.7: Simulation of Chapter 3

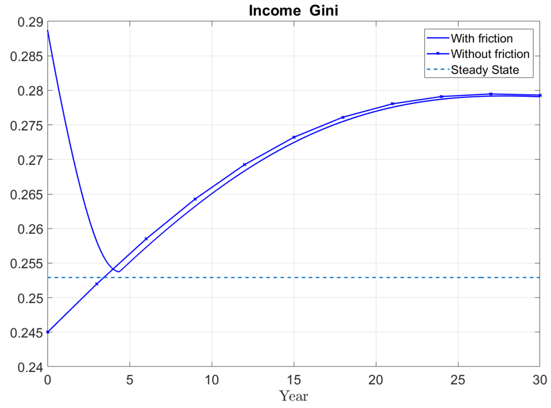


Figure B.8: Simulation of Chapter 3

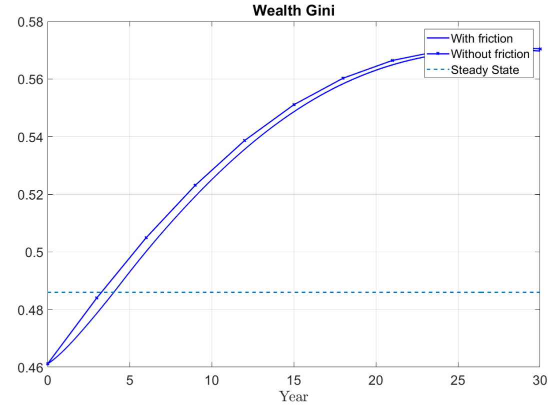


Figure B.9: Simulation of Chapter 3

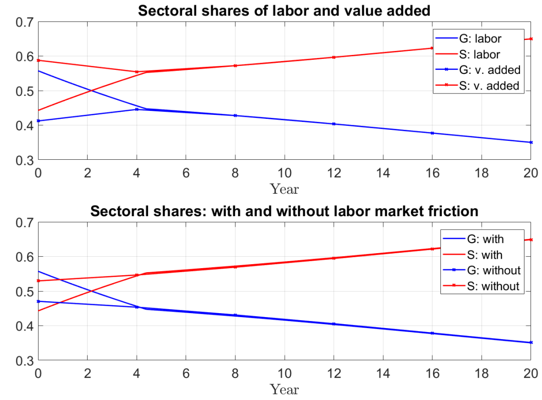


Figure B.10: Simulation of Chapter 3

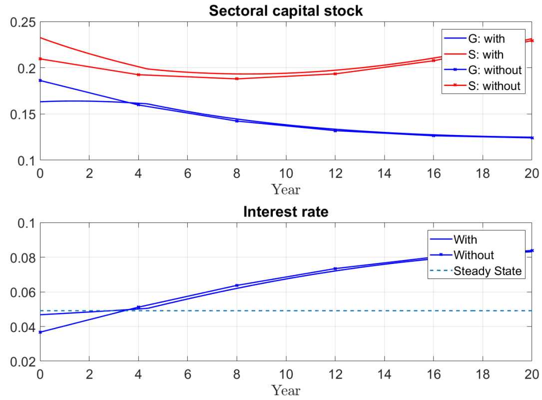


Figure B.11: Simulation of Chapter 3

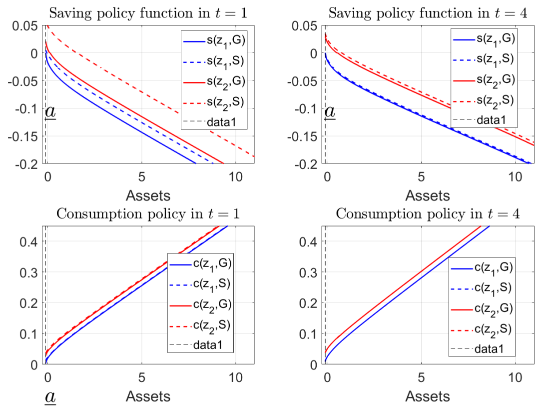


Figure B.12: College/high school educated wage ratio by (7)

Figure B.13: Calibrated model

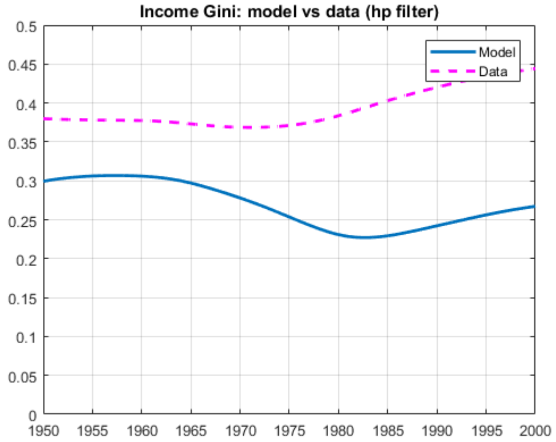


Figure B.14: Calibrated model (Exercise 1)

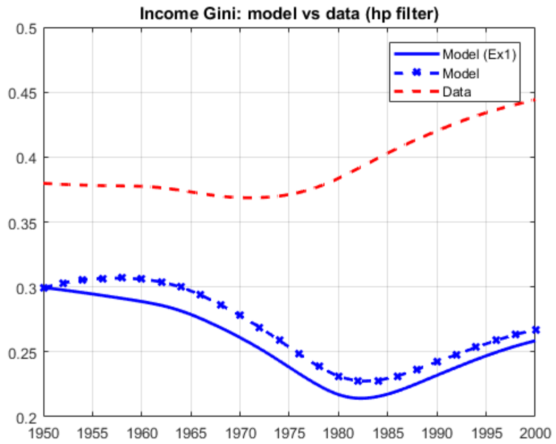


Figure B.15: Calibrated model (Exercise 2)

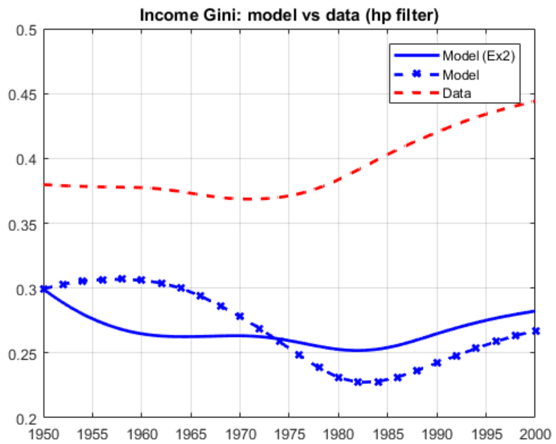


Figure B.16: Calibrated model (Exercise 4)

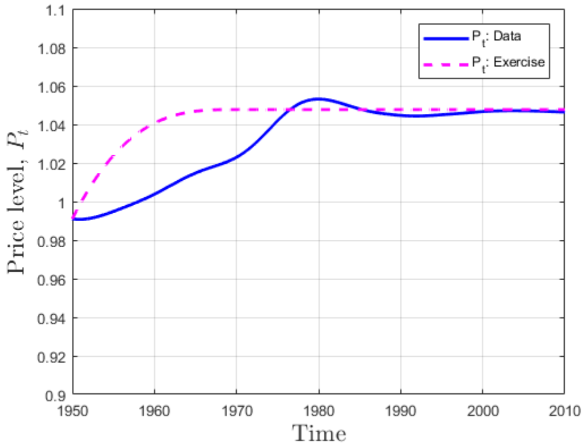


Figure B.17: Calibrated model (Exercise 4)

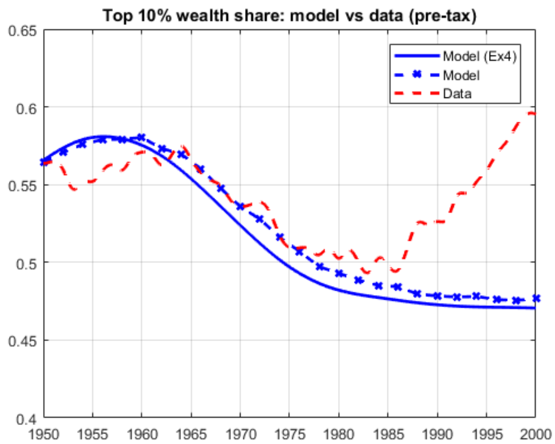


Figure B.18: Calibrated model (Exercise 4)

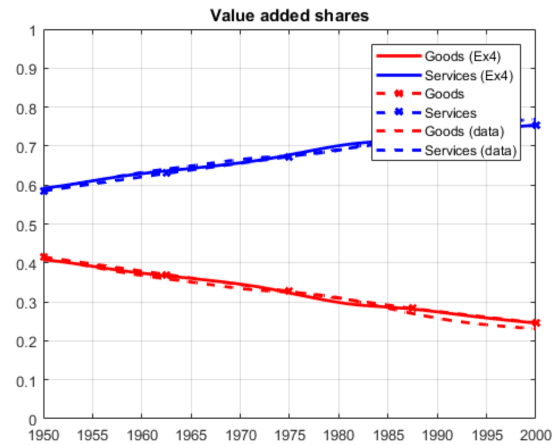


Figure B.19: Calibrated model (Exercise 4)