3
Robust Control Strategies

In this chapter, a new approach to modeling systems called chained form system is presented, this technique will help to design the control strategy. Additionally a new theorem is presented, this theorem is a robust technique stabilize the TMR to the origin overcoming the problems in the polar coordinates controller.

3.1
Chained Form System

This section presents, a well known transformation, called Chained form system in which formulation is carried out using Lie's Algebra. This form will us some advantages in the control design.

3.1.1
Preliminary Definitions

Definition 3.1 (Chained Form System.) *A general two-input system can be defined in its chained form as follows [71]:*

$$
\begin{aligned}
\dot{x}_1 &= v_1 \\
\dot{x}_2 &= v_2 \\
\dot{x}_3 &= x_2 v_1 \\
\dot{x}_4 &= x_3 v_1 \\
&\vdots \\
\dot{x}_n &= x_{n_1} v_1 \ .
\end{aligned}
\tag{3-1}
$$

In order to formulate the Chained Form System, a few definitions of Lie's Algebra [71, 72] are presented:

Definition 3.2 (Lie Brackets.) *Given two functions f and g, that depend of a vector q, there is an operator Lie bracket as follows:*

$$[\,f, g\,] = \frac{\partial g}{\partial q} f(q) - \frac{\partial f}{\partial q} g(q) \ . \tag{3-2}$$

Example: In order to demonstrate the definitions of Lie's Algebra, some parts of the Chained Form formulation for the Unicycle model will be presented, whose generalized coordinates are defined by $q = [\, x \ y \ \theta \,]^{\mathsf{T}}$:

$$\dot{q} = g_1\, v + g_2\, \omega \ ,$$
$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} v \ + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \ .$$

Now, we can implement the Lie Brackets for g_1 and g_2 as :

$$[\, g_1, g_2 \,] = \frac{\partial g_2}{\partial q} g_1(q) - \frac{\partial g_1}{\partial q} g_2(q) \,,$$

$$[\, g_1, g_2 \,] = \begin{bmatrix} 0 & 0 & -\sin\theta \\ 0 & 0 & \cos\theta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \ ,$$

which shows the process to find the Lie derivatives of two vectors.

Definition 3.3 (Lie derivatives) *Lie derivatives define the time derivative of a given V function along the flow of the function "f" and it is denoted as $L_f V$:*

$$L_f V = \frac{\partial V}{\partial q} f(q) \ , \tag{3-3}$$

which results always will be a scalar function.

Example: Using the last example, applying the same kinematic model and an arbitrary function $h_2 = x \sin\theta - y \cos\theta$ and the g_1 vector, it is possible to calculate the Lie derivatives:

$$L_{g_1} h_2 = \frac{\partial h_2}{\partial q} g_1 \ ,$$

$$L_{g_1} h_2 = \begin{bmatrix} \sin\theta & -\cos\theta & (x\cos\theta + y\sin\theta) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ ,$$

$$L_{g_1} h_2 = x\cos\theta + y\sin\theta \ .$$

Definition 3.4 (Nested Lie Derivatives) *Some operations requires a nested Lie Derivative, and it is defined as:*

$$L_f^{(n)} V = L_f \left(L_f^{(n-1)} V \right) \ . \tag{3-4}$$

Definition 3.5 (Iterated Lie Products) *In some cases is necessary to use the Lie Brackets iteratively and is defined as follows:*

$$ad_{g_1}g_2 = [\,g_1,\, g_2\,]\ ,$$

$$ad_{g_1}^{(k)}g_2 = [\,g_1,\, ad_{g_1}^{(k-1)}g_2\,]\ , \tag{3-5}$$

$$ad_{g_1}^{k}g_2 = [g_1, \cdots, [g_1, \cdots, [\,g_1,\, g_2\,]\cdots]]\ .$$

Definition 3.6 (Differential of smooth Function) *If we have a smooth function β defined in $\mathbb{R}^n \to \mathbb{R}$ its differential is defined by:*

$$d\beta = \begin{bmatrix} \frac{\partial \beta}{\partial q_1} & \frac{\partial \beta}{\partial q_2} & \cdots & \frac{\partial \beta}{\partial q_n} \end{bmatrix}\ . \tag{3-6}$$

Example: Given the same smooth function $h_2 = x \sin\theta - y \cos\theta$ in the space of $q = [\,x\ y\ \theta\,]^{\mathsf{T}}$ its differential is defined by:

$$dh_2 = \begin{bmatrix} \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} & \frac{\partial h_2}{\partial \theta} \end{bmatrix}\ ,$$
$$dh_2 = \begin{bmatrix} \sin\theta & \cos\theta & (x\cos\theta + y\sin\theta) \end{bmatrix}\ .$$

Definition 3.7 (Linear Span distribution) *The linear span of a set of vectors in a vector space, is the intersection of all linear sub-spaces whose contain vector in that set.*

$$span(S) = \left\{ \sum_{i=1}^{k} \lambda_i v_i \mid k \in N,\ v_i \in S,\ \lambda_i \in K \right\}\ . \tag{3-7}$$

Example: Having the next span distribution:

$$\Delta = span\,\{v_1\,,\, v_2\,,\, v_3\}\ ,$$
$$= span \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}\ .$$

The span set Δ is the one that generates the $\mathbb{R}^3$ sub space, having the three components v_1, v_2, v_3 the canonical basis, capable to generate any vector in

$\mathbb{R}^3$ by a linear combination of them. For example, the aleatory vector in $\mathbb{R}^3$ $u = [15\ 10\ 27]^\mathsf{T}$ can be generated as follows using the span set.

$$\begin{aligned} u &= \lambda_1 v_1 \,+\, \lambda_2 v_2 \,+\, \lambda_3 v_3 \ , \\ &= 15 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \,+\, 10 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \,+\, 27 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ . \end{aligned}$$

Definition 3.8 (Involutive Distribution) *A distribution is involutive if it is closed under the Lie bracket, i.e.,*

$$\Delta \ involutive \iff \forall\, f, g \in \Delta, [\, f\, g\,] \in \Delta \ . \tag{3-8}$$

3.1.2 Conversion to Chained Form

In this section, the whole process of conversion of any controllable system to the chained form is described, for this process all of the definitions on the previous section are required.

Given a controllable system:

$$\dot{x} \;=\; u_1 g_1(x) \,+\, u_2 g_2(x) \ , \tag{3-9}$$

where g_1 and g_2 are linearly independent and smooth. Then, exist a matrix $\beta(x) \in \mathbb{R}^{n \times n}$ and a diffeomorphism $\phi : \mathbb{R}^n \to \mathbb{R}^n$ such that :

$$v = \beta(x) u \qquad\qquad z = \phi(x) \ . \tag{3-10}$$

Step I: Define the constant full rank and involutive distributions:

$$\begin{aligned} \Delta_0 &= span \left\{ g_1 \,,\, g_2 \,,\, ad_{g_1} g_2 \,,\, \cdots \,,\, ad_{g_1}^{n-2} g_2 \right\} \ , \\ \Delta_1 &= span \left\{ g_2 \,,\, ad_{g_1} g_2 \,,\, \cdots \,,\, ad_{g_1}^{n-2} g_2 \right\} \ , \\ \Delta_2 &= span \left\{ g_2 \,,\, ad_{g_1} g_2 \,,\, \cdots \,,\, ad_{g_1}^{n-3} g_2 \right\} \ . \end{aligned} \tag{3-11}$$

Step II: Exist the function $h_1(x)$ and $h_2(x)$ such that

- h_1 follows the next conditions:

$$\begin{aligned} dh_1 \cdot \Delta_1 &= 0 \ , \\ dh_1 \cdot g_1 &= 1 \ . \end{aligned} \tag{3-12}$$

- h_2 follows the condition:

$$dh_2 \cdot \Delta_2 = 0 \ . \tag{3-13}$$

Step III: Once the functions h_1 and h_2 are found, then it is required to map $\phi : x \rightarrow z$ by a transformation given by:

$$\begin{aligned} z_1 &= h_1 \ , & v_1 &:= u_1 \ , \\ z_2 &= L_{g_1}^{n-2} h_2 \ , & v_2 &:= \left(L_{g_1}^{n-1} h_2\right) u_1 + \left(L_{g_2} L_{g_1}^{n-2} h_2\right) \ u_2 \ , \\ &\vdots \\ z_{n-1} &= L_{g_1} h_2 \ , \\ z_n &= h_2 \ . \end{aligned} \tag{3-14}$$

This yields to the known chained form system:

$$\begin{aligned} \dot{z}_1 &= v_1 \ , \\ \dot{z}_2 &= v_2 \ , \\ \dot{z}_3 &= z_2 v_1 \ , \\ &\vdots \\ \dot{z}_n &= z_{n_1} v_1 \ . \end{aligned} \tag{3-15}$$

A complete example for the conversion from a system to the chained form system using kinematic model of the Unicycle Mobile Robot is in Appendix B,in which all the steps are explained.

3.1.3 Chained Form for Tracked Mobile Robot

In this section, a new formulation of the chained form system for the Tracked Mobile robot is presented, as it was explained in the above section the kinematic model is required in equation (2-3), but it is necessary to see another representation shown in equation (3-9):

$$\dot{q} = u_1 g_1 \ + u_2 \, g_2 \ , \tag{3-16}$$

$$= u_1 \begin{bmatrix} d \sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix} + u_2 \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \ . \tag{3-17}$$

Remark 1 *Notice that the assignment of g_1 and g_2 plays an important rule* *in the chained form system, because they have effects in the conversion to the chained form system, making it non-viable or viable.*

In the case of chained system for TMR there are two possible choices:

1. $g_1 = [\cos\theta \;\; \sin\theta \;\; 0]^\mathsf{T}$ and $g_2 = [d\sin\theta \;\; -d\cos\theta \;\; 1]^\mathsf{T}$,
2. $g_1 = [d\sin\theta \;\; -d\cos\theta \;\; 1]^\mathsf{T}$ and $g_2 = [\cos\theta \;\; \sin\theta \;\; 0]^\mathsf{T}$.

For the first choice, it can be verified that the chained system formulation is non-viable, because in the formulation of the **Step I**, we can realize that all the distributions are not involutive. Now, it is clear according to the equation (3-17) that we choose the second choice, next process of formulation is continued.

Step I: Defining the distributions:

$$\begin{aligned}
\Delta_0 &= span\,\{g_1\,,\; g_2\,,\; ad_{g_1}g_2\}\;\;, \\
\Delta_1 &= span\,\{g_2\,,\; ad_{g_1}g_2\}\;\;, \\
\Delta_2 &= span\,\{g_2\}\;\;.
\end{aligned}$$

Finding $ad_{g_1}g_2$:

$$ad_{g_1}g_2 = [\,g_1, g_2\,] = \frac{\partial g_2}{\partial q}g_1(q) - \frac{\partial g_1}{\partial q}g_2(q)\;,$$

$$[\,g_1, g_2\,] = \begin{bmatrix} 0 & 0 & -\sin\theta \\ 0 & 0 & \cos\theta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d\sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & d\cos\theta \\ 0 & 0 & d\sin\theta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}\;.$$

Replacing the above calculations in the span distribution definition :

$$\begin{aligned}
\Delta_1 &= span\left\{ \begin{bmatrix} d\sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix}\,,\; \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}\,,\; \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \right\}\;, \\
\Delta_2 &= span\left\{ \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix}\,,\; \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \right\}\;, \\
\Delta_3 &= span\left\{ \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \right\}\;.
\end{aligned}$$

It is important to verify whether that the distribution is involutive, it is clear that the Δ_1 and Δ_3 are involutive, next the verification with Δ_2 is done. For this objective, we need to check if $[ad_{g_1}\ , g_2] \in \Delta_2$, finding $[ad_{g_1}\ , g_2]$:

$$[\, g_2, [ad_{g_1}\ , g_2]\,] = \begin{bmatrix} 0 & 0 & -\cos\theta \\ 0 & 0 & -\sin\theta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -\sin\theta \\ 0 & 0 & \cos\theta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ,$$

the result vector is in Δ_2.

Step II: The functions h_1 and h_2 are required:

- The function h_1 is chosen as $h_1 = \theta$, then dh_1 is computed:

$$dh_1 = \frac{\partial h_1}{\partial q} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} .$$

  Verifying each condition :

  - Condition $dh_1 \cdot \Delta_1 = 0$ for $\Delta_1 = \{g_2\}$:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = 0 .$$

  - Condition $dh_1 \cdot \Delta_1 = 0$ for $\Delta_1 = \{ad_{g_1} g_2\}$:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} = 0 .$$

  - Condition $dh_1 \cdot g_1 = 1$ is verified:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix} = 1 .$$

- The function h_2 is chosen as: $h_2 = x\sin\theta - y\cos\theta$, now finding dh_2:

$$dh_2 = \frac{\partial h_2}{\partial q} = \begin{bmatrix} \sin\theta & -\cos\theta & (x\cos\theta + y\sin\theta) \end{bmatrix} .$$

  We verify that the condition $dh_2 \cdot \Delta_2 = 0$:

$$\begin{bmatrix} \sin\theta & -\cos\theta & (x\cos\theta + y\sin\theta) \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = 0 .$$

Step III: Mapping $\phi : x \to z$ by a transformation given by:

$$
\begin{aligned}
z_1 &= h_1 \ , & v_1 &:= u_1 \ , \\
z_2 &= L_{g_1} h_2 \ , & v_2 &:= \left(L_{g_1}^2 h_2\right) u_1 + \left(L_{g_2} L_{g_1} h_2\right) u_2 \ , \\
z_3 &= h_2 \ .
\end{aligned}
$$

Having the function h_1 , it can be observed that $z_1 = h_1 = \theta$, next we will calculate $z_2 = L_{g_1} h_2$ as follows:

$$
\begin{aligned}
z_2 = L_{g_1} h_2 &= \frac{\partial h_2}{\partial q} \cdot g_1 \ , \\
&= \begin{bmatrix} \sin\theta & -\cos\theta & (x\cos\theta + y\sin\theta) \end{bmatrix} \begin{bmatrix} d\sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix} \ , \\
&= x\cos\theta + y\sin\theta + d \ .
\end{aligned}
$$

Having the expression for h_2 , having $z_3 = h_2 = x\cos\theta - y\cos\theta$, the computation of the expression $v_2 = \left(L_{g_1}^2 h_2\right)$ is done:

$$
\begin{aligned}
\left(L_{g_1}\left(L_{g_1} h_2\right)\right) &= \left(L_{g_1}\left(x\cos\theta + y\sin\theta + d\right)\right) \ , \\
&= \begin{bmatrix} \cos\theta & \sin\theta & (-x\sin\theta + y\cos\theta) \end{bmatrix} \begin{bmatrix} d\sin\theta \\ -d\cos\theta \\ 1 \end{bmatrix} \ , \\
&= -x\sin\theta + y\cos\theta \ .
\end{aligned}
$$

And we find the other part for the expression of v_2:

$$
\begin{aligned}
\left(L_{g_2}\left(L_{g_1} h_2\right)\right) &= \left(L_{g_2}\left(x\cos\theta + y\sin\theta + d\right)\right) \ , \\
&= \begin{bmatrix} \cos\theta & \sin\theta & (-x\sin\theta + y\cos\theta) \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \ , \\
&= 1 \ .
\end{aligned}
$$

The final expression for the mapping is :

$$
\begin{aligned}
z_1 &= \theta \ , & v_1 &= \omega \ , \\
z_2 &= x\cos\theta + y\cos\theta + d \ , & v_2 &= (-x\sin\theta + y\sin\theta)\,\omega + v \ , \\
z_3 &= x\sin\theta - y\cos\theta \ .
\end{aligned}
\tag{3-18}
$$

In order to find the chained form, we derive the states z_1, z_2, z_3 in function of the time, taking account the kinematic model in equation (2-3):

$$\begin{aligned}
\dot{z}_1 &= \omega \ , \\
\dot{z}_2 &= (-x \sin\theta + y \sin\theta)\,\omega + v \ , \\
\dot{z}_3 &= \omega(x\cos\theta + y\cos\theta) + \omega d \ .
\end{aligned} \tag{3-19}$$

This yields to the chained form system:

$$\begin{aligned}
\dot{z}_1 &= v_1 \ , \\
\dot{z}_2 &= v_2 \ , \\
\dot{z}_3 &= z_2 v_1 + d v_1 \ ,
\end{aligned} \tag{3-20}$$

as can be seen the chained form system has a drift into the model $f = \omega d$.

It is also clear to see that there is a relationship among the input signal control in the chained system $u = [\,v_1\ v_2\,]^{\mathsf{T}}$ and the input control in the Kinematic model (2-3),

$$\begin{aligned}
v_1 &= \omega \ , \\
v_2 &= (-z_3)\,\omega + v \ .
\end{aligned} \tag{3-21}$$

Then, going to the matrix form:

$$V = Tu = \begin{bmatrix} 0 & 1 \\ 1 & -z_3 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} . \tag{3-22}$$

As can be seen, we can express the input controls for the TMR u as a function of T and V as follows:

$$\begin{aligned}
u &= T^{-1} V \ , \\
\begin{bmatrix} v \\ \omega \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 1 & -z_3 \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} .
\end{aligned} \tag{3-23}$$

3.2 Sliding Mode based Controller

In this section, the robust control technique called *Sliding Mode* is presented as well as the development of a new theorem to control the TMR. This new theorem stabilizes the TMR to the origin in despite of some disturbances.

Sliding Mode is being widely used in practical control problems since always exists discrepancies among the actual physical system and the mathematical representation, these discrepancies come from unknown disturbances, uncertain plant parameters, or parasitic/unmodeled dynamics. The Sliding Model Controller provides different advantages such as [73]:

- Robustness.
- Finite-time convergence.
- Reduced order compensated Dynamics.

In order to formulate the controller, the chained form system defined in equation (3-20) is considered. Then we use Corollary 3.9 based on sliding mode controller [72], that aim the control law stabilizes the chained system defined on (3-20).

Corollary 3.9 *If the initial conditions of the system on* (3-20) *will be defined on the region* Υ *defined by:*

$$\Upsilon = \left\{ z \in \Upsilon \,|\, \frac{\alpha}{2}\left(z_1^2 + z_2^2\right) > \sigma \right\} \ , \tag{3-24}$$

where:

$$\sigma(z) = 2z_3 - z_1 z_2, \tag{3-25}$$

is a sliding surface.

Having the corollary, next step is formulate a theorem that is in charge to control the TMR using the Chained Form System as a new representation of the entire system.

Theorem 3.10 *(Sliding Mode Control for Tracked Mobile Robot)*
Consider the kinematic model of the Tracked Mobile Robot in (2-3)*, and the coordinate transformations defined in* (3-19) *the chained form* (3-20) *holds. Then the following stabilizing control laws:*

$$\begin{aligned} v_1 &= -z_1 - \alpha z_2 sign(\sigma) \ , \\ v_2 &= -z_2 + \alpha z_1 sign(\sigma) \ . \end{aligned} \tag{3-26}$$

where $\alpha > 0$*, ensure the stabilization of the posture error* e_q *to zero, where the control inputs* $v_1 \in \mathbb{R}$ *and* $v_2 \in \mathbb{R}$*, has a transformation for the control inputs in the kinematic model for Tracked Mobile Robot* (3-23)*.*

Proof. For proof, please see the Appendix A.3. ∎

3.2.1 Verification

In this section,it will present the numerical simulations for the SMC controller, these tests were performed using the following parameters :

$$\alpha = 0.5 \ , \qquad h = 0.01 \ , \qquad k = 0.1 \ , \tag{3-27}$$

where α is a control gain, h is the sampling time and k is the constant of the soil.

Having initial configurations for the robot as are shown in table 3.1.

Table 3.1: Initial configurations for TMR

Initial Configuration	x**(m)**	y **(m)**	θ **(rad)**
C_1	1.5	1.5	$\frac{\pi}{2}$
C_2	-1.5	-1.5	$\frac{\pi}{2}$
C_3	1.5	-1.5	$\frac{\pi}{2}$

Then the desired configuration will be as the other controllers $q_d = [0\ 0\ 0]^{\mathsf{T}}$

First Test Theorem 2.1 shows the control law for regulation of the Tracked Mobile Robot using Cartesian approach, this controller drives the TMR to any configuration in the inertial frame, the kinematic model (2-3) has a factor to indicate the slippage d (2-8), in this test it will be studied the variation of the value of k.

Table 3.2: Different parameters on slippage gain of TMR in SMC

Configuration	k
C_1	0.01
C_2	0.1
C_3	0.5
C_4	1.0

In the first test, the parameter k in equation (2-8) will vary according to the Table 3.2 and the robot will go from the same initial configuration $q_0 = \begin{bmatrix} 1.5 & 1.5 & \frac{\pi}{2} \end{bmatrix}^{\mathsf{T}}$.

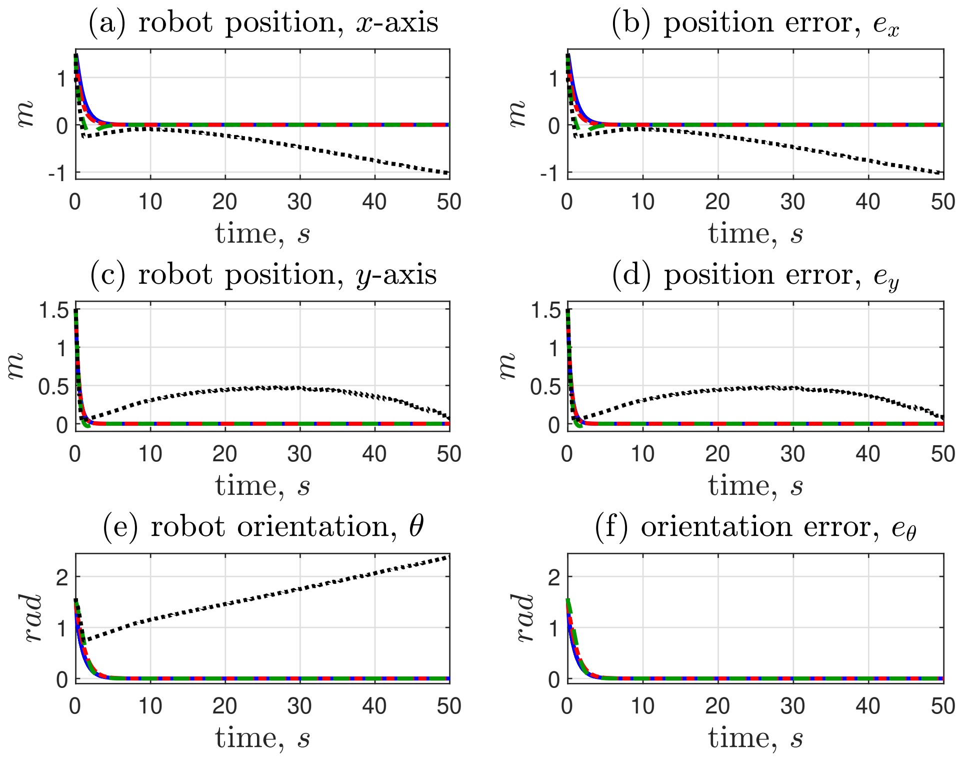

Figure 3.1: Simulation results: (a) robot position in the x-axis over time; (b) robot position in the y-axis over time. Legend: C1 (−), C2 (−.), C3 (− −), C4 (. .).

As a result of the simulation, in Figure 3.1 can be observed the position and error of the different coordinates of the TMR $q = [\,x\ y\ \theta\,]^\mathsf{T}$. It can be observed as greater is the parameter k the results in the simulation become unstable.

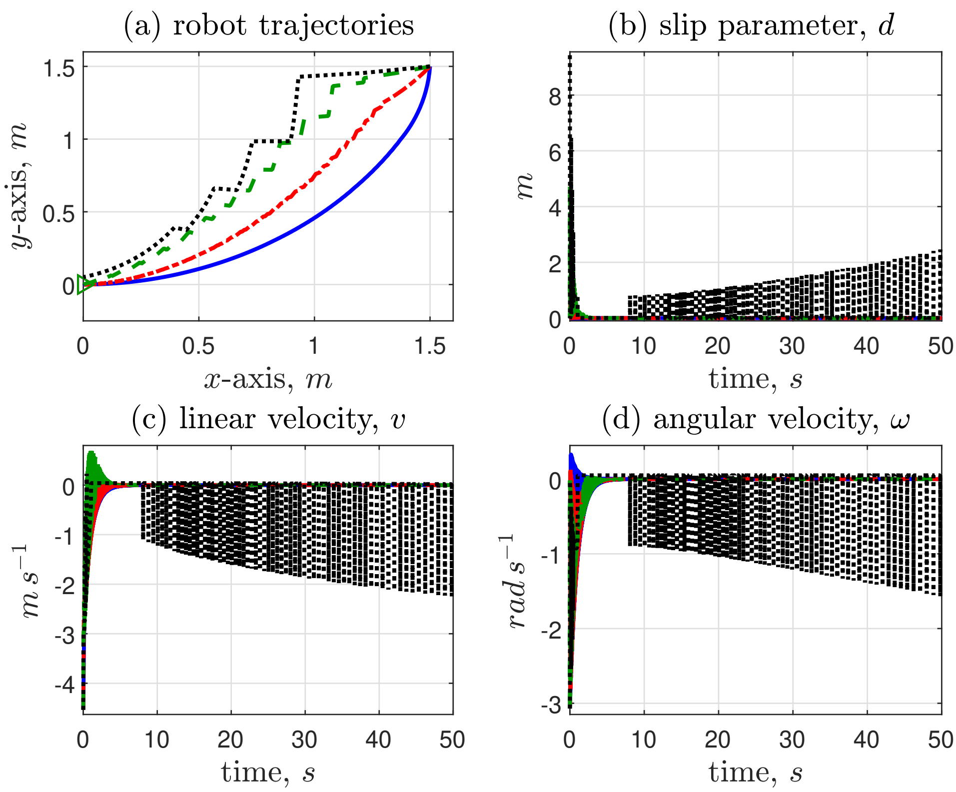

Figure 3.2: Simulation results: (a) robot trajectories in the xy plane; (b) robot orientation θ over time; (c) linear velocity v over time; (d) angular velocity ω over time. Legend: C1 (−), C2 (−.), C3 (− −), C4 (. .).

In Figure 3.2, it is observed that there are different results in the variation of the parameters: in the 3.2 (a) can be seen the trajectories generated for the robot, can be seen as the parameter k is bigger the trajectory becomes irregular. The Figure 3.2 (b) shows the behavior of the slip parameter it can be seen the variation of the parameter k having the chattering and in the highest value can be observed the value becoming unstable, in Figure 3.2 (c), (d) it is shown the control inputs having the chattering and the unstable values of them.

As a conclusion can be observed that the variation of the parameter k has influence in the trajectory, velocities and slip parameter d.

Second Test In the second test, three different initial configurations of the Tracked Mobile Robot are considered, according the to the Table 3.1.

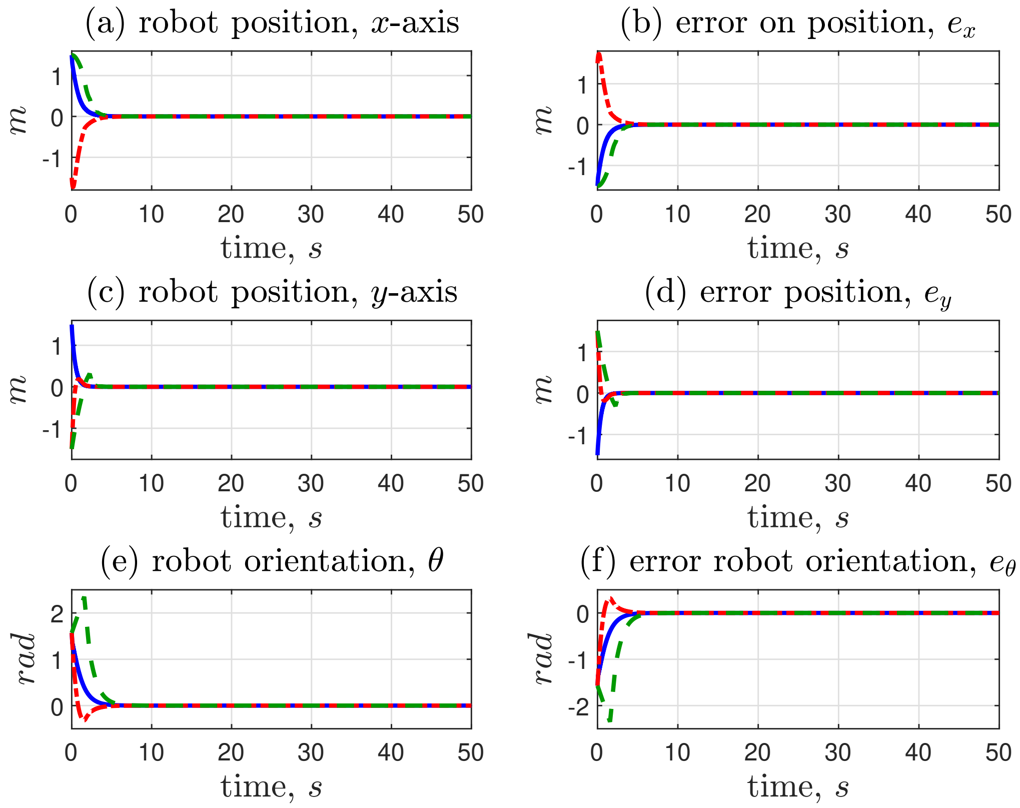

Figure 3.3: Simulation results: (a) robot position in the x-axis over time; (b) robot position in the y-axis over time. Legend: C1 (−), C2 (−.), C3 (− −).

Figure 3.3 shows the behavior of the position and error of the different coordinates of the TMR $q = [x\ y\ \theta]^\mathsf{T}$, it can be observed that the coordinates reach the desired configuration and the error is close to zero, and the convergence time is higher than the other controllers.

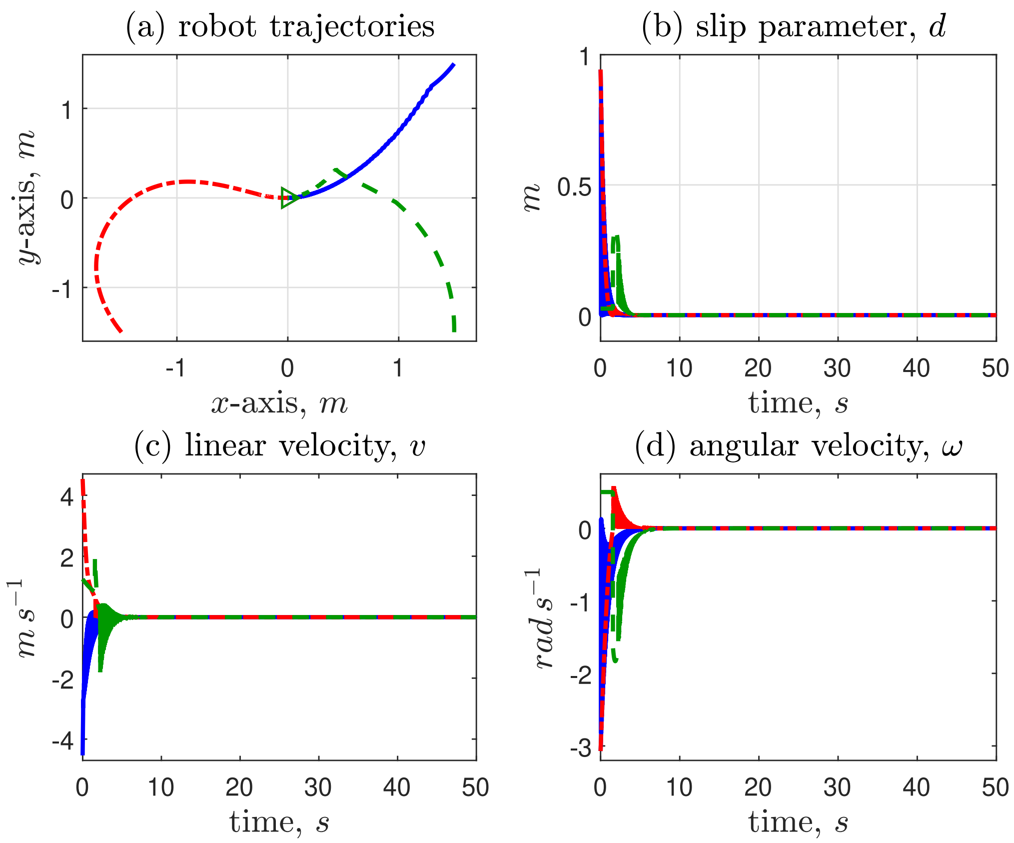

Figure 3.4: Simulation results: (a) robot trajectories in the xy plane; (b) robot orientation θ over time; (c) linear velocity v over time; (d) angular velocity ω over time. Legend: C1 (—), C2 (–.), C3 (– –).

In Figure 3.4 (a) the robot trajectories can be observed: the robot reaches the desired configuration, although having different trajectories based on the initial configuration. Figure 3.4 (b) shows the slip parameter having a chattering phenomena. In Figure 3.4(c) (d) is shown the control inputs, the linear velocity converges to zero as was mentioned in Theorem 2.1.

It can be observed that the control inputs has the chattering, this is a inherent problem in the SMC controller it can be solved using a second order controller.

Third Test In the third test, three different initial configurations of the Tracked Mobile Robot are considered, and we use the same uncertainty of polar coordinates, as is shown in the equation (2-21).

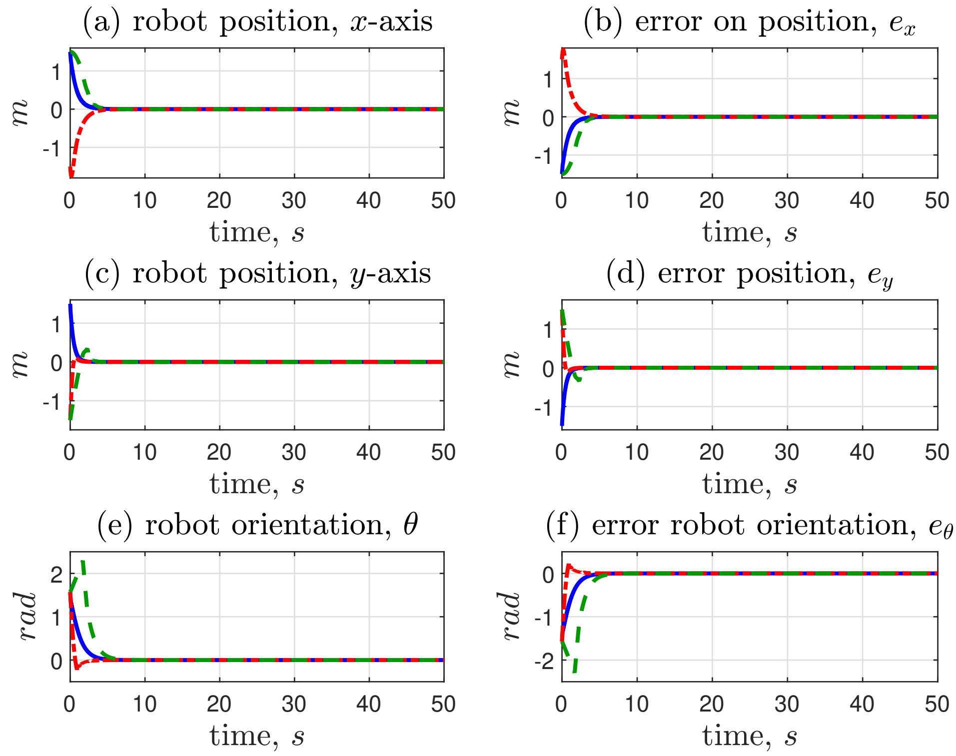

Figure 3.5: Simulation results: (a) robot position in the x-axis over time; (b) robot position in the y-axis over time. Legend: C1 (—), C2 (–.), C3 (– –).

Figure 3.5 shows the behavior of the position and error of the different coordinates of the TMR $q = [\,x\ y\ \theta\,]^{\mathsf{T}}$, it can be observed that the coordinates reach the desired configuration and the error is close to zero, in despite of the uncertainty the controller is capable to have a good performance.

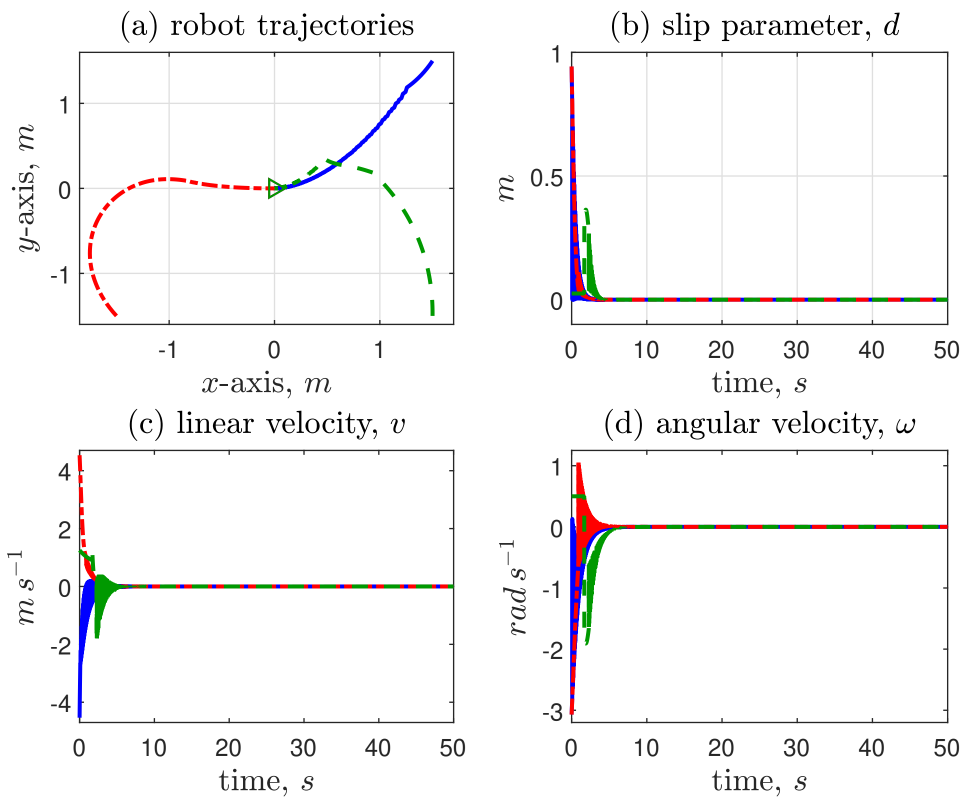

Figure 3.6: Simulation results: (a) robot trajectories in the xy plane; (b) robot orientation θ over time; (c) linear velocity v over time; (d) angular velocity ω over time. Legend: C1 (−), C2 (−.), C3 (− −).

In Figure 3.6 (a) the robot trajectories can be observed: the robot reaches the desired configuration. Figure 3.6 (b) shows the slip parameter as the robot describes the chattering phenomena which is a problem in the SMC controller. In Figure 3.6(c) (d) is shown the control inputs, the linear velocity converges to zero as was mentioned in Theorem 2.1, but it has the problem of chattering.

As a conclusion, can be observed that the robot reaches the desired configuration in all the initial cases. It is worth to remark that the problem of chattering is still present, this problem could be solved using a second order sliding mode.