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Apêndice
Tensores e espaços de tensões e deformações

Tensor de tensões. Entende-se por tensor um ente matemático que é independente do referencial. As constantes são tensores de ordem zero e os vetores são tensores de ordem 1.

A figura A1 apresenta as componentes positivas de tensão que atuam em um elemento diferencial de solo.

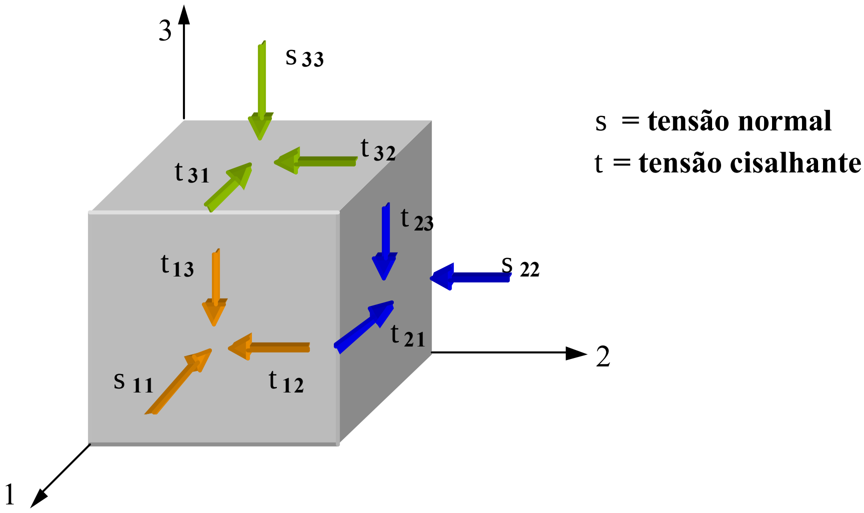

Figura A1: Convenção de tensões num elemento diferencial de solo.

O estado de tensões esquematizado na figura A1 pode ser expresso através do tensor de tensões,

$$T_s = \begin{bmatrix} s_{11} & t_{12} & t_{13} \\ t_{21} & s_{22} & t_{23} \\ t_{31} & t_{32} & s_{33} \end{bmatrix} \quad \text{ou} \quad T_s = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \qquad \text{(A1)}$$

Para um dado estado de tensões, o tensor T_s pode ser expresso como um somatório dos tensores *hidrostático* e *de desvio*, respectivamente, onde $p = \frac{1}{3}(s_{11} + s_{22} + s_{33})$

$$T_s = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} + \begin{bmatrix} s_{11} - p & t_{12} & t_{13} \\ t_{21} & s_{22} - p & t_{23} \\ t_{31} & t_{32} & s_{33} - p \end{bmatrix} \qquad \text{(A2)}$$

Empregando-se a notação do delta de Kronecker, a equação A2 também pode ser expresso como abaixo, onde S_{ij} representa o tensor de desvio.

$T_s = pd_{ij} + S_{ij}$ (A3)

Invariantes do tensor de tensões - Existem duas maneiras para se obter os invariantes de tensão. A utilizada neste trabalho, constrói os invariantes a partir do próprio tensor, de acordo com as definições seguintes:

$$J_1 = tr(s) = s_{11} + s_{22} + s_{33} \quad \text{(A4)}$$

$$J_2 = \tfrac{1}{2} tr(s)^2 \quad \text{(A5)}$$

$$J_3 = \tfrac{1}{3} tr(s)^3 \quad \text{(A6)}$$

Quanto aos invariantes do tensor de tensões de desvio, estes são definidos como:

$$J_{1D} = tr(S) = 0 \quad \text{(A7)}$$

$$J_{2D} = \tfrac{1}{2} tr(S)^2 = \tfrac{1}{2}\left[(s_{11} - p)^2 + (s_{22} - p)^2 + (s_{33} - p)^2 + 2S_{12}^2 + 2S_{23}^2 + 2S_{13}^2\right] \quad \text{(A8)}$$

$$J_{3D} = \tfrac{1}{3} tr(S)^3 = J_3 - \tfrac{2}{3} J_1 J_2 + \tfrac{2}{27} J_1^3 \quad \text{(A9)}$$

Tensor de Deformações - O tensor de deformações pode ser obtido por analogia com o tensor das tensões, substituindo-se s por e, e $\frac{1}{2}g$ por t. Logo, temos

$$T_e = \begin{bmatrix} e_{11} & \frac{1}{2}g_{12} & \frac{1}{2}g_{13} \\ \frac{1}{2}g_{21} & e_{22} & \frac{1}{2}g_{23} \\ \frac{1}{2}g_{31} & \frac{1}{2}g_{32} & e_{33} \end{bmatrix} \quad \text{ou} \quad T_e = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \quad \text{(A10)}$$

Também o tensor T_e pode ser expresso como um somatório de um tensor *hidrostático* mais outro *de desvio*, ou seja

$$T_e = \tfrac{1}{3} e_v d_{ij} + \mathrm{E}_{ij} \quad \text{(A11)}$$

onde $e_v = e_{11} + e_{22} + e_{33}$ é a deformação volumétrica. Assim,

$$T_e = \tfrac{1}{3} e_v d_{ij} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \mathrm{E}_{11} & \mathrm{E}_{12} & \mathrm{E}_{13} \\ \mathrm{E}_{21} & \mathrm{E}_{22} & \mathrm{E}_{23} \\ \mathrm{E}_{31} & \mathrm{E}_{32} & \mathrm{E}_{33} \end{bmatrix} \quad \text{(A12)}$$

Invariantes do tensor de tensões – Obtidos a partir do próprio tensor das deformações de acordo com

$$I_1 = tr(e) = e_{11} + e_{22} + e_{33} = e_v \quad \text{(A13)}$$

$$I_2 = \tfrac{1}{2} tr(I)^2 \qquad \text{(A14)}$$

$$I_3 = \tfrac{1}{3} tr(I)^3 \qquad \text{(A15)}$$

Planos e tensões octaédricas - Os planos octaédricos são determinados pela aplicação da condição de que os cossenos dos ângulos diretores, formados com as direções principais σ_1, σ_2, σ_3, são iguais em valor absoluto. Existem oito planos que satisfazem esta condição, chamados de planos octaédricos (figura A2).

A tensão normal atuante nestes planos é calculada como

$$s_{oct} = \tfrac{1}{3}(s_{11} + s_{22} + s_{33}) = \tfrac{1}{3} J_1 \qquad \text{(A16)}$$

e a tensão cisalhante octaédrica por

$$t_{oct} = \tfrac{1}{3}\left[(s_{11} - s_{22})^2 + (s_{11} - s_{33})^2 + (s_{22} - s_{33})^2\right] = \sqrt{\tfrac{2}{3} J_{2D}} \qquad \text{(A17)}$$

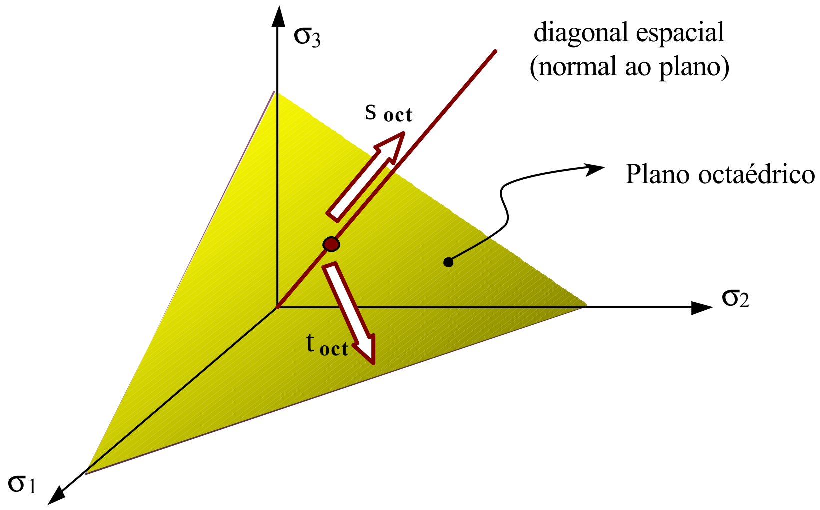

Figura A2: Um dos oito planos octaédricos e as tensões octaédricas nele atuantes.

Espaço de tensões e trajetórias de tensão - O espaço Cartesiano definido pelos eixos das tensões principais σ_1, σ_2, σ_3 é conhecido como *espaço das tensões principais* ou *espaço de Westergaard*, onde são representados planos relevantes à análise das trajetórias de tensão (figura A3).

Um plano relevante contido no espaço de tensões é o *plano triaxial*, ou de *Rendulic*, definido pela condição $s_2 = s_3$, própria dos ensaios triaxiais convencionais. Nesse plano, são descritas todas as trajetórias de tensão levadas a cabo nestes ensaios. Outro plano relevante é o *octaédrico* cuja normal é paralela à diagonal principal do espaço de tensões ($s_1 = s_2 = s_3$). Particularmente, o plano octaédrico que passa pela origem, denominado de *plano p* ($s_1 + s_2 + s_3 = 0$), é bastante utilizado como plano de projeção no estudo das trajetórias de tensão espaciais.

A figura A3 apresenta também as trajetórias de tensão mais empregadas em ensaios geotécnicos, as quais são detalhadas a seguir:

- Trajetórias contidas no plano triaxial:

  CH: Compressão hidrostática ($\Delta s_1 = \Delta s_2 = \Delta s_3$).

  CTC: Compressão triaxial convencional ($\Delta s_1 > 0$, $\Delta s_2 = \Delta s_3 = 0$).

  RTC: Compressão triaxial reduzida ($\Delta s_1 = 0$, $\Delta s_2 = \Delta s_3 < 0$).

  CTE: Extensão triaxial convencional ($\Delta s_1 = 0$, $\Delta s_2 = \Delta s_3 > 0$).

  RTE: Extensão triaxial reduzida ($\Delta s_1 < 0$, $\Delta s_2 = \Delta s_3 = 0$).

- Trajetórias contidas no plano octaédrico:

  CS: cisalhamento simples ($\Delta s_2 = 0$, $\Delta s_1 = -\Delta s_3$).

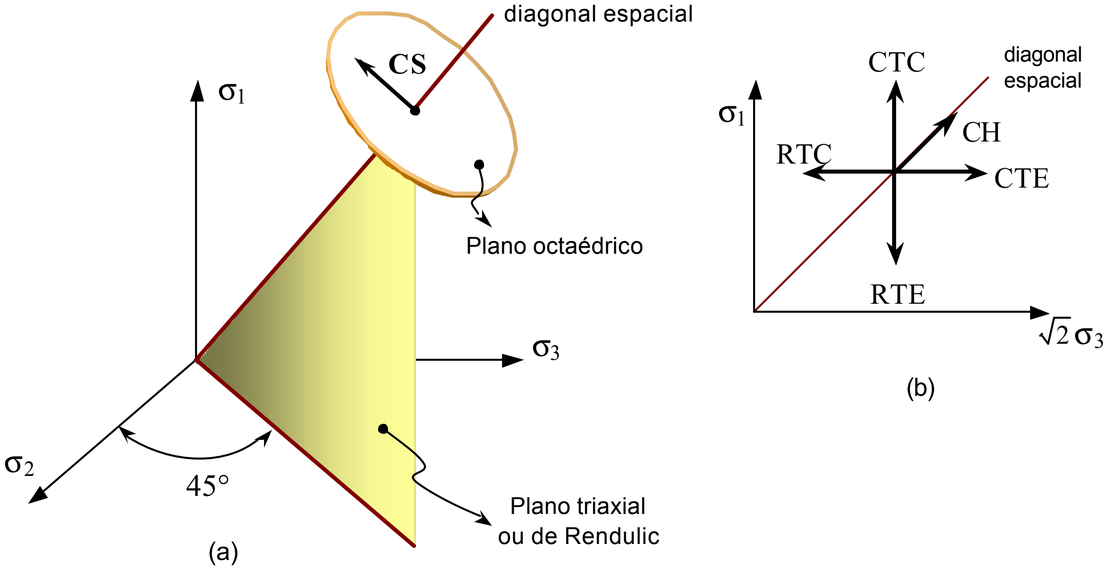

Figura A3: Espaço de tensões: a) Espaço de Westergaard, plano triaxial e octaédrico; b) Trajetórias de tensão no plano triaxial.