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A

Coefficients of the characteristic polynomial

The coefficients of the characteristic polynomial of the Jacobian matrix of the test rig system around the reference velocity is

$$\begin{aligned}
 a_1 &= 34.6260dT_{r_1} + 304.2343 \\
 a_2 &= 1.0510(10^4)dT_{r_1} + 1.2236(10^5) \\
 a_3 &= 4.2283(10^6)dT_{r_1} + 5.3915(10^5) \\
 a_4 &= 1.5317(10^7)dT_{r_1} + 1.6501(10^7) \\
 a_5 &= 4.0190(10^8)dT_{r_1} + 2.4730(10^7) \\
 a_6 &= 4.2755(10^8)dT_{r_1} + 1.1503(10^8) \\
 a_7 &= 1.1475(10^8),
 \end{aligned} \tag{A-1}$$

The principal minors are

$$\begin{aligned}
 \Delta_1 &= |a_1|, & \Delta_2 &= \begin{vmatrix} a_1 & a_3 \\ 1 & a_2 \end{vmatrix}, \\
 \Delta_3 &= \begin{vmatrix} a_1 & a_3 & a_5 \\ 1 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}, & \Delta_4 &= \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ 1 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \end{vmatrix}, \\
 \Delta_5 &= \begin{vmatrix} a_1 & a_3 & a_5 & 0 & 0 \\ 1 & a_2 & a_4 & a_6 & 0 \\ 0 & a_1 & a_3 & a_5 & 0 \\ 0 & 1 & a_2 & a_4 & a_6 \\ 0 & 0 & a_1 & a_3 & a_5 \end{vmatrix}, & \Delta_6 &= \begin{vmatrix} a_1 & a_3 & a_5 & a_7 & 0 & 0 \\ 1 & a_2 & a_4 & a_6 & 0 & 0 \\ 0 & a_1 & a_3 & a_5 & a_7 & 0 \\ 0 & 1 & a_2 & a_4 & a_6 & 0 \\ 0 & 0 & a_1 & a_3 & a_5 & a_6 \end{vmatrix}, \\
 \Delta_7 &= |\mathbb{H}|.
 \end{aligned} \tag{A-2}$$