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Conclusion

We believe that the SAA method is a valuable tool to obtain good candidate solutions for chance constrained problems. An important advantage of the method is the fairly general assumption on the underlying distribution of the random variables of the problem: SAA only requires the ability to sample from the given distribution. The related numerical analysis is tractable and stable, allowing for practical implementations. The thesis bridges the gap between theory and application, by presenting theoretical foundations, techniques for parameter calibration and a collection of applications, which exemplify as benchmarks and as new amenable problems.

SAA was used in a portfolio chance constrained problem with random returns. In the normal case, the *efficient frontier* can be computed explicitly and we used it as a benchmark solution. Fixing the parameters $\varepsilon = 0.10$ and $\gamma = 0$ in the method, we concluded that the sample size suggested by Campi-Garatti inequality (4-6) was too conservative for our problem: a much smaller sample gave rise to better feasible solutions. Similar results were obtained for the lognormal case, where upper bounds were computed using a method developed in [NS].

As another illustration of the SAA method, we modeled a two dimensional blending problem as a joint chance constrained problem. Due to the independence assumption, one can again solve the problem explicitly and find the optimal solution for any given reliability level. This served as a benchmark for SAA on the class of approximate joint chance constrained problems.

In both examples, the choice $\gamma = \varepsilon/2$ obtains very good candidate solutions. Even though it generated more infeasible points if compared to the choice $\gamma = 0$, the feasible ones were of better quality. Using the γN-plot (Figures (4.4) and (4.9)) we were able to confirm these empirical findings for our two test problems. Relatively small sample sizes (e.g., if compared to (4-6) estimates) can yield good candidate solutions, which is crucial since for $\gamma > 0$ the SAA problem is an integer program. Upper bounds were constructed for the portfolio problem with $\gamma = 0$ and continuous linear programs were solved to obtain the estimates. According to approximation (4-8), the number of samples N should be of order $1/\varepsilon$. Since no closed solution is available for the portfolio problem with lognormal returns, having an upper bound is an important information about the variability of the solution.

Finally we described the *hurdle-race problem*, proposed in [VDGK]. We proposed a more adequate formulation in which the hurdles were taken jointly. We obtained good candidate solutions and lower bounds for the true optimal value using SAA. An additional extension, assuming stochastic hurdles instead of deterministic ones, is handled by SAA at almost no extra cost.

Future work will include writing a SAA solver. For this goal, the empirical findings in this text are crucial. Together with H. Bortolossi, we started to write a program in C++ using Osi, a uniform API for interacting with callable solver libraries. At present, we handle SAA problems with SYMPHONY, an open-source generic MILP developed by T.K. Ralphs. Both Osi and SYMPHONY can be freely downloaded at www.coin-or.org. The idea is to make our solver freely available for use with SLP-IOR[1], a user-friendly interface for stochastic programs developed by P. Kall and J. Mayer. As of yet, problems such as the portfolio problem of Chapter 4, where the uncertainty is multiplying the decision variables, cannot be solved by any solver available at SLP-IOR.

The first Chapters of the thesis, in a condensed form, are the content to [PAS]. A text with S. Vanduffel about the hurdle-race problem is in preparation. We plan to extend the results described here by assuming stochastic liabilities of the type

$$\alpha_i = a_i + f_i(E_i), \qquad i = 1, \ldots, n,$$

where a_i are constants, $(E_1, \ldots, E_n)$ is a multivariate normal vector and f_i are given functions. Such a setting may be appropriate in the context of life insurance activities. The a_i would be the total amount of fixed guaranteed benefits to be paid to the policy holders. The insurance company may also provide a profit sharing mechanism linked to some return process $(E_1, \ldots, E_n)$ of an external benchmark (e.g a stock index).

[1] www.ior.uzh.ch/Pages/English/Research/StochOpt/index_en.php