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No free lunch for fiscal inflations:  
a fiscal-induced stagflation

DISSERTAÇÃO DE MESTRADO

DEPARTAMENTO DE ECONOMIA  
Programa de Pós-graduação em Economia

Rio de Janeiro  
Abril 2016

**Moisés Shalimay de Souza Andrade**

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a fiscal-induced stagflation**

**Dissertação de Mestrado**

Dissertation presented to the Programa de Pós-graduação em Economia of the Departamento de Economia , PUC-Rio as a partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Tiago Couto Berriel

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Rio de Janeiro, Abril 11, 2016

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Bibliographic data

Andrade, Moisés Shalimay de Souza

No free lunch for fiscal inflations: a fiscal-induced stagflation / Moisés Shalimay de Souza Andrade; advisor: Tiago Couto Berriel. — Rio de Janeiro : PUC-Rio, Departamento de Economia, 2016.

(em Inglês)

v., 57 f: il. ; 29,7 cm

1. Dissertação (mestrado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia.

Inclui referências bibliográficas.

1. Economia — Tese. 2. Política fiscal. 3. Fricções financeiras. 4. Teoria fiscal do nível de preços. 5. Estagflação. 6. Crise de dívida. 7. Crise bancária. I. Berriel, Tiago Couto. II. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. III. Título.  
(em Português)

CDD: 330

## Acknowledgments

I thank my girlfriend for the emotional and unconditional support, my mother for allowing me to get so far and my professors, especially my dear advisor Tiago Berriel for all the advices and friendship

## Abstract

Andrade, Moisés Shalimay de Souza; Berriel, Tiago Couto(advisor).  
**No free lunch for fiscal inflations: a fiscal-induced stagflation.** Rio de Janeiro, 2016. 57p. MSc. Dissertation —  
Departamento de Economia, Pontifícia Universidade Católica do  
Rio de Janeiro.

Expansionary fiscal policies have been advocated to induce output expansions and inflation in deep recession or deflationary episodes. We show that, in a fiscalist setup, an increase in deficits can trigger a stagflation by negatively affecting financial intermediation of resources to investments. Financial intermediaries collect deposits to buy government bonds and lend through nominal long-term loans. When intermediaries face financial frictions and a maturity mismatch on their assets and liabilities, a surprise inflation and/or a revaluation of bonds prices impair their net-worth reducing lending, investments, and output. Recession comes with inflation in a fiscal expansion because the fall on capital triggered on the financial sector rises production firms marginal costs. The probability of a recession is higher the greater is the maturity mismatch, the sensitivity of bonds prices to the policy rate, and the share of bonds on banks balances. These results: (1) give theoretical support for the negative relation documented between financial sector performance and inflation (2) help explaining high debt, high inflation environments coinciding with banking crisis and, more importantly, (3) expose drawbacks of fiscal inflation policies proposed to inflate and stimulate low inflation economies, where the fiscalist setup stressed in this paper is more probable to be present.

## Keywords

Fiscal policy; Financial frictions; Fiscal theory of the Price Level;  
Stagflation; Debt crisis; Banking crisis;

## Resumo

Andrade, Moisés Shalimay de Souza; Berriel, Tiago Couto(orientador). **Sem Almoço Grátis para Inflações Fiscais: uma Estagflação Induzida por Déficits**. Rio de Janeiro, 2016. 57p. Dissertação de Mestrado — Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Expansões fiscais têm sido propostas como solução para economias passando por fortes recessões e episódios de deflação. Mostramos em uma arcabouço fiscalista que um aumento dos deficits pode iniciar uma estagflação por afetar negativamente a intermediação de recursos para investimentos. Intermediários financeiros coletam depósitos para comprar títulos do governo e realizar empréstimos através de contratos nominais de longo-prazo. Quando intermediários enfrentam fricções financeiras e um descasamento entre seus ativos e passivos, uma inflação surpresa e/ou uma reavaliação dos preços dos títulos prejudica seus balanços, reduzindo os empréstimos, investimentos e produção. Em uma expansão fiscal, a recessão vem com inflação porque a queda na oferta de capital iniciada no setor financeiro aumenta os custos marginais das firmas produtoras de bens. A probabilidade de uma recessão é maior quanto maior for o descasamento de maturidade, a sensibilidade dos preços dos títulos às taxas de juros e quanto maior a participação dos títulos no balanço dos bancos. Esses resultados: (1) dão suporte teórico para a relação negativa entre a performance do setor financeiro e alta inflação; (2) ajudam a explicar episódios de alto endividamento público, alta inflação e crises bancárias e, mais importante, (3) expõem desvantagens de políticas fiscais inflacionárias propostas para inflacionar e estimular economias com baixa inflação, onde o arcabouço proposto neste artigo é mais provável de estar presente.

## Palavras-chave

Política fiscal; Fricções financeiras; Teoria fiscal do nível de preços; Estagflação; Crise de dívida; Crise bancária;

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# 1

## Introduction

Fiscal related issues are on the spotlight of academics and practitioners. US and Japan have been accumulating debt at an unprecedented rate, posing a few analysts to voice concerns about fiscal consolidation. Europe has been experiencing a severe sovereign debt crisis for a few years. Brazil is again going through fiscal distress, deadlocked by political and structural restrictions to tax and spending adjustments. Many DSGE models neglect the fiscal side, assuming (sometimes very implicitly) that the fiscal authority is ready to rise revenues (and that people expect it is ready) to support any policy decision made by the monetary authority. Real-world examples of fiscal authorities troubled to pay its debts make clear that such hypothesis is not realistic for every scenario.

Not surprisingly, there has been a renewed interest on the Fiscal Theory of the Price Level (FTPL).<sup>1</sup> This literature recognizes the relevance of the fiscal side of the economy on price determination, combining in one framework the discussion about fiscal and monetary policies. In short, these theories argue that prices are a fiscal phenomena: the value of money reflects the expected path of primary surpluses covering government liabilities. If surpluses become low relative to liabilities, there will be too much money chasing too few goods, driving prices up.

Although fiscal theory could be applied for several contexts, it is specially attractive to explain two scenarios: economies going through some type of fiscal distress and inflation financing - particularly, fiscal stimulus financed through inflation. In a high debt environment, with a potential high tax burden to the people, rising surpluses to cover additional debt obligations (arising, for example, from monetary policy) can be too costly and politically unfeasible.<sup>2</sup> In these circumstances, concerns about debt sustainability arises and people becomes more sensitive to fiscal shocks. Additional deficits, for example, are more likely to trigger a run from government bonds to other assets and goods, driving prices up, than keeping the economy unaffected, as Ricardian equivalence would predict. Indeed, views of FTPL proponents suggest a close link between the FTPL and fiscal distressed economies. For example,

<sup>1</sup>See for example recent efforts of Cochrane (2011), Sims (2013), Bianchi e Melosi (2014) and Leeper e Leith (2016).

<sup>2</sup>Moreover, such environments are more prone to attain the top of a Laffer curve, posing limits to the amount of revenues can be achieved.

Sims (2013) when explaining the necessity of fiscal support for monetary policy effectiveness says "This is easiest to understand in high-inflation, high nominal debt economies where fiscal policy is frozen by political deadlock or chicanery" and Loyo (1999) offers a FTPL explanation for Brazilian hyperinflation in a similar environment. Leeper e Leith (2016) also refer to the recent Brazilian experience of unsustainable fiscal policy and high debts as suggestive evidence of the FTPL.<sup>3</sup>

However, FTPL models have a sharp and robust prediction in the presence of price rigidities: a fall on surpluses generates a boom of economic activity. The traditional explanation relies on wealth effects: if taxes fall and there is no expectation of an equivalent rise in the future, households expend the additional resources on goods and services, stimulating production. But if fiscal accounts are messy, it is not obvious that negative surplus news must induce a boom. Again, own proponents of FTPL seems not to believe this is necessarily the case. As says Cochrane (2011): "The larger history of fiscal inflations and currency collapses does not inspire hope that a fiscal inflation always results in prosperity. The hyperinflations that follow wars (Sargent, 1992), Latin American fiscal collapses, currency crashes, or the recent hyperinflation in Zimbabwe were associated with sharp declines in economic conditions, not the spectacular booms Phillips curve would predict".

One possible explanation for recession on these economies could be a rise in default probability and risk-premia. However, it is not obvious why a government would choose (and people would expect) default if it has the option of deflating nominal debt. Indeed, inflation can be a very good way to pay off debt: unlike outright default, there is no breach of contract, the proceedings are effectively paid, so there would be little reason for the government being excluded from credit markets.

This takes us to the second scenario: inflation financing of fiscal deficits. According to what we stressed above, a good policy prescription for economies going through deflationary spirals goes as: peg interest rates and convince people that surpluses will not be sufficient to pay off public debt; inflation will come to do the work along with an output boom. This policy may seem reckless, but similar options are already being discussed given the difficulties to inflate economies through conventional policies.<sup>4</sup> A relevant question is: would such a policy generate the so desired demand-inflation boomlet or only

<sup>3</sup>Additionally, Leeper e Leith (2016) cite the Euro Area sovereign crisis as other good example of countries where the fiscal limit had been reached and the surpluses necessary to assure monetary policy actions are not guaranteed

<sup>4</sup>See for example some recent articles on The Economist February edition dedicated to the theme: Economist (2016a), Economist (2016b).

stagflation?

In this paper, we build a New Keynesian model with financial frictions, and show that a fiscal deterioration can trigger a stagflation when we take account for the costs of inflation to financial intermediation. In our model, financial intermediaries/banks face an agency problem similar in spirit to Gertler e Kiyotaki (2010) and Gertler e Karadi (2011) which ties its ability of financing to its balance-sheet conditions. Departing from the model of Gertler e Karadi (2011), here intermediaries collect short-term deposits to buy long-term nominal bonds and finance capital through long-term nominal loans. This introduces a maturity mismatch on banks' balance-sheets which gives rise to an inflation exposure: although both assets and liabilities are nominal, the former are more sensible to inflation due to higher maturity.

When surpluses fall and are not expected to rise again, agents expect a rise on inflation to cover additional deficits, generating inflationary pressures today. Surprise inflation reduces the ex-post return on banks' loans, but if banks lend only on short-term this is not much of a problem, since deposit ex-post rates also falls. However, when banks lend through long-term contracts, they cannot adjust the whole portfolio according to the new path of inflation and has to keep bad loans on balance-sheets, leading to greater losses. Additionally, the rise on interest-rates following inflation further damages banks' balance-sheets by reducing the value of their long-term assets - in particular, their holdings of long-term government bonds. There is, thus, a 'credit deflation channel', the mirror of Fischer's debt deflation channel: while borrowers are better off with the surprise devaluation of debts, lenders are worse off by the same reason. Contrary to the debt deflation effect, the credit deflation makes investing entrepreneurs worse off: the fall on the value of banks' assets tightens their agency problem, reduces capital finance and prompts a rise on spreads and lending rates.

Following a deficit shock, surprise inflation and bonds revaluation impair banks net-worth, reducing capital finance and investments. For a reasonable calibration, we show that a fall on lump-sum taxes leads to a recession and that government spending multiplier, typically greater than one in FTPL models, is less than one. Recession comes with inflation because of a supply-side type shock to the economy. From the point of view of producing firms, the fall on capital is an exogenous force coming from the financial sector, not an optimal decision, as its productivity did not change. Hence, lower capital raises firms' marginal costs, engendering a rise in optimal prices and inflation, which helps to restore government solvency. Hence, deficits remain financed by inflation, although at the cost of output.

Limited arbitrage between policy rates and lending rates is key for these results. Without financial frictions, real interest rates typically fall to restore government solvency in the presence of price rigidities. By arbitrage, all other rates fall leading to a rise on investments and consumption. In our model, while real policy rates fall, lending real rates are higher and often rises because of spreads, consistent with the fall on investments.

While the fall on banks' net-worth induces a fall on investments, the fall on surpluses also raises aggregate demand. The prevailing force will depend on the degree of maturity mismatch, the sensitivity of bond prices to interest-rates and the share of bonds on banks' total assets. The stronger each one of these factors, the greater the chance of a recession following a deficit shock. We further investigate the role of surprise inflation and asset-price surprises in driving a recession, finding that alone each one is strong enough to generate a downturn of output.

In sum, our results show that a change on expectations of primary surpluses can be damaging to the financial sector, even without an outright default. The 'inflation default' and revaluation of bond prices reduces intermediaries' net-worth and this lead to a rise in spreads, fall on investments and decline on output. Therefore, we propose theoretical support for works finding a negative relationship between banks' performances and inflation and a positive relation between banking crises and episodes of high inflation and high government debt. Additionally, we offer a complementary explanation to sovereign debt-banking crisis, often related to the damage of outright defaults of government bonds on banks' balance-sheets. In these crises, negative surplus news can feed inflation, impacting intermediaries' balance-sheets and contributing to banks' fragility and recession. As inflation grows too high, desirability and probability of a default rises, depleting even more banks' net-worth, intensifying the recession.

Moreover, our work raises additional warnings about fiscal inflationary policies, adding to the points made in Cochrane (2011). Our results stress drawbacks of such policies that are more prone to occur just in economies experiencing low inflation, where the presence of long-term nominal contracts tend to be more common, banks tend to be more exposed to public debt as a result of QE and where protection to inflation is less needed.

## Empirical relevance

In a recent paper, Cao (2014) quantifies banks' losses to unanticipated inflation in the US. Using data from commercial bank reports spanning from

1997Q3 to 2009Q3, she finds that on average 70% of banks' assets and liabilities are denominated in nominal terms and have a maturity mismatch of about five years. She finds that a rise of 1% on inflation trigger losses of about 10% to 15% of Tier 1 capital, even for large banks and banks holding interest-rate derivatives. Such losses arises mainly from loans and leases made to the private sector, that entails more than 50% of nominal assets on banks' portfolios. Another substantive amount comes from government bonds, which composed roughly 10% of banks' assets at the end of her dataset. In a similar analysis, the Bank of Japan (2015) estimates that a 1% parallel rise in the yield curve would lead to losses of 20% of Tier 1 capital for Japanese banks, of which around 13 to 14 percentage points come from bond holdings losses.

In addition, there are many papers documenting that inflation is damaging to the financial sector. Boyd et al. (2001) and Boyd e Champ (2003) find a negative relationship between inflation and financial sector performance. Demirgüç-Kunt e Detragiache (1997) and Demirgüç-Kunt e Detragiache (2005) find strong correlation between banking crisis and inflation. In a more recent paper, Boyd et al. (2015) construct cleaner measures of banking crises, disentangling adverse shocks to the banking sector from restorative policies and finding that higher inflation rises the probability of systemic banking shocks.

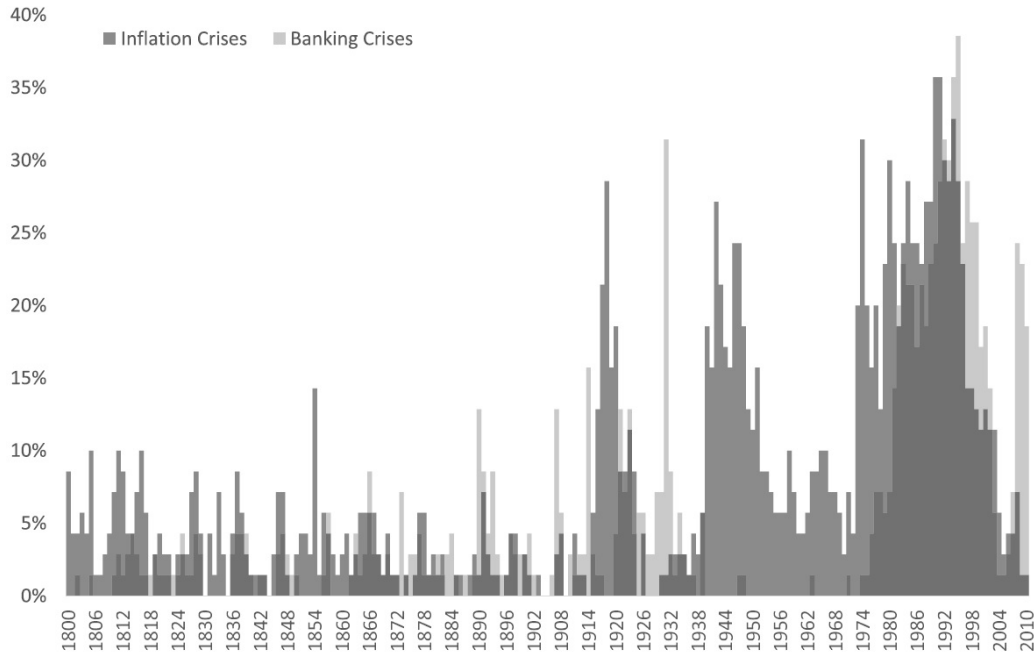
In a comprehensive survey, Reinhart e Rogoff (2011) document a close link between high government debt, sovereign debt crisis and banking crisis in a cross-country dataset. They also find association of high indebtedness with inflation crisis, particularly for years after the first world war, when links between money and gold were weakened. We take their dataset to explore the coincidence of inflation and banking crisis. Figure 1.1 shows the share of countries going through each type of crises. Light shadowed areas represent banking crisis, intermediary shadowed areas inflation crises and darker areas the coincidence of both crises. There is a relative high coincidence of the two types, particularly since the second half of the seventies. This is interesting, as this period coincides with the end of the linkages between money and gold prices and the start of the era of fiat money and nominal debt, turning the option for inflation financing more viable and direct.<sup>5</sup>

Summarizing, there is evidence that inflation can be damaging to the

<sup>5</sup>During the Bretton Woods period, currencies worldwide were convertible to gold, so although debt could be issued on the own countries currencies, it were in practice almost real, as governments could not emit gold. Some maneuvering was possible because conversions were costly, but clearly there was a limit to the amount of cash governments could introduce in the economy. With the end of currency conversion, debt effectively becomes nominal, as the government issues it in their own currency. In the lens of fiscal theory, the value of money then starts to depend on the amount of surpluses covering government liabilities.

financial sector by its effects on maturity mismatched bank balances. There is also evidence for a relation between high inflation and poor financial sector performance and high-inflation, high-debt environments to the occurrence of banking crisis. Our paper shows these histories can be tied together by the standard models used for policy evaluation, considering an appealing framework to study inflation financing, the fiscal determination of prices.

Figure 1.1: Banking and Inflation crises



## Related works

Our paper fits in the extensive literature of the Fiscal Theory of the Price Level.<sup>6</sup> This literature typically focus on inflation and monetary policy implications of fiscal determination, relegating output implications to the side. Therefore, our paper adds to this literature by expliciting further output and inflation consequences of fiscal policy and inflation financing. In this literature, Cochrane (2011) is the work closest to ours. He shows that in the event of a fiscal shock, the government can use long-term bonds to trade all current inflation for future inflation. He then plugs the postponed inflation path in a textbook New Keynesian model, showing that deficits can be stagflationary on the short-run because of supply-side like effects: with the rise in future inflation and no current inflation, the Phillips Curve shifts to the left. We differ from Cochrane (2011) by the mechanism through which stagflations could arise

<sup>6</sup>For seminal references, see Leeper (1991), Woodford (1994), Sims (1994), Cochrane (2001) and Cochrane (2005).



and by taking the full general equilibrium approach to generate our impulse response functions.<sup>7</sup>

By considering models with financial frictions, we follow Gertler e Kiyotaki (2010) and Gertler e Karadi (2011).<sup>8</sup> We depart from the traditional models by allowing banks to hold government bonds and long-term assets. In this sense, our paper relates to a recent literature which studies the cost of sovereign default on financial intermediaries. In a neoclassical model with financial frictions, Bocola (2016) finds that sovereign default, or even expectations of a default, can negatively affect banks' net-worth and trigger a recession. Bi et al. (2014) extend Bocola's analysis to a New Keynesian model with bank runs, finding that defaults can also induce a disruption of interbank markets. Albeit our mechanism is similar to these papers, our work differs by focusing on surprise inflation, a more indirect form of default.

The idea that inflation can be damaging to the financial sector is present in previous works. Additionally to the empirical works cited above, Kumhof e Tanner (2005) stress the importance of stable government debt to facilitate financial intermediation, contrasting policy prescriptions from optimal fiscal policy literature, who advocates for debt devaluation through inflation, and practitioners, who see it only as a last resort. Our paper adds to this discussion by showing that the negative relationship between inflation and financial intermediation can be rationalized by standard DSGE models and analyzing under which circumstances the damage is more severe.

Optimal fiscal policy literature advocates for the use of state contingent inflation to absorb government shocks and smooth distortionary taxation. But when inflation negatively affects the financial sector, it stops being a costless way to pay debt, even in the presence of flexible prices. A normative analysis for the option of inflation financing vs taxation is given by Cao (2015). In her paper, bankers face collateral constraints to finance capital. A surprise inflation, by reducing the value of nominal government debt tightens banks' constraints and negatively affects capital financing. She finds that the government must balance inflation and distortionary taxation in the case of flexible prices and that the financial costs of inflation are substantive even in the presence of price rigidities, provided that government debt is sufficiently long-term. Different from Cao (2015), we make no normative analysis. Instead, we focus on the positive aspects of inflation financing

<sup>7</sup>Daniel (2001) and Corsetti e Mackowiak (2006) relate FTPL models to exchange-rate crisis, but the focus is also on prices and exchange-rate determination, with no mention to output consequences. Loyo (2000) studies stagflation through the lens of the FTPL, but focus on monetary policy shocks.

<sup>8</sup>See also Bernanke et al. (1999) for frictions on the borrower side of contracts.

and fiscal deficits, exploring more carefully the consequences for financial intermediation and economic activity. Our model economy also differ in some aspects. First, we adopt the more common approach to introduce financial frictions, following the lines of Gertler e Karadi (2011). Second, we add other long-term nominal assets besides long-term bonds. Our model is thus more flexible to accommodate different cases of maturity mismatch and allow inflation surprises to have substantial effects on the financial sector even when they do not affect asset prices - that is, when the monetary authority pegs interest rates or when maturity mismatch in bonds is null. This helps to connect the model to different countries' experiences, like Japan, where banks holds great amounts of government debt and the US, where banks holds a lesser amount of government debt but nevertheless has substantial maturity mismatch and exposition to a inflation default.

## Guideline

In section 2 we outline a few results of fiscal theory in a simplified New Keynesian model, focusing on key points relevant for our analysis. In section 3 we present our model. In section 4 we build intuition to understand numerical results. In section 5 we do numerical exercises and discussion. Section 6 concludes.

In this section we stress key points from the FTPL relevant for our analysis. We focus on three issues: (1) the debt valuation equation with short-term debt and fully flexible prices; (2) output and real rates consequences of adding price rigidities and (3) the role of long-term bonds.

## 2.1

### The debt valuation equation and output booms

We start with the government budget constraint written in present value form. With short-term debt only, it reads as

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}/\pi_{t+s+1}} \quad (2-1)$$

where  $B_{t-1}$  is the stock of government debt,  $P_t$  is the overall price index,  $s_t$  is government primary surplus,  $R_t$  is the gross interest-rate on government debt and  $\pi_t$  is the gross inflation rate. This is the key equation for the Fiscal Theory of Price Level (FTPL). Cochrane (2005) interprets it as a market clearing condition: the real value of outstanding government debt reflects the expected value of primary surpluses covering it. If there is a fall on surpluses, the real value of government debt falls reflecting the less resources covering for it. This revaluation can happen by two (non-excludable) ways: (1) a rise in prices  $P_t$  and (2) a fall on expected real rates. Flexible and rigid price environments will differ in the ways that this equation is satisfied under fiscal determination. To make distinctions clear, we add a little structure to our economy towards a New Keynesian environment.

**Households.** Households consume, work and save through government bonds to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \log(C_{t+j}) - \frac{H_{t+j}^{1+\psi}}{1+\psi} \right]$$

subject to the budget constraint

$$C_t + \frac{B_t}{R_t P_t} = \frac{W_t}{P_t} H_t + \Upsilon_t + \frac{B_{t-1}}{P_t} - s_t$$

where  $C_t$  is consumption,  $W_t$  is the nominal wage,  $H_t$  is the work effort and  $\Upsilon_t$  are firms' profits.  $\beta$  is the household discount factor and  $1/\psi$  is the Frisch elasticity of labor supply. The first order conditions of this problem lead to the

usual euler and supply of labor equations:

$$C_t^{-1} = \beta E_t \frac{R_t}{\pi_{t+1}} C_{t+1}^{-1} \quad (2-2)$$

$$C_t H_t^\psi = \frac{W_t}{P_t} \quad (2-3)$$

**Firms and price setting.** Firms produce differentiated goods through the function

$$Y_t = H_t \quad (2-4)$$

where  $Y_t$  is the firm's production and also the aggregate level of output.<sup>1</sup> Firms sell their goods on competitive monopolistic markets and can face nominal price rigidities or not. In a fully flexible environment, firms will chose prices as a markup  $\mu$  over marginal costs

$$P_t = \mu W_t \quad (2-5)$$

If they face nominal rigidities, however, they will not be able to choose prices freely. For simplicity, we assume the following structure of price rigidities: in period  $t$  firms are not allowed to adjust prices; from  $t + 1$  onwards, prices are fully flexible and firms will chose (optimally) to set them as in (2-5).

**Monetary and fiscal policy and market clearing.** The monetary authority acts passively, setting a constant interest-rate every period and the fiscal authority sets a constant stream of surpluses

$$R_t = 1/\beta \quad (2-6)$$

$$s_t = s > 0 \quad (2-7)$$

and markets clear

$$Y_t = C_t \quad (2-8)$$

We now investigate the following experiment: the government unexpectedly decides to reduce surpluses  $s$  to  $s' < s$ . We first look to the flexible-price case.

**Flexible prices.** Firms can choose freely to adjust prices, so by equation (2-5)

$$W_t/P_t = 1/\mu$$

That is, real wages are constant. Combining with labor supply (2-3) and market clearing (2-8), we find that consumption and output are also constant. By the euler equation (2-2), we see that real interest-rates are constant and the monetary policy rule (2-6) then implies a constant inflation. All real variables are constant, so what changes? Prices. Combining the debt valuation equation

<sup>1</sup>By symmetry and our hypothesis about price setting, all firms will choose the same production.

(2-1) and the fiscal policy (2-7), we find that

$$P_t = \frac{(1 - \beta)B_{t-1}}{s} = \frac{s}{s'} \quad (2-9)$$

where the second equality uses the stock of debt implied by fiscal policy *before* the shock:  $B_{t-1} = \frac{s}{1-\beta}$ . As the numerator is predetermined and  $s$  is falling, the result is a rise on overall prices. Prices are determined, even under an interest-rate peg. This is the fiscal determination of the price level: prices go up or down depending on the amount of surpluses expected to retire government debt from the economy. The economic forces at play behind equilibrium work as follows. With the fall on surpluses, consumers are initially wealthier. As they try to expend additional resources on goods, firms rise prices reducing the value of consumers' assets  $B_{t-1}/P_t$  and mitigating demand pressures. With flexible prices, this process goes on until all demand pressures are eliminated and real variables return to their pre-shock values.

**Rigid prices.** If prices are rigid, current prices are not sufficient to lead to equilibrium. In our simplified case, this is even more pronounced: as prices are pre-set, they cannot help at all. As prices are fully flexible from  $t + 1$  onwards, real variables will be the same as the flexible case from  $t + 1$  onwards. On  $t$  prices are fixed, so output is demand determined. From the government debt valuation, we find that

$$\pi_{t+1} = \left[ \frac{B_{t-1}/P}{s} - 1 \right] \frac{1 - \beta}{\beta} = 1 + \beta^{-1} \left[ \frac{s}{s'} - 1 \right] \quad (2-10)$$

Inflation is rising with the fall on surpluses. As the nominal rate is constant, real rates fall inducing a rise of consumption and output.<sup>2</sup>

The traditional way for explaining the economic forces at play is again through wealth effects: the fall on surpluses rises households' wealth and consumption demand. As prices are rigid (here, fixed), firms adjust quantities rather than prices leading to an output boom. Prices rise in the subsequent periods when firms are allowed to adjust due to the increase in nominal costs, so inflation goes up. This interpretation has strong appeal, but may lead to confusion because households' budget constraints do not enter explicit in the equilibrium computation and is less direct to apply when we add investment to the model. An alternative way to understand this result is as follows. When

<sup>2</sup>To be precise, there is a rise on the *growth rate* of consumption. In this simple model, this coincides to a rise in the *level* of consumption because consumption converges to the (same) steady-state immediately after the shock and the fall of surpluses are equivalent to a fall on taxes. This coincidence could not occur if, for example, the fall on surpluses were induced by a rise of government purchases and the Frisch elasticity of labor supply was low (a high  $\psi$ ). In this case, consumption could be crowded-out by government purchases since households would not be willing to work much more despite the higher wages. But in all case, output would rise because of government purchases.

surpluses fall and are not expected to rise again, by the debt valuation equation (2-1) agents expect a rise in inflation to cover the additional deficits, as current prices do not adjust fully (here, not adjust at all). The rise in inflation with no (or little) response of monetary policy implies a fall on expected real interest rates, stimulating consumption and, with rigid prices, output.

It is easy to see then what could happens if one added investment to the model: with the fall on real rates investment also rises, contributing to the rise in output. As we will show, this will no longer be necessarily true when we add financial frictions to the model. In this case, endogenous limits to arbitrage emerge and the policy real rate is nonetheless the same as the associated to capital. In this case, investments can fall even with the a fall on the policy real rate, as a result of reduced capital financing, higher spreads and lending rates coming from the banking sector and higher lending rates. But first, we turn to the case with long-term government bonds.

## 2.2

### Long-term bonds and inflation postponement

In the presence of long-term bonds, the debt valuation equation reads as

$$\frac{Q_t B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s}/\pi_{t+s+1}}$$

where  $Q_t B_{t-1}$  is the nominal value of outstanding bonds. Now there is a third way in which the debt valuation equation can be satisfied: changes in the market price of outstanding long-term bonds.

Analyzing the full model with long-term bonds is a little cumbersome and difficult to generate explicit analytical results. As the main messages of the last section does not change with the introduction of these assets, we will focus just on the consequences for inflation.<sup>3</sup> For this purpose, we will assume that prices are fully flexible at all periods, so real variables do not change. To make the analysis clear, we will also impose a specific structure to the bond market, assuming only two bonds: a short-term one, paying \$1 on the following period as in the last section and a long-term one, paying \$1 two periods ahead. We include a short-term debt just to define a policy rate, so we additionally impose that it is in zero net-supply. Given this market structure, there is an arbitrage relation that defines the price of outstanding long-term bonds

$$Q_t^1 = 1/R_t \quad (2-11)$$

<sup>3</sup>See Leeper e Leith (2016) for a more complete analytical treatment.

where  $Q_t^N$  is the price of a long-term bond with  $N$  periods to mature. Note that *outstanding* long-term bonds are equivalent to one-period short-term, as they will pay exactly \$1 next period. To see the difference from the case of short-term debt only, it is useful to look to the households' budget constraint in this environment

$$C_t + Q_t^2 \frac{B_t^2}{P_t} = \frac{W_t}{P_t} H_t + \Upsilon_t + Q_t^1 \frac{B_{t-1}}{P_t} - s_t$$

where we imposed market clearing for one-period bonds. Different from the case of short-term debt only, changes in the policy interest-rate,  $R_t$ , affects directly the price of assets households already holds, not only assets they could buy. That way, changes in policy rates have a direct wealth effect on households by reducing the value of their portfolio. This falling value of long-term debt, in turn, is mirrored in the government budget constraint, which accordingly reads as

$$\frac{Q_t^1 B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{s_{t+j}}{\prod_{s=0}^{j-1} R_{t+s} / \pi_{t+s+1}} \quad (2-12)$$

To see the change on the inflation trade off, we now allow nominal rates responding to inflation according to the following rule

$$R_t = \beta^{-1} \pi_t^\phi, \quad \phi < 1 \quad (2-13)$$

Again, suppose the economy was in zero inflation steady-state equilibrium until period  $t$  and the government unexpectedly reduce surpluses  $s$  to a new value  $s'$  with  $s' < s$ . Before the shock, we had by (2-12) that outstanding debt equals to

$$B_{t-1} = \frac{s}{\beta(1-\beta)}$$

where we normalize the pre-shock prices to unity. Substituting this expression, the arbitrage relation (2-11) and the policy rate (2-13) in the debt valuation equation (2-12) *after* the shock and using the fact that real rates are constant, we find the following expression for inflation at  $t$

$$\pi_t = \left[ \frac{(1-\beta)\beta B_{t-1}}{s'} \right]^{\frac{1}{1+\phi}} = \left[ \frac{s}{s'} \right]^{\frac{1}{1+\phi}} \quad (2-14)$$

as  $s' < s$ , gross inflation-rate is greater than one and is higher the greater the fall on surpluses. Using again the fact that real rates are constant and substituting the policy rate (2-13), we also find an expression for future inflation

$$\pi_{t+j} = (\pi_t)^{\phi^j} \quad (2-15)$$

Expressions (2-14) and (2-15) show the differences on inflation dynamics relative to the short-term debt case. Now the government has the option of trading inflation today for inflation in the future by altering the value of outstanding long-term bonds (that is, by altering  $R_t$ ). A higher value of  $\phi$  reduce inflation at  $t$  at the cost of higher future inflation. We also see what happens if the government tries to respond too strongly to inflation, for example, following the taylor principle: if  $\phi > 1$ , by (2-15), inflation takes an explosive path. If on the contrary, the government does not let long-term debt value fluctuate, pegging the short-term interest-rate ( $\phi = 0$ ), we come back to the flexible price short-term debt case: all revaluation happens in the moment of the shock and there is no future inflation (compare (2-9) to (2-14) with  $\phi = 0$ ).

The mechanism goes as follows. The fall on surpluses generates demand pressures. The government then starts selling long-term debt at lower prices (that is, it permits  $R_t$  to rise), leading demand towards government bonds, alleviating pressures on goods and current inflation. But by selling long-term debt, the government transfer these demand pressures for the future, as there will be more maturing debt to chase goods tomorrow. Hence, expectations of inflation rises today and if nothing else happens, actual future inflation rises as well.<sup>4</sup> If on the other hand, the government trades any amount of debt at a fixed rate, as in an interest-rate peg, it keeps prices of government bonds stable and this channel dies.

<sup>4</sup>Again, it is possible to interpret this mechanism through wealth effects: when government starts selling long-term debt at lower prices, the portfolio value of households holding these assets goes down, mitigating the expansion on the budget constraint caused by the fall on surpluses and reducing demand pressures.



### 3

## Model description

Our economy is a version of Gertler e Kiyotaki (2010) and Gertler e Karadi (2011) in which government bonds and long-term nominal loans composes banks' balances. Long-term loans are introduced using a framework similar to Andreasen et al. (2013). Relative to the last section, we add capital, financial frictions and a more complex maturity structure for bonds.

### 3.1

#### Households

Households consume, work and save to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{(C_{t+j} - hC_{t+j-1})^{1-\sigma}}{1-\sigma} - \frac{H_{t+j}^{1+\psi}}{1+\psi} \right] \quad (3-1)$$

subject to the budget constraint

$$C_t + d_t = w_t H_t + \frac{R_{t-1}^d}{\pi_t} d_{t-1} + (1 - \varsigma) \frac{R_t^b}{\pi_t} Q_{t-1} b_{t-1} - \tau_t + \Upsilon_t \quad (3-2)$$

where  $d_t$  are real deposits on banks,  $R_t^d$  is the nominal rate on these deposits,  $H_t$  is the work effort,  $w_t$  is the real wage,  $\pi_t$  is the inflation rate,  $\tau_t$  are lump-sum taxes,  $\Upsilon_t$  are firms' profits,  $b_t$  are government bonds and  $Q_t$  and  $R_t^b$  the price and return of government bonds. The parameter  $\sigma$  is the inverse of the intertemporal elasticity of consumption,  $h$  is the habit persistence on consumption,  $1/\psi$  is the Frisch elasticity and  $\varsigma$  indexes government bond holders (see the financial intermediaries section). Consumption  $C_t$  is a CES aggregate over differentiated goods obtained from an expenditure minimization problem and is given by

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (3-3)$$

### 3.2

#### Wholesale firms

Wholesale firms operate in competitive markets, renting capital  $K_t$  and labor  $H_t$  to produce output  $Y_t$  according to the technology

$$Y_t = K_{t-1}^\alpha H_t^{1-\alpha} \quad (3-4)$$

Let  $p_t^w$  be the relative price of wholesale goods,  $r_t^k$  the rental price of capital and  $w_t$  the real wage. Optimal factor allocations are then given by

$$\alpha p_t^w Y_t / K_{t-1} = r_t^k \quad (3-5)$$

$$(1 - \alpha) p_t^w Y_t / H_t = w_t; \quad (3-6)$$

### 3.3

#### Entrepreneurs

At the beginning of period  $t$ , entrepreneurs take loans from financial intermediaries to finance capital for use at  $t + 1$ . A loan signed at period  $t$  specifies the amount of capital  $\tilde{k}_t$  the entrepreneur wants to finance and a fixed nominal rate  $R_t^l$  to hold until the loan matures. Following Andreasen et al. (2013), we introduce long-term loans by imposing a random maturity for debt contracts assuming that each period loans mature with probability  $1 - \theta_k$ . Entrepreneurs choices are aligned to the horizon of its debt contract. While the loan does not mature, entrepreneurs keep its capital choice constant. When the loan matures, entrepreneurs reoptimize taking a new loan.

There are two points of this structure worth to note. First, it implies an average duration of loan contracts of  $\frac{1}{1-\theta_k}$  and allow us to parametrize the model through the single parameter  $\theta_k$ . Second, it implies infrequent capital adjustments as the adopted in Kiyotaki e Moore (1997) and Sveen e Weinke (2007) on the context of firm-specific capital. As showed by Andreasen et al. (2013), when capital is homogeneous, infrequent capital adjustments have no effect on prices and aggregate variables in a broad range of DSGE models, including ours when we eliminate financial frictions. Thus, adopting this structure allow us to isolate the effects of maturity mismatch and financial frictions on the economy.

When ready for use, entrepreneurs rent capital to wholesale firms and sell non-depreciated capital for capital producers which is bought again at the start of the following period. All profits from these operations are rebated lump-sum to households so entrepreneurs do not accumulate net-worth. Accordingly, entrepreneurs adjusting at  $t$  choose capital  $\tilde{k}_t$  to maximize the present value

of profits<sup>1</sup>

$$E_t \sum_{j=1}^{\infty} \beta^j \theta_k^{j-1} \frac{\lambda_{t+j}}{\lambda} \left[ r_{t+j}^k \tilde{K}_t - \frac{R_t^L}{\prod_{s=0}^j \pi_{t+s}} p_t^k \tilde{K}_t + (1 - \delta) p_{t+j}^k \tilde{K}_t \right]$$

where  $\lambda_t$  is the Lagrange multiplier on households' budget constraint and  $\delta$  is the depreciation rate. As entrepreneurs take nominal debt to finance real assets, there is a Fischer's debt-deflation channel which makes entrepreneurs better off with inflation. This effect is reflected on the negative term above and is more intense the longer the maturity of the loan contract.

For every entrepreneur optimizing, first order condition for capital choice is given by

$$E_t \sum_{j=1}^{\infty} \beta^j \theta_k^{j-1} \frac{\lambda_{t+j}}{\lambda} \left[ r_{t+j}^k - \frac{R_t^L}{\prod_{s=0}^j \pi_{t+s}} p_t^k + (1 - \delta) p_{t+j}^k \right] = 0 \quad (3-7)$$

Entrepreneurs not optimizing simply make  $\tilde{K}_t = \tilde{K}_{t-1}$ . As all optimizing entrepreneurs are homogeneous and face the same probability of adjusting capital, aggregate demand for capital is given by

$$K_t = (1 - \theta_k) \tilde{K}_t + \theta_k K_{t-1} \quad (3-8)$$

### 3.4

#### Financial Intermediaries

Financial intermediaries/banks are owned by households. Let  $l_t$  be the real value of loans to entrepreneurs and  $n_t$  financial intermediaries' net-worth. Intermediaries' balance-sheet is then given by

$$l_t + \varsigma Q_t b_t = d_t + n_t \quad (3-9)$$

The indicator parameter  $\varsigma$  is introduced only to vary government bond holders. We study two polar cases: when  $\varsigma = 1$ , financial intermediaries hold government bonds and when  $\varsigma = 0$  households are the holders of government debt. We could allow for intermediate cases, but for simplicity and to highlight the main mechanisms at play, we focus only on these two cases.<sup>2</sup>

Intermediaries' revenues  $rev_t$  from loans depend on the whole portfolio

<sup>1</sup>Implicitly, we assume that every period entrepreneurs sell capital to capital producers at a price  $p_{t+j}^k$  and buy back at price  $p_t^k$  signed in the beginning of the loan contract, such that all capital gains goes to entrepreneurs. We do this for two reasons. First, buying at a fixed price prevents the loan value to vary with the price of capital, resembling more real-life loan arrangements. Second, allowing capital gains enables us to recover standard financial-friction models when we impose one-period loans ( $\alpha_k = 0$ ).

<sup>2</sup>See section 5.4 for more on this issue.

of loans lent to entrepreneurs. Given the maturity structure imposed, it can be written in the following recursive form

$$rev_t = (1 - \theta_k)R_t^l p_t^k \tilde{K}_t + \theta_k rev_{t-1}/\pi_t \quad (3-10)$$

A similar recursive form can be obtained for the total amount of loans

$$l_t = (1 - \theta_k)p_t^k \tilde{K}_t + \theta_k l_{t-1}/\pi_t \quad (3-11)$$

These expressions simply say that revenues and loans are a function of contracts signed in the current period, plus the real value of contracts brought from the past.

As in Gertler e Karadi (2011), there is an agency problem between bankers and depositors which requires bankers to supply own net-worth to provide credit, giving rise to endogenous liquidity constraints and spreads. As banks will lend at rates at least higher than borrowing rates, net-worth could grow to the point that banks do not need deposits to provide credit. To prevent this, we assume that each period a fraction  $1 - \theta_b$  of banks exit the market and transfer accumulated net-worth to its household. At the beginning of  $t$ , banks' net-worth is given by

$$n_t = \frac{1 - \chi}{\pi_t} [rev_{t-1} + \varsigma R_t^b Q_{t-1} b_{t-1} - R_{t-1}^d d_{t-1}] \quad (3-12)$$

Following Andreasen et al. (2013), we introduce the proportional contribution  $\chi$  on banks' net-worth that is paid to insurance agencies. This is done for two reasons. First, these agencies guarantee that entering banks starts with the same composition of assets and liabilities of existing banks, simplifying aggregation and allowing the use of a representative bank. Second, this helps to ensure that banks' net-worth does not grow without bounds.<sup>3</sup> In any case, by matching leverage and spreads to conventional values on the literature, we calibrate  $\chi$  to a quite small value, so it has little impact on our results.

Bankers' objective function is to maximize the present-value of expected net-worth that will be transferred to households. Hence, their value function can be written in the following recursive form

$$V_t = \max E_t \beta \frac{\lambda_{t+1}}{\lambda_t} [(1 - \theta_b)n_{t+1} + \theta_b V_{t+1}] \quad (3-13)$$

That is, the value function of a banker optimizing at  $t$  is the present value of expected net-worth one period ahead in the case of exiting the industry plus

<sup>3</sup>This setup is different from Gertler e Karadi (2011), who assume that retired banks are replaced by new banks with a sufficiently low net-worth

the value of continuation of remaining a banker.

To the extent that households obtain a higher risk-adjusted return by lending to the bank than saving itself, bankers will want to expand its funds by taking deposits indefinitely to the point that all arbitrages are exploited. To motivate a limit in their ability to do so, we assume the following agency problem. Each period, a banker can choose to divert a fraction of total assets to its own household. In that case, households can force intermediaries to bankruptcy, but can recover only a fraction of assets. Following Gertler e Karadi (2013), we assume this agency friction can be less severe for government bonds than private loans, reflecting the fact that the latter is more difficult for depositors to monitor. Therefore, each period the banker has the option to divert a fraction  $\omega$  of private loans and  $\omega\iota$  of bonds, with  $0 \leq \iota \leq 1$ . Accordingly, for households be willing to supply funds to the banker, the following incentive compatibility constraint must hold

$$V_t \geq \omega [l_t + \iota \varsigma Q_t b_t] \quad (3-14)$$

This expression says that the value of keeping operating (left side) must be at least greater than the value of diverting assets and going bankrupt. Banks then have to limit its demand for deposits relative to own net-worth to keep incentives aligned to that of depositors. Throughout the paper, we assume this incentive constraint is always binding and ensure this is the case within a local region of the steady-state.

The optimization problem of bankers is then to choose  $l_t, Q_t b_t$  and  $d_t$  to solve (3-13) subject to the balance-sheet condition (3-9), the laws of motion (3-10)-(3-12) and the incentive constraint (3-14). To solve this problem, we first guess that the value function has the following form

$$V_t = \nu_t^k l_t + \nu_t^b \varsigma Q_t b_t - \mu_t d_t \quad (3-15)$$

where each coefficient represents the marginal value of varying the respective variable. By guess and verify, we find these coefficients to be

$$\nu_t^k = E_t \Omega_{t+1} \frac{rev_t / l_t}{\pi_{t+1}} \quad (3-16)$$

$$\nu_t^b = E_t \Omega_{t+1} \frac{R_{t+1}^b}{\pi_{t+1}} \quad (3-17)$$

$$\mu_t = E_t \Omega_{t+1} \frac{R_t^d}{\pi_{t+1}} \quad (3-18)$$

where  $\Omega_{t+1}$  is an adjusted stochastic discount factor, given by

$$\Omega_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\beta}{(1-\chi)} [(1-\theta_b) + \theta_b ((lev_{t+1}^k + \iota lev_{t+1}^b)(\nu_{t+1}^k - \mu_{t+1}) + \mu_{t+1})] \quad (3-19)$$

where  $lev_t^k = \frac{l_t}{n_t}$  and  $lev_t^b = \frac{\varsigma Q_t b_t}{n_t}$  are the leverage ratios for private loans and bonds, respectively. These expressions simply says that the marginal gain of financing capital or the government is the present-value of expected average real returns of doing so. The marginal cost of collecting more deposits, in turn, is the expected payment by doing so. By the presence of financial frictions, bankers' stochastic factor is not the same as households' and must be adjusted by the term in brackets.

Letting  $\vartheta_t$  denote the Lagrange multiplier on the incentive constraint, substituting the balance-sheet condition (3-9) for  $d_t$  in the guess for the value function and using the expressions above, the first order conditions for  $l_t$  and  $Q_t b_t$  imply

$$E_t \Omega_{t+1} \left( \frac{rev_t/l_t}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \omega \frac{\vartheta_t}{1 + \vartheta_t} \quad (3-20)$$

$$E_t \Omega_{t+1} \left( \frac{R_{t+1}^b}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \varsigma \omega \iota \frac{\vartheta_t}{1 + \vartheta_t} \quad (3-21)$$

So when the compatibility constraint binds ( $\vartheta_t > 0$ ), a spread between deposit rates and lending rates arise. The spread on government bond returns will be lower if the agency friction is less intense on these assets ( $\iota < 1$ ) and absent if they are owned by households ( $\varsigma = 0$ ).<sup>4</sup> We can combine these two equations to obtain the following arbitrage relation that must hold whenever banks hold bonds

$$E_t \Omega_{t+1} \left( \frac{R_{t+1}^b}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) = \varsigma \iota E_t \Omega_{t+1} \left( \frac{rev_t/l_t}{\pi_{t+1}} - \frac{R_t^d}{\pi_{t+1}} \right) \quad (3-22)$$

Finally, we can use the definitions for leverage and the guess for the value function to rewrite the incentive constraint as

$$lev_t^k + \iota lev_t^b = \frac{\mu_t}{\gamma - (\nu_t^k - \mu_t)} \quad (3-23)$$

### 3.5

#### Capital producers

At the end of period  $t$ , capital producers buy non-depreciated capital and investment goods to produce new capital. In the beginning of  $t+1$  and before shocks are realized, capital is sold again to entrepreneurs. As noted before, in order to keep capital gains with entrepreneurs and to prevent the value of loans

<sup>4</sup>To be precise, arbitrage between bonds and deposits when households are the holders of government bonds is found by solving the households' problem. In this case, the arbitrage relation would involve households' discount factor, not the bankers one. In a linearized model there is no risk, so using the setup with  $\varsigma$  do not affect results and we choose it for concision.

to vary with capital prices, capital producers buys capital for the market price and resell it to the price signed in the start of the entrepreneurs loan contract. Thus, the present-value of profits is given by

$$E_t \sum_{j=0}^{\infty} \frac{\lambda_{t+1}}{\lambda_t} \beta^j [v_{t+j} - (1 - \delta)p_{t+j}^k K_{t+j-1} - I_{t+j}] \quad (3-24)$$

where aggregate investment  $I_t$  is a bundle of retail goods similar to the consumers',  $p_t^k$  is the real price of capital and  $v_t$  is an aggregate given by

$$v_t = (1 - \theta_k) \sum_{j=0}^{\infty} \theta_k^j \tilde{K}_{t-j} \frac{P_{t-j}^k}{P_t} = (1 - \theta_k) p_t^k \tilde{K}_t + \theta_k \frac{v_{t-1}}{\pi_t} \quad (3-25)$$

As in Christiano et al. (2005), capital production involves investment flow adjustment costs and follows the technology

$$K_t = (1 - \delta)K_{t-1} + I_t - S(I_t/I_{t-1})I_t \quad (3-26)$$

where  $S(\cdot)$  is the adjustment cost function, satisfying:  $S(0) = 1$ ,  $S'(0) = 0$  and  $S''(0) = \kappa > 0$ .<sup>5</sup>

Capital producers' objective is to choose  $K_t, \tilde{K}_t, v_t$  and  $I_t$  to maximize the discounted value of profits (3-24) subject to (3-25), (3-26) and capital demand from entrepreneurs (3-8). The optimization is described on the appendix.

### 3.6 Retailers

Retailers buy goods from wholesale producers and differentiate it through a linear technology to sell at competitive monopolistic markets. We introduce price rigidities following Calvo (1983): each period, only  $1 - \theta_p$  firms are allowed to choose prices freely. Firms who are able to optimize choose prices to maximize discounted profits

$$E_t \sum_{j=0}^{\infty} (\beta \xi)^j \frac{\lambda_{t+1}}{\lambda_t} Y_{t+j}(i) (\tilde{p}_t - mc_{t+j})$$

subject to the demand curves

$$Y_t(i) = (\tilde{p}_t)^{-\eta} (C_t + G_t + I_t)$$

where  $mc_t = p_t^w$  is the real marginal cost and  $\tilde{p}_t$  is the (real) new price. Solution to this problem leads to

<sup>5</sup>Particularly, we follow Christiano et al. (2015) adopting the following functional form for  $S(\cdot)$ :  $S\left(\frac{I_t}{I_{t-1}}\right) = 1/2 [\exp(\sqrt{\kappa}(I_t/I_{t-1} - 1)) + \exp(-\sqrt{\kappa}(I_t/I_{t-1} - 1)) - 2]$ . On a linearized solution, any function with the properties in the text gives identical results.

$$\tilde{p}_t = \frac{\eta}{\eta - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta\xi)^j \frac{\lambda_{t+j}}{\lambda_t} Y_{t+j} m c_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\xi)^j \frac{\lambda_{t+j}}{\lambda_t} Y_{t+j}} \quad (3-27)$$

By the Calvo scheme and agents' baskets of retail goods, inflation is given by

$$\pi_t^{1-\eta} = (1 - \theta_p)(\tilde{p}_t \pi_t)^{1-\eta} + \theta_p \quad (3-28)$$

### 3.7

#### Government, policies and market clearing

The government collects taxes and issues debt to finance purchases and repay maturing debt. Following Cochrane (2001), Woodford (2001) and Eusepi e Preston (2011), we introduce long-term bonds imposing a geometric payment structure: a bond sold at the price  $Q_t$  pays \$1 in  $t$ ,  $\$1+\rho$  in  $t+1$ ,  $\$1+\rho^2$  in  $t+2$  and so on. As Leeper e Leith (2016) show, this is equivalent to assuming a geometric decaying rate for a portfolio of zero-coupon bonds. This setup implies an average maturity of outstanding bonds of  $\frac{1}{1-\beta\rho}$ , allowing us to control maturity through  $\rho$ . Given this maturity structure, we can define the ex-post return for bonds as

$$R_t^b = \frac{1 + \rho Q_t}{Q_{t-1}} \quad (3-29)$$

and write the period government budget constraint as

$$G_t + Q_t b_t = \tau_t + \frac{1 + \rho Q_t}{\pi_t} b_{t-1} \quad (3-30)$$

where government consumption  $G_t$  is a bundle equal to the consumers' and follows the exogenous process

$$\log(G_t) = (1 - \rho_g) \log(G) + \rho_g \log(G_{t-1}) + \varepsilon_t^G \quad (3-31)$$

where  $G$  is the steady-state value of government purchases,  $\rho_g$  is an autoregressive coefficient and  $\varepsilon_t^g$  is a zero mean normal distributed random shock.

Monetary policy is described by the following rule:

$$\log(R_t^d) = \log(1/\beta) + \rho_r \log(R_{t-1}) + (1 - \rho_r)(\phi_\pi \log(\pi_t)) \quad (3-32)$$

As we will focus on the case for fiscal price-level determination, rather than interpreting the above rule as an explicit inflation-targeting, we will interpret it as an 'inflation postponement' rule. As we showed on section 2, letting  $\phi_\pi > 0$  the government can trade inflation along time by allowing asset prices to restore government solvency. The higher the value of  $\phi_\pi$ , the lower the inflation coming



today and the higher the inflation tomorrow. On the other hand, when  $\phi_\pi = 0$ , the government is willing to trade all quantities of bonds at a constant rate so asset prices remains constant, killing this channel. Therefore, we let  $\phi_\pi > 0$  whenever we want to study the cases of inflation affecting bond prices directly.<sup>6</sup>

Following Leeper (1991), the fiscal rule is specified by a tax response to the stock of government debt

$$\log(\tau_t) = \log(\tau) + \phi_\tau \log(Q_{t-1}b_{t-1}) + \epsilon_t \quad (3-33)$$

where  $\tau$  is the steady-state value of taxes and  $\epsilon_t$  is a stochastic process following

$$\epsilon_t = \rho_\tau \epsilon_{t-1} + \varepsilon_t^\tau \quad (3-34)$$

Finally, goods market clearing requires that

$$Y_t = (C_t + G_t + I_t)_t \quad (3-35)$$

where  $_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\eta} di$  is a measure of price dispersion.

<sup>6</sup>In the model with financial frictions, because of spreads government bond prices can vary even in a riskless economy with  $\phi_\pi = 0$ .

## 4

### Forces at play

Before going to our numerical exercises, we build intuition for key features of our model. There are two offsetting forces at play in a fall on surpluses financed through inflation. First, the usual aggregate demand effect described in section 2: when surpluses fall, households start expecting negative rates to cover the additional deficits and raise consumption, stimulating production. On the other hand, as we will make clear shortly, inflation default negatively affects financial intermediaries' net-worth, reducing capital finance, raising spreads and discouraging investments. Which of these forces prevail depends on: the maturity mismatch of the banking sector, the size of government debt and relevance on banks' portfolio and how the government time this inflation.

Suppose that banks hold the government debt and that agency problem is the same between private loans and bonds. Then, we can express the incentive constraint as

$$l_t + Q_t b_t \leq lev_t n_t \quad (4-1)$$

where  $lev_t = \frac{\mu_t}{\omega - (\nu_t^k - \mu_t)}$  is the overall leverage of financial intermediaries. This expression is basically a liquidity constraint. A fall on banks' net-worth  $n_t$  tightens this constraint, reducing credit for capital purchases and for the government. Intuitively, the fall on banks' net-worth rises the incentive to divert assets relatively to keep operating, so banks must reduce finance to satisfy the incentive constraint and remain attracting deposits. The tightening of the constraint rises the value of relaxing it and by equations (3-20)-(3-21), rises the spread on lending rates. Therefore, this limit to arbitrage implies that lending real rates can be positive even if deposit/policy real rates remains negative. The fall in credit and the rise on real rates, in turn, generate a decrease of investments.

To see how surprise inflation affects banks' net-worth, it is useful to look at the net-worth accumulation equation, modified and reproduced here for convenience

$$n_t = \frac{1 - \chi}{\pi_t} [rev_{t-1} + \varsigma(1 + \rho Q_t)b_{t-1} - R_{t-1}^d d_{t-1}] \quad (4-2)$$

There are two ways by which a surprise inflation can negatively affect banks' net-worth: reducing the value of loan revenues and the value of long-term bonds. But at the same time, deposits' value fall with inflation, offsetting

these effects. As banks are leveraged, typically the negative effect prevails.<sup>1</sup> And the longer the maturity of assets relative to liabilities, the stronger it will prevail. To make this clear, look to the net-worth equation when banks hold no bonds and loans are only one-period:

$$n_t = \frac{1 - \chi}{\pi_t} [R_{t-1}^l p_{t-1} K_{t-1} - d_{t-1}] = \frac{(1 - \chi)n_{t-1}}{\pi_t} [lev_{t-1}(R_{t-1}^l - R_{t-1}^d) + R_{t-1}^d] \quad (4-3)$$

In the second equality, as spreads are positive and rates predetermined, we see that a surprise inflation negatively affect banks' net-worth. But for one-period loans and a reasonable calibration, we find these effects to be almost irrelevant to the rest of the economy. The reason is that with one-period loans, in response to inflation banks can immediately revise all their loans to entrepreneurs, switching to new loans with higher rates. When contracts are long-term, however, this will not be possible and banks will have to stick with some contracts even if inflation erodes their value. This can be clearly seen by the law of motion of banks' revenues when contracts have more than one period:

$$rev_t = (1 - \theta_k) R_t^l p_t^k \tilde{K}_t + \theta_k (1 - \theta_k) \frac{R_{t-1}^l}{\pi_t} p_{t-1}^k \tilde{K}_{t-1} + \theta_k^2 (1 - \theta_k) \frac{R_{t-2}^l}{\pi_t \pi_{t-1}} p_{t-2}^k \tilde{K}_{t-2} + \dots \quad (4-4)$$

This expression shows the other side of the traditional Fischer's debt-deflation channel for borrowers: if intermediaries engage in long-term nominal loans, revenues are diminished when there is a surprise inflation and banks cannot immediately adjust their contracts with the private sector. The longer the average duration of loans, the greater the impact of a surprise inflation. If financial intermediaries (or lenders in general) have leverage constraints limiting the credit they can provide, this fall on profits lead to higher spreads which, in contrast to the conventional effect of the debt-deflation channel, makes entrepreneurs worse off and discourage investments. So the presence of a debt-deflation channel comes with a 'credit-deflation channel' which, as we will show, can even countervail the positive effect on entrepreneurs.

If inflation surprises are the problem, the government can trade it for bond-price surprises by allowing interest rates to vary, reducing the value of debt outstanding. In this case, although the total amount of inflation is

<sup>1</sup>This is not true if loans are denominated in real terms, as in Gertler e Karadi (2011) where banks buy equity stakes from firms. As a great portion of banks financing is made through nominal loans, we will stick with this case to stress its consequences. But we note that, even in the case of real loans, revaluation of nominal assets (long-term bonds, for example) still have relevant effects on banks' positions in the event of a surprise inflation.

similar, a great portion of it is anticipated, allowing intermediaries to adjust better to the shock. However, as we see by (4-2), when banks hold long-term government bonds, or assets with comparable payment structure, the fall on bond prices reduces banks' net-worth, leading to the same negative effects on capital financing. In our model, the fall on banks' assets comes from the fall in the price of government bonds but we emphasize this is only because, for simplicity, we did not introduce other types of long-term nominal assets. If banks were allowed to hold other types of long-term assets, by arbitrage (even if limited) there would be a fall on their value as well. Nonetheless, note again the role of maturity: when assets are one-period only ( $\rho = 0$ ), this channel becomes irrelevant and if loans are relatively short-term, impact of inflation on intermediaries' net-worth are mild.

If the fall on investment is such that the economy enters a recession, one could think that this would lead to a deflation, contradicting the reason for the recession to start with. This is not true: the fall on capital acts a supply side shock to the economy, rising the present value of firms' marginal costs. Specifically, the fall on capital is not an optimal allocation, as there is no change on its productivity. Rather, from the point of view of producing firms, it is a result of external factors coming from the financial sector and the government. Therefore, the inflation necessary to finance government deficits still comes, but at the cost of output.

## 5

### Numerical analysis

Numerical solution to the model is obtained through a first-order approximation around a deterministic steady-state in which leverage constraints bind. We explore impulse-response functions to deficit shocks coming from taxes and government purchases. We first show and comment the impulse responses for the full model for each case. To isolate each channel detailed in the last section, we further separate the analysis in two cases: (1) banks holding only long-term loans and no bonds and (2) holding one-period loans and long-term bonds.

#### 5.1

##### Calibration

Table 5.1 describes our calibration. We take conventional values for the discount factor  $\beta$ , the capital share on the production function  $\alpha$ , the depreciation rate  $\delta$  and the government expenditure share. As in Gertler e Karadi (2011) and ?, the parameters pertaining to the banking sector  $\chi, \theta_b, \omega$  and  $\iota$  are calibrated to imply a leverage ratio of four and a steady-state spread for loans and bonds of one hundred basis-point and fifty basis-point, respectively. The parameters from the real economy are obtained from Christiano et al. (2015), who estimate a model with financial and labor frictions using US data. The only exception is the Frisch elasticity  $1/\psi$ , for which we use the value proposed by Chetty et al. (2011) based on macro and micro estimates.<sup>1</sup> Conditional on fiscal price-level determination, the lump-sum taxes response to debt  $\phi_\tau$  does not have relevant effects on the economy, so we keep it at zero. For exogenous processes, we assume an autoregressive coefficient of 0.9. Finally, we keep free the values for the parameters controlling the duration of loans and bonds,  $\theta_k$  and  $\rho$ , the inflation response  $\phi_\pi$  and the debt-to-GDP ratio and analyze the consequences of changing each one.

#### 5.2

##### Full model

For the full model, we calibrate the free parameters as follows. According to the evidence in Cao (2014), we choose  $\alpha_k$  and  $\rho_b$  to generate an average duration of four years for loans and five years for bonds. The debt-to-GDP

<sup>1</sup>There is no direct analogue for this parameter in Christiano et al. (2015).

Table 5.1: Calibration

Parameter	Value	Description
$\beta$	0.99	Households discount factor
$h$	0.89	Habits persistence
$1/\psi$	0.75	Frisch elasticity
$\alpha$	0.33	Capital share
$\delta$	0.025	Depreciation rate
$\kappa$	12.07	Investment adjustment costs
$\theta_p$	0.74	Probability of keeping prices fixed
$\eta$	4.36	Elasticity of substitution
$\chi$	0.018	Insurance agencies contribution
$\omega$	0.33	Fraction of assets can be diverted
$\theta_b$	0.92	Probability of remaining a banker
$\iota$	0.5	Intensity of bonds agency friction
$\phi_\tau$	0	Taxes response to debt
G/Y	0.2	Steady-state share of government purchases
$\rho_r$	0.8	Policy rate smoothing parameter
$\phi_\pi$	Free	Policy rate response to inflation
$\theta_k$	Free	Loans maturity parameter
$\rho$	Free	Bonds maturity parameter
$\frac{Qb}{4Y}$	Free	Steady-state debt-to-GDP ratio

ratio is set at 80%, the average level for G7 economies at the end of 2015 and the policy rate coefficient is set at  $\phi_\pi = 0.2$ .

Figure 5.1 shows the impulse-response functions to a fall on taxes of one percent of GDP. The surprise rise on inflation following a fall on taxes has two negative effects on banks' net-worth. First, it reduces banks' real revenues on impact (the inflation term in (3-12)) and even more as they cannot adjust a fraction of loans according to the new inflation path (the cumulative inflation in (4-4)). Second, the rise in the policy rate push bond prices down, reducing the value of banks' assets. The result is a rise on spreads and a fall on capital financing, which lead to a fall on investments and output. The rise on bond spreads acts as a financial accelerator channel by pushing bond prices down even more. Consumption rises on impact because of the fall on taxes and real deposit rates, but fall later as output (and hence, income)

keeps below steady-state.<sup>2</sup> Despite the recession, inflation comes up as result of a supply-side like shock to the economy: the fall on capital coming from the financial sector rises the costs of capital for producing firms, increasing marginal costs and inflation.

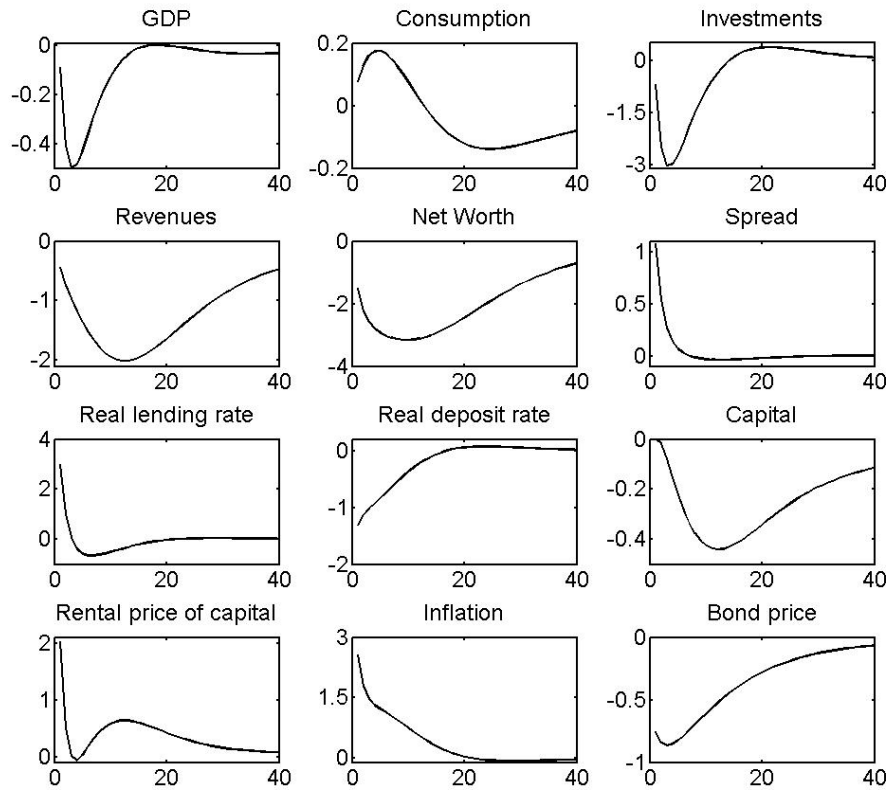
Figure 5.2 shows impulse response functions to a rise in government purchases of one percent of GDP. Not surprisingly, a direct aggregate stimulus from the government combined with rigid prices leads to a rise on output. But all the effects on the financial sector behind the fall on investments in the previous case are also present here. This explain why the fiscal multiplier, typically well above one in models with fiscal price-level determination, is less than one here. Government purchases crowds-out investments, as the rise in inflation and fall on bond prices reduce capital finance. Consumption rises little and fall relatively faster than the previous case, also as a result of a crowding out effect.<sup>3</sup> Accordingly, fiscal multiplier becomes less than one and eventually turns negative as government purchases return to steady-state while consumption and investment remain below steady-state. Inflation rises more than in the previous case since there is also the rise in government demand pressing prices upwards.

These exercises show that a change on expectations of surpluses covering the public debt can be damaging to the financial sector, even if they do not imply an outright default (which we explicitly excluded here). The surprise 'inflation default' and revaluation of bond prices reduces intermediaries' net-worth and this alone can lead to a rise in spreads, fall on investments and recession. This gives theoretical support for works finding a negative relationship between banks' performance and inflation and a positive relation between banking crises and episodes of high inflation and high government debt. Outright sovereign default, as in Bocola (2016), is the common channel associated with the impairing of the financial sector coming from an unsustainable fiscal policy. Instead of claiming that our channel is the right explanation, we think these alternatives can reinforce each other: before or even

<sup>2</sup>The rise in consumption following this type of shock is a common result under fiscal price-level determination. Here it is reinforced by an also common result of negative net-worth shocks in the presence of financial frictions: following the fall on net-worth, banks reduce demand for deposits to satisfy the compatibility constraint, pressing deposit rates down. There are ways to address this countercyclical movement of consumption (see for example Bigio (2015)), but as this is not central for our analysis we abstract from these features here.

<sup>3</sup>The initial response of consumption is sensitive to the Frisch elasticity. The intuition is as follow: the rise in output following the government spending shock rises prices and the demand for work. Due to the rise in prices, consumers will have to work more to expand consumption. But if the elasticity is low, they will not be willing to vary much of their work even with higher wages, so they optimally choose to work little and consume less (relative to a case of high elasticity).

Figure 5.1: Impulse response to a tax shock, full model



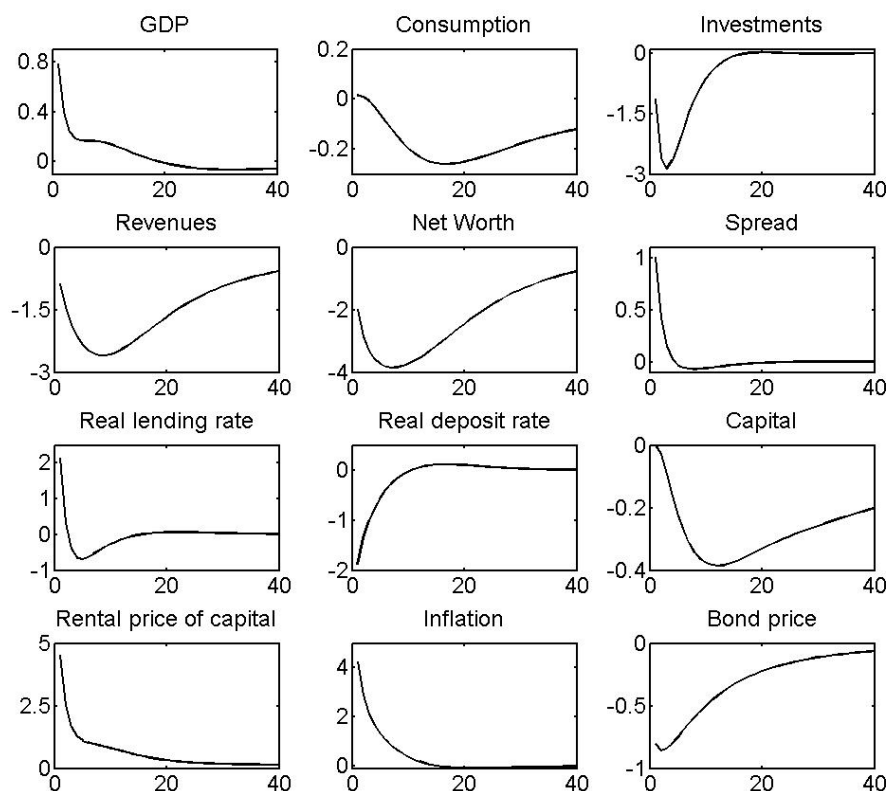
jointly with an outright default, negative news about surpluses in the presence of high debt and unsustainable fiscal policy feeds inflation, which hurts banks' balance-sheets contributing to or even triggering a recession. Furthermore, as inflation grows intolerable, default starts being a more desirable or even the only option (see Uribe (2006)) to reduce debt, rising its probability and impairing banks' net-worth even more, as in Bocola (2016).

One may say the channels we stress here are not important, as in high-inflation environments agents tend to reduce the maturity of contracts. On the contrary, we think this can be an anecdotal evidence of the costs of surprise inflation and asset price revaluations: as these costs grows high, agents (optimally) shorten the maturity of contracts. Although we do not model the choice of the length of contracts here, their shortening can be another cost to the economy, as this restrain firms of matching the horizons of its liabilities and its long-term projects, can raise probability of default from the private sector and raise the costs of portfolio management for intermediaries.

These results are also additional warnings against fiscal policy financed through inflation, adding to the points made in Cochrane (2011). This includes the 'helicopter drop' policies being proposed more recently as a solution to



Figure 5.2: Impulse response to a government spending shock, full model



low-inflation environments. A 'helicopter drop' is similar to our exercise for taxes: the government makes a promise that it will be reckless, transferring money to people and (at least implicitly) promising to not rise taxes to retire this money - or to not issue bonds to be paid with greater taxes on the future. So by definition, for such a policy to 'stimulate' at all it must involve a non-Ricardian fiscal policy as in the FTPL.<sup>4</sup> We do not argue that such a policy would lead necessarily to a recession like here, as there are other countervailing forces we do not consider in our model. But these results nevertheless stress drawbacks of such a policy that are more prone to happen just in economies experiencing low inflation, where the presence of long-term nominal contracts tend to be more common, banks tend to be more exposed to public liabilities as a result of QE and where protection to inflation is less needed.

<sup>4</sup>There are at least other three possibilities worth noting by which such a policy could not be innocuous. First, if consumers had a finite horizon they would react if the taxes rise happened beyond their expected horizon (see Richter (2015)). Second, if consumers were rule-of-thumb or liquidity constrained, such a policy would be a relief to their income and could stimulate consumption (see Gali et al. (2004) for a treatment on rule-of-thumb consumers). And third, if people had less than rational expectations about fiscal policy. A very interesting work on this third possibility is Eusepi e Preston (2013), in which non-Ricardian behavior arises as a result of learning on expectation formation.

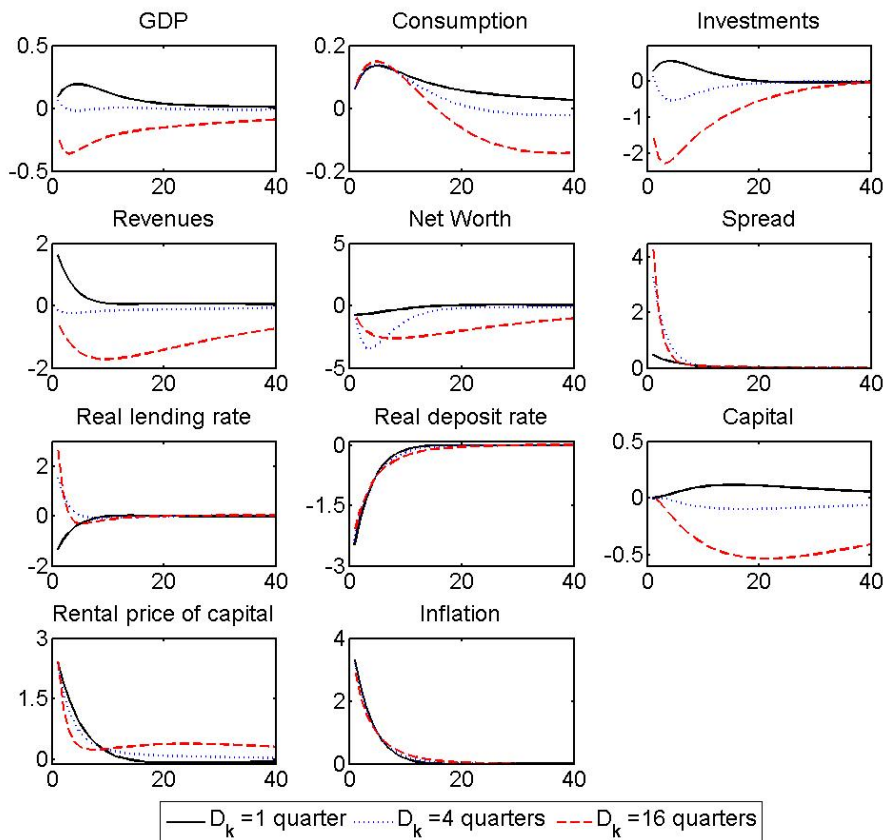
### 5.3

#### The credit deflation channel

We isolate the effect of inflation on banks' loan revenues (the 'credit-deflation' channel) by removing bonds from banks' balance-sheets, setting the policy response to  $\phi_\pi = 0$  and assuming one-period bonds only ( $\rho = 0$ ). This way, asset prices does not vary and the financing is done only through surprise inflation. The debt-to-GDP ratio is irrelevant here since asset prices are fixed and banks hold no bonds, so we keep the value of 80% of GDP.

Figure 5.3 shows the impulse-response to a fall of taxes of one percent of GDP for different scenarios of loans' average duration. These figures makes clear the role played by the long-term structure of banks' loans in driving our results: if loans are relatively short-term, the aggregate demand effect of the fall on taxes more than offset the negative effect on banks' net-worth, generating a rise on output, consumption and investments. With an average maturity of four years, the negative effects are predominant and output falls by the same reasoning as stressed on the previous section.

Figure 5.3: Impulse response to a tax shock with different loan durations and no bonds



The government can try to reduce inflation surprises by raising policy

rates. Figure 5.4 shows the exercise of tax reduction keeping loans' duration at four years and varying bonds' maturity. We suppose the government devotes some effort to postpone inflation, which we capture by letting  $\phi_\pi = 0.9$ . In the thick line government bonds are one-period only. The dotted and dashed lines denote scenarios of bonds' average maturity of 4 and 10 years, respectively. From the figure, we see that interest-rate rises when debt is one-period is not very effective: investment and output falls greatly and inflation rises sharply. Intuition goes as follows. When debt is one-period, bond prices are predetermined and inflation comes as a surprise. Due to fiscal price-level determination, rising rates pressures inflation, not the opposite. With more surprise inflation and higher rates, capital finance and investments fall, triggering a recession and a rise on firms' marginal costs. The figure is different if there are long-term bonds on the market, as the dashed and dotted lines show. In this case, the value of bonds fall and the government trades current for future inflation. As a greater portion of inflation is expected, banks adjust loans more effectively, reducing the negative effects of the surprise inflation. As a result, the fall on net-worth, investments and output is greatly mitigated relative to the inflation surprise case. However, a key hypothesis in this analysis is that banks hold no bonds or more broadly, hold no assets exposed to surprise revaluation. When they do, the fall on asset prices following the surplus shock negatively affect banks' net-worth and hence, capital financing. We explore this channel in the next section.

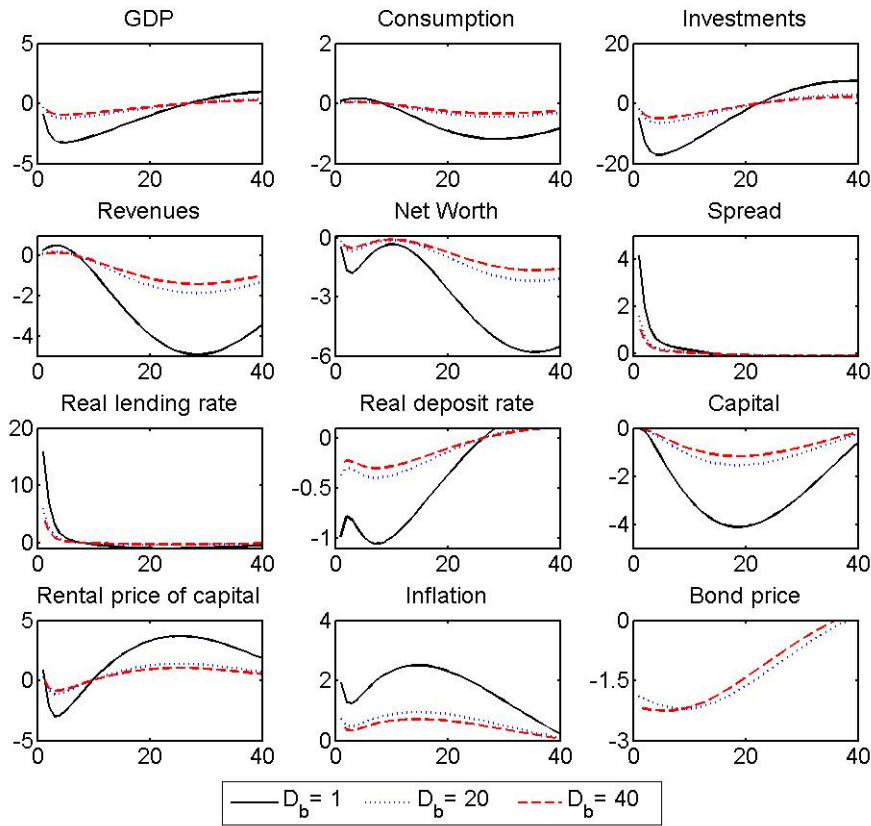
## 5.4

### The asset price channel

In this section, we assume only short-term loans  $\alpha_k = 0$  and an interest-rate response of  $\phi_\pi = 0.9$  - again, we think this as a scenario in which the government is devoting some effort to postpone inflation. As before, we initially fix the debt-to-GDP ratio at 80%. For our model, this implies that bonds composes 28% of banks' assets. We will discuss and test other values for this parameter shortly.

Figure 5.5 shows the impulse response for a fall on taxes of one percent GDP for different bond maturities. With bonds of relative short maturity, the aggregate demand effect predominates and output rises. There is a fall on net worth and rise in spreads, but insufficient to generate a rise in real lending rates and a fall on capital financing great enough to compensate the demand effect. Inflation is higher due to the rise in output and the small response of bond prices. The figure changes when government bonds becomes more long-term: with five and ten year average maturity, the fall on net-worth following the

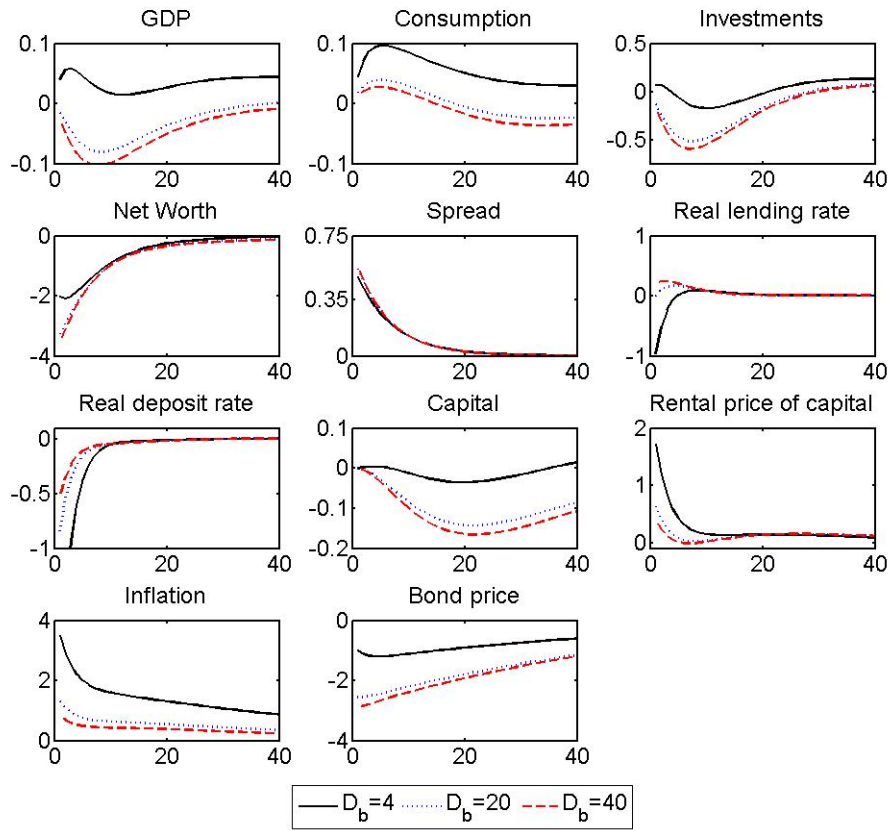
Figure 5.4: Impulse response to a tax shock, inflation postponement, different bond maturities



surprise revaluation of asset prices reduces capital financing and investments. Spreads and real lending rates rise, consistent with the fall on investments. Inflation is lower and smoother, but comes with recession.

These results are sensitive to the debt-to-GDP ratio, or more precisely, to the fraction of bonds composing intermediaries' assets. The average share of bank holdings of government bonds vary among countries. Gennaioli et al. (2014) find they compose on average 13% of banks' assets for emerging economies and are typically greater for countries who went through banking crises. Cao (2014) estimate such holdings to be approximately 10% for US banks at the end of 2008. In Japan, such holdings reached more than 20% of assets on 2013, but are smoothly being reduced as a result of efforts from Japanese authorities. In Figure 5.6 and 5.7 we do the tax reduction exercise for varying debt-to-GDP ratio scenarios, chosen to imply different fractions of bonds on banks' assets. We also consider average maturity of government bonds of five and ten years. As these figures show, the extent of the damage of bond-price revaluation to the financial sector depends on its share of banks' assets and the sensitivity of prices to interest-rates. With a low fraction of assets, the damage in banks' balance-sheets is not sufficient to compensate

Figure 5.5: Impulse response to a tax shock, different bond maturities, no long-term loans



the aggregate demand effect coming from the fall on taxes, even with a high-duration portfolio of bonds. As the share and average maturity grows, the negative effects coming from the financial sector starts predominating, even for reasonable share of assets.

Two points are worth noting about this analysis. First, in our model we assumed that banks hold no other assets beyond bonds. If assets with payment structure similar to those of government bonds were introduced (corporate bonds, for example), similar effects on banks' net-worth would arise. For this reason, in the full model we stuck with the higher calibration for the bonds share (28%) implied by the debt-to-GDP ratio of 80%. Second, in our model either banks or households hold government bonds. It is possible to make the two of them holding public debt at the same time, but we note that, depending on the way household holdings are introduced, the sensitivity of bonds prices tend to be higher, as arbitrage relations tends to be stronger. So, for the same maturity and share of assets, bond prices will tend to have more effect on banks' net-worth.<sup>5</sup>

<sup>5</sup>A similar reasoning applies to the degree of agency friction on bonds  $\iota$ .

Figure 5.6: Impulse response to a tax shock different debt-to-GDP, no long-term loans, average bond duration of 5 years

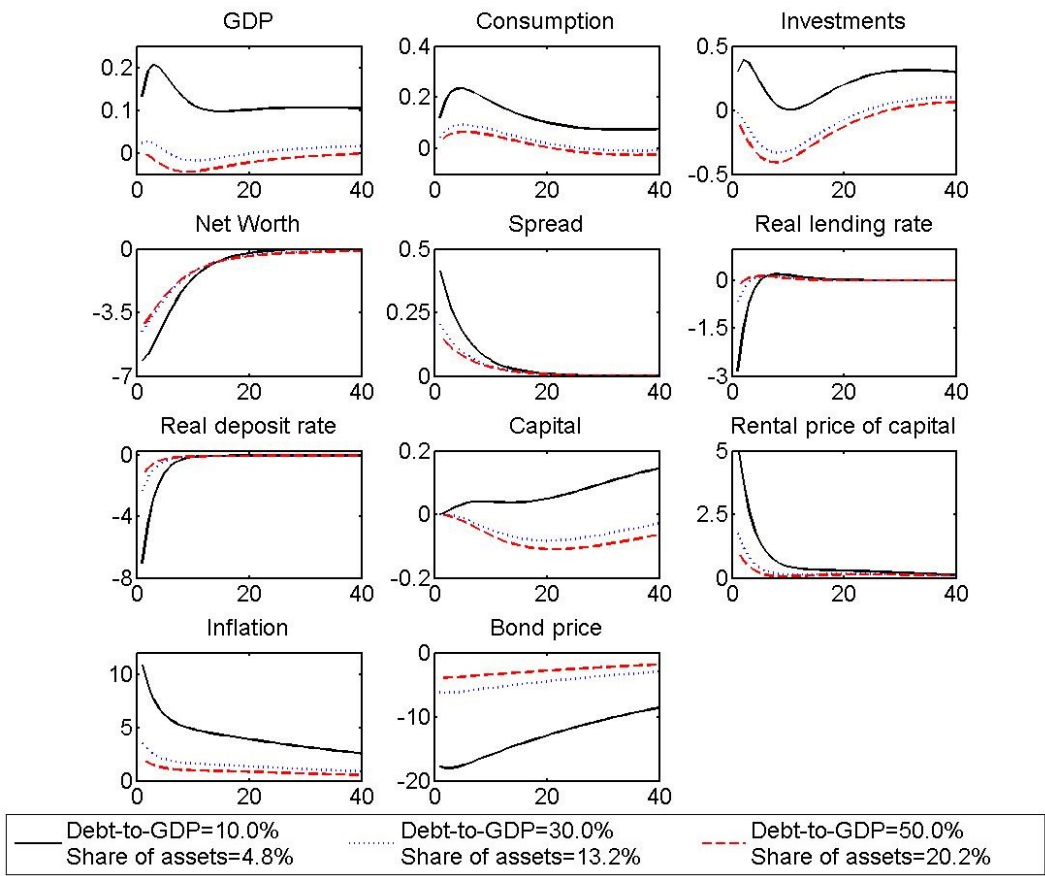
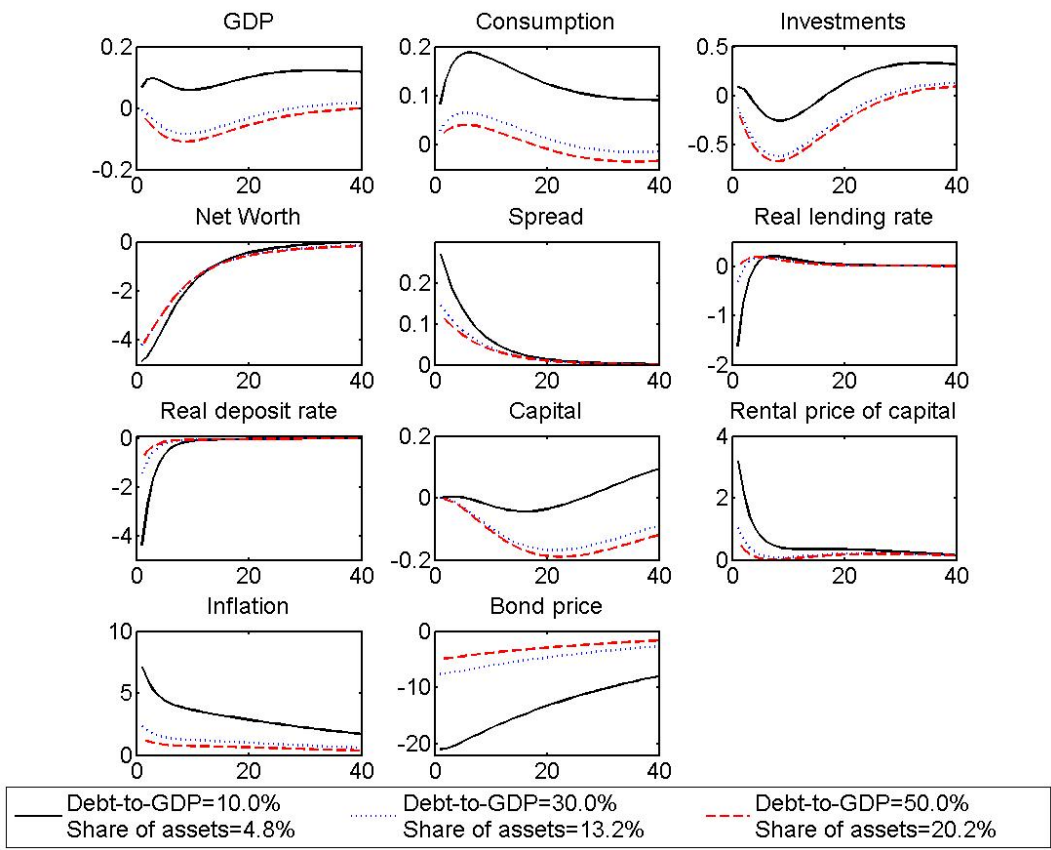


Figure 5.7: Impulse response to a tax shock different debt-to-GDP, no long-term loans, average bond duration of 10 years



## 6

### Conclusion

We based our analysis on the Fiscal Theory of Price Level to study fiscal-induced inflationary processes. We showed that surprise inflation engendered by a deficit shock damages financial intermediation when intermediaries have a maturity mismatch on assets and liabilities. This reduce capital finance and raise spreads, leading the economy to stagflation. The results help explaining experiences of high-debt, high-inflation environments who suffered from banking crisis and also exposes potential drawbacks of fiscal inflationary polices proposed to low-inflation countries as a way of stimulus.



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## A

### Capital producers' optimization problem

Capital producers' objective is to choose  $v_t$ ,  $I_t$ ,  $K_t$  and  $\tilde{K}_t$  to maximize expected flow of profits

$$\sum_{j=0}^{\infty} \frac{\lambda_{t+1}}{\lambda_t} \beta^j [v_{t+j} - (1 - \delta)p_t^k K_{t-1+j} - I_{t+j}]$$

subject to

$$\begin{aligned} v_t &= (1 - \theta_k)p_t^k \tilde{K}_t + \theta_k \frac{v_{t-1}}{\pi_t} \\ K_t &= (1 - \delta)K_{t-1} + I_t - S(I_t/I_{t-1})I_t \\ K_t &= (1 - \theta_k)\tilde{K}_t + \theta_k K_{t-1} \end{aligned}$$

for each  $t$ . Associating multipliers  $u_{1,t}, q_t, u_{2,t}$  for each restriction, first order conditions are given by:

$$\begin{aligned} \partial v_t : 1 &= u_{1t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \theta_k u_{1t+1} \\ \partial I_t : 1 &= q_t \left( 1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \\ \partial K_t : q_t - u_{2t} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (q_{t+1}(1 - \delta) - \theta_k u_{2t+1} - (1 - \delta)p_{t+1}^k) \\ \partial \tilde{K}_t : u_{1t} p_t^k &= u_{2t} \end{aligned}$$

Here,  $q_t$  is the Tobin's  $q$ , representing the marginal value of changing the stock of capital to be used at  $t + 1$ . It differs from the price of installed capital  $p_t^k$ , because of infrequent capital adjustments, but both coincide at steady-state.

## **B**

### **Model summary**

Equilibrium in the model can be computed from the following set of equations:



**Households**

$$(C_t - hC_{t-1})^{-1} = \lambda_t + h\beta(E_t C_{t+1} - C_t)^{-1} \quad (\text{B.0.1})$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t^d}{\pi_{t+1}} \quad (\text{B.0.2})$$

$$H_t^\psi = w_t \lambda_t \quad (\text{B.0.3})$$

**Wholesale firms**

$$r_t^k = p_t^w \alpha Y_t / K_{t-1} \quad (\text{B.0.4})$$

$$w_t = p_t^w (1 - \alpha) Y_t / H_t \quad (\text{B.0.5})$$

$$Y_t = H_t^{1-\alpha} K_{t-1}^\alpha \quad (\text{B.0.6})$$

**Entrepreneurs**

$$z_{1t} = E_t \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1}^k + (1 - \delta)p_{t+1}^k + \beta\theta_k z_{1t+1}) \quad (\text{B.0.7})$$

$$z_{2t} = E_t \frac{\lambda_{t+1}}{\lambda_t} (1 + \beta\theta_k z_{2t+1}) / \pi_{t+1} \quad (\text{B.0.8})$$

$$z_{1t} = p_t^k R_t^l z_{2t} \quad (\text{B.0.9})$$

$$K_t = (1 - \theta_k) \tilde{K}_t + \theta_k K_{t-1} \quad (\text{B.0.10})$$

**Capital producers**

$$1 = u_{1t} - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \theta_k u_{1t+1} \quad (\text{B.0.11})$$

$$1 = q_t \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \quad (\text{B.0.12})$$

$$q_t - u_{2t} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (q_{t+1}(1 - \delta) - \theta_k u_{2t+1} - (1 - \delta)p_{t+1}^k) \quad (\text{B.0.13})$$

$$u_{1t} p_t^k = u_{2t} \quad (\text{B.0.14})$$

$$K_t = (1 - \delta) K_{t-1} + I_t - S \left( \frac{I_t}{I_{t-1}} \right) I_t \quad (\text{B.0.15})$$

**Financial intermediaries**

$$\nu_t^k = E_t \Omega_{t+1} \frac{rev_t / l_t}{\pi_{t+1}} \quad (\text{B.0.16})$$

$$\nu_t^b = E_t \Omega_{t+1} \frac{R_{t+1}^b}{\pi_{t+1}} \quad (\text{B.0.17})$$

$$\mu_t = E_t \Omega_{t+1} \frac{R_t^d}{\pi_{t+1}} \quad (\text{B.0.18})$$

$$\Omega_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} \frac{\beta}{(1 - \chi)} [(1 - \theta_b) + \theta_b [(lev_{t+1}^k + ulev_{t+1}^b)(\nu_{t+1}^k - \mu_{t+1}) + \mu_{t+1}]] \quad (\text{B.0.19})$$

$$\nu_t^b - \mu_t = \varsigma \iota (\nu_t^k - \mu_t) \quad (\text{B.0.20})$$

$$lev_t^k + ulev_t^b = \frac{\mu_t}{\gamma - (\nu_t^k - \mu_t)} \quad (\text{B.0.21})$$

$$lev_t^k = \frac{l_t}{n_t} \quad (\text{B.0.22})$$

$$lev_t^b = \frac{Q_t b_t}{n_t} \quad (\text{B.0.23})$$

$$n_t = \frac{1 - \chi}{\pi_t} [rev_{t-1} + \varsigma R_t^b Q_{t-1} b_{t-1} - R_{t-1}^d d_{t-1}] \quad (\text{B.0.24})$$

$$rev_t = (1 - \theta_k) R_t^l p_t^k \tilde{K}_t + \theta_k rev_{t-1} / \pi_t \quad (\text{B.0.25})$$

$$l_t = (1 - \theta_k) p_t^k \tilde{K}_t + \theta_k l_{t-1} / \pi_t \quad (\text{B.0.26})$$

$$l_t + Q_t b_t = d_t + n_t \quad (\text{B.0.27})$$

### Retailers

$$\tilde{p}_t = \frac{\eta}{\eta - 1} \frac{x_{1t}}{x_{2t}} \quad (\text{B.0.28})$$

$$x_{1t} = \lambda_t y_t p_t^w + \beta \theta_p E_t(\pi_{t+1})^\eta x_{1t+1} \quad (\text{B.0.29})$$

$$x_{2t} = \lambda_t y_t + \beta \theta_p E_t(\pi_{t+1})^{\eta-1} x_{2t+1} \quad (\text{B.0.30})$$

$$\pi_t^{1-\eta} = (1 - \theta_p)(\tilde{p}_t \pi_t)^{1-\eta} + \theta_p \quad (\text{B.0.31})$$

### Government and Market clearing

$$R_t^b = \frac{1 + \rho Q_t}{Q_{t-1}} \quad (\text{B.0.32})$$

$$G_t + Q_t b_t = \tau_t + \frac{1 + \rho Q_t}{\pi_t} b_{t-1} \quad (\text{B.0.33})$$

$$\log(R_t^d) = \log(1/\beta) + \rho_r \log(R_{t-1}) + (1 - \rho_r)(\phi_\pi \log(\pi_t)) \quad (\text{B.0.34})$$

$$\log(\tau_t) = \log(\tau) + \phi_\tau \log(Q_{t-1} b_{t-1}) + \epsilon_t \quad (\text{B.0.35})$$

$$Y_t = (C_t + G_t + I_t)_t \quad (\text{B.0.36})$$

$$_t = (1 - \theta_p) \tilde{p}_t^{-\eta} + \theta_p \pi_t^\eta {}_{t-1} \quad (\text{B.0.37})$$

### Exogenous process

$$\log(G_t) = (1 - \rho_g) \log(G) + \rho_g \log(G_{t-1}) + \varepsilon_t^G \quad (\text{B.0.38})$$

$$\epsilon_t = \rho_\tau \epsilon_{t-1} + \varepsilon_t^\tau \quad (\text{B.0.39})$$