



Roberta de Queiroz Lima

**Modeling and simulation in nonlinear
stochastic dynamics of coupled systems and
impacts**

TESE DE DOUTORADO

Thesis presented to the Programa de Pós-Graduação em Engenharia Mecânica of the Departamento de Engenharia Mecânica, PUC-Rio as partial fulfillment of the requirements for the degree of Doutor em Engenharia Mecânica.

Adviser: Prof. Rubens Sampaio
Co-Adviser: Prof. Christian Soize

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Abstract

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Modeling and simulation in nonlinear stochastic dynamics of coupled systems and impacts. Rio de Janeiro, 2015. 89p.
PhD Thesis — Departamento de Engenharia Mecânica, Pontifícia Universidade Católica do Rio de Janeiro.

In this Thesis, the robust design with an uncertain model of a vibro-impact electromechanical system is done. The electromechanical system is composed of a cart, whose motion is excited by a DC motor (motor with continuous current), and an embarked hammer into this cart. The hammer is connected to the cart by a nonlinear spring component and by a linear damper, so that a relative motion exists between them. A linear flexible barrier, placed outside of the cart, constrains the hammer movements. Due to the relative movement between the hammer and the barrier, impacts can occur between these two elements. The developed model of the system takes into account the influence of the DC motor in the dynamic behavior of the system. Some system parameters are uncertain, such as the stiffness and the damping coefficients of the flexible barrier. The objective of the Thesis is to perform an optimization of this electromechanical system with respect to design parameters in order to maximize the impact power under the constraint that the electric power consumed by the DC motor is lower than a maximum value. To choose the design parameters in the optimization problem, a sensitivity analysis was performed in order to define the most sensitive system parameters. The optimization is formulated in the framework of robust design due to the presence of uncertainties in the model. The probability distributions of random variables are constructed using the Maximum Entropy Principle and statistics of the stochastic response of the system are computed using the Monte Carlo method. The set of nonlinear equations are presented, and an adapted time domain solver is developed. The stochastic nonlinear constrained design optimization problem is solved for different levels of uncertainties, and also for the deterministic case. The results are different and this shows the importance of the stochastic modeling.

Keywords

Coupled systems; Embarked system; Vibro-impact; Stochastic analysis; Robust design optimization; Nonlinear dynamics.

Résumé

Lima, Roberta de Queiroz; Sampaio, Rubens and ; Soize, Christian.
Modélisation et simulation en dynamique stochastique non linéaire des systèmes couplés avec phénomènes d'impact.
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Dans cette Thèse, nous étudions l'optimisation robuste avec un modèle incertain d'un système électromécanique avec vibro-impact. Le système électromécanique est constitué d'un chariot dont le mouvement est généré par un moteur à courant continu, et d'un marteau embarqué dans ce chariot. Le marteau est relié au chariot par un ressort non linéaire et par un amortisseur linéaire, de façon qu'un mouvement relatif existe entre eux. Une barrière flexible linéaire, placée à l'extérieur du chariot limite les mouvements du marteau. En raison du mouvement relatif entre le marteau et la barrière, des impacts peuvent se produire entre ces deux éléments. Le modèle du système développé prend en compte l'influence du moteur à courant continu dans le comportement dynamique du système. Certains paramètres du système sont incertains, tels que les coefficients de rigidité et d'amortissement de la barrière flexible. L'objectif de la Thèse est de réaliser une optimisation de ce système électromécanique en jouant sur les paramètres de conception. Le but est de maximiser la puissance d'impact sous la contrainte que la puissance électrique consommée par le moteur à courant continu soit inférieure à une valeur maximale. Pour choisir les paramètres de conception dans le problème d'optimisation, une analyse de sensibilité a été réalisée afin de définir les paramètres du système les plus sensibles. L'optimisation est formulée dans le cadre de la conception robuste en raison de la présence d'incertitudes dans le modèle. Les lois de probabilités des variables aléatoires du problème sont construites en utilisant le Principe du Maximum d'Entropie. Les statistiques de la réponse stochastique du système sont calculées en utilisant la méthode de Monte Carlo. L'ensemble des équations non linéaires est présenté, et un solveur temporel adapté est développé. Le problème d'optimisation non linéaire stochastique est résolu pour différents niveaux d'incertitudes, ainsi que pour le cas déterministe. Les résultats sont différents, ce qui montre l'importance de la modélisation stochastique.

Mots-clés

Systèmes couplés; Système embarqué; Vibro-impact; Analyse stochastique; Optimisation robuste; Dynamique non-linéaire.

Resumo

Lima, Roberta de Queiroz; Sampaio, Rubens and ; Soize, Christian.
Modelagem e simulação em dinâmica estocástica não-linear de sistemas acoplados e impactos. Rio de Janeiro, 2015. 89p.
Tese de Doutorado — Departamento de Engenharia Mecânica,
Pontifícia Universidade Católica do Rio de Janeiro.

Nesta Tese, o design robusto, com um modelo incerto de um sistema de vibro-impacto eletromecânico é feito. O sistema é composto de um carrinho, cujo movimento é acionado por um motor de corrente contínua e um martelo embarcado neste carrinho. O martelo é ligado ao carrinho por um mola não linear e por um amortecedor linear, de modo que existe um movimento relativo entre eles. Uma barreira linear flexível, colocada fora do carrinho, restringe aos movimentos do martelo. Devido ao movimento relativo entre o martelo e a barreira, impactos podem ocorrer entre estes dois elementos. O modelo matemático desenvolvido para sistema leva em conta a influência do motor no comportamento dinâmico do sistema. Alguns parâmetros do sistema são incertos, tais como a rigidez e os coeficientes de amortecimento da barreira flexível. O objectivo da Tese é realizar uma otimização deste sistema electromecânico com respeito a parâmetros de projeto, a fim de maximizar a potência de impacto sob a restrição de que a potência elétrica consumida pelo motor seja menor do que um valor máximo. Para escolher os parâmetros de projeto no problema de otimização, uma análise de sensibilidade foi realizada a fim de definir os parâmetros mais sensíveis do sistema. O problema de otimização é formulado no âmbito de otimização robusta, devido à presença de incertezas no modelo. As distribuições de probabilidades das variáveis aleatórias são construídas através do Princípio da Máxima Entropia e estatísticas da resposta estocástica do sistema são calculadas pelo método de Monte Carlo. O conjunto de equações não-lineares é apresentado, e um integrador temporal adaptado é desenvolvido. O problema de otimização não-linear estocástico com restrição é resolvido para diferentes níveis de incertezas e também para o caso determinístico. Os resultados são diferentes e isto mostra a importância da modelagem estocástica.

Palavras-chave

Sistemas acoplados; Sistema embarcado; Vibro-impacto; Análise estocástica; Otimização robusta; Dinâmica não linear.

Contents

1	Introduction	14
1.1	Motivation of the Thesis	14
1.2	Percussive systems	16
1.3	Hierarchical electromechanical systems analyzed	17
1.4	Organization of the Thesis	19
2	Motor-cart system: a parametric excited nonlinear system due to electromechanical coupling	20
2.1	Dynamics of the motor-cart system	20
	Electrical system: DC motor	20
	Cart-motor system: a master-slave relation	22
2.2	Dimensionless cart-motor system	23
2.3	Numerical simulations of the dynamics of the motor-cart system	24
2.4	Asymptotically stable periodic orbit	29
2.5	Summary of the Chapter	32
3	Motor-cart-pendulum system: introduction of a mechanical energy reservoir	33
3.1	Dynamics of the motor-cart-pendulum system	33
3.2	Dimensionless cart-motor-pendulum system	34
3.3	Numerical simulations of the dynamics of the motor-cart-pendulum system	36
3.4	Pumping Leads To Revolution	39
3.5	Summary of the Chapter	43
4	Electromechanical system with internal impacts and uncertainties	44
4.1	Dynamics of the motor-cart-pendulum-barrier system	45
4.2	Dimensionless motor-cart-pendulum-barrier system	46
4.3	Impact energy	48
4.4	Numerical simulations of the dynamics of the coupled system	48
	No coupling between the motor and the mechanical system	48
	Coupled system	49
4.5	Probabilistic model	51
4.6	Numerical simulations of the stochastic vibro-impact electromechanical system	52
4.7	Summary of the Chapter	56
5	Robust design optimization with an uncertain model of a nonlinear percussive electromechanical system	58
5.1	Dynamics of the vibro-impact electromechanical system	58

5.2	Dimensionless vibro-impact electromechanical system	61
5.3	Measure of the system performance	63
5.4	Sensitivity analysis and choice of the design parameters	63
5.5	Construction of the probability model	68
5.6	Robust design optimization problem	69
5.7	Results of the robust optimization problem	69
5.8	Summary of the Chapter	71
6	Summary, future works and publications	75
6.1	Future works	76
6.2	Publications	76
	Bibliography	81

List of Figures

1.1	First system: cart-motor system.	17
1.2	Second system: cart-motor-pendulum system.	18
1.3	Third system: motor-cart-pendulum-barrier system.	18
1.4	Fourth system: motor-cart-hammer coupled system.	19
2.1	Electrical DC motor.	21
2.2	Coupled cart-motor system.	22
2.3	Motor-cart system with $\Delta = 0.001$ m: (a) angular speed of the motor shaft over time and (b) Fast Fourier Transform of the cart displacement.	25
2.4	Motor-cart system with $\Delta = 0.01$ m: (a) angular speed of the motor shaft over time and (b) Fast Fourier Transform of the cart displacement.	26
2.5	Motor-cart system: Fast Fourier Transform of the current (a) when $\Delta = 0.001$ m and (b) when $\Delta = 0.01$ m.	26
2.6	Motor-cart system with $\Delta = 0.01$ m: (a) cart displacement and (b) motor current over time.	27
2.7	Motor-cart system with $\Delta = 0.01$ m: (a) horizontal force f and (b) torque τ during one cycle of the cart movement.	27
2.8	Motor-cart system with $\Delta = 0.01$ m: (a) current variation during one cart movement cycle and (b) torque variation as function of the current.	28
2.9	Motor-cart system with $\Delta = 0.01$ m: (a) angular velocity of the motor shaft during one cart movement cycle and (b) current variation as function of the angular velocity of the motor shaft.	28
2.10	Motor-cart system $\Delta = 0.01$ m: (a) torque variation as function of the horizontal force f and (b) horizontal force variation as function of the angular velocity of the motor shaft.	29
2.11	Motor-cart system: period of one cart movement cycle (a) as function of Δ with $m = 5.0$ kg and (b) as function of m with $\Delta = 0.005$ m.	29
2.12	Comparison between numerical findings and the asymptotic approximation.	31
2.13	Coupled cart-motor-spring-damper system.	32
3.1	Cart-motor-pendulum system.	33
3.2	Motor-cart-pendulum system with $\Delta = 0.001$ m: (a) angular velocity of the motor shaft and (b) current over time.	37
3.3	Motor-cart-pendulum system with $\Delta = 0.001$ m: (a) pendulum displacement and (b) cart displacement over time.	37
3.4	Motor-cart-pendulum system with $\Delta = 0.001$ m: Fast Fourier Transform of (a) cart and pendulum displacements and (b) of current.	38
3.5	Motor-cart-pendulum system with $\Delta = 0.01$ m: (a) angular velocity of the motor shaft and (b) current over time.	38

3.6	Motor-cart-pendulum system with $\Delta = 0.01$ m: (a) pendulum and (b) cart displacement over time.	39
3.7	Motor-cart-pendulum system with $\Delta = 0.01$ m: Fast Fourier Transform of (a) pendulum and (b) cart displacements.	39
3.8	Motor-cart-pendulum system with $\Delta = 0.01$ m: Fast Fourier Transform of (a) current and (b) angular speed of the motor shaft over time.	40
3.9	Motor-cart-pendulum system with $\Delta = 0.01$ m: (a) angular velocity of the motor shaft and (b) current over time.	40
3.10	Motor-cart-pendulum system with $\Delta = 0.01$ m: (a) pendulum and (b) cart displacement over time.	41
3.11	Motor-cart-pendulum system with $\Delta = 0.01$ m: portrait graphs of (a) $\ddot{\alpha}$ graph as function of $\dot{\alpha}$ and (b) $\dot{\alpha}$ graph as function of x .	41
3.12	Motor-cart-pendulum system with $\Delta = 0.01$ m: portrait graphs of (a) $\dot{\theta}$ graph as function of $\dot{\alpha}$ and (b) θ as function of $\dot{\alpha}$.	42
3.13	Motor-cart-pendulum system with $\Delta = 0.01$ m: portrait graphs of (a) tangent $\dot{\theta}$ graph as function of $\dot{\alpha}$ and (b) τ as function of $\dot{\alpha}$.	42
4.1	Coupled motor-cart-pendulum-barrier system.	44
4.2	No coupling ($\Delta = 0$ m): normalized average of the maximum impact energy as function of the parameter gap/ l_p for different values of k_i N/m.	49
4.3	Coupled system ($\Delta > 0$): normalized average of the impact energy as function of the parameter gap/ l_p for different values of Δ (units in meters).	51
4.4	Coupled system ($\Delta > 0$): normalized average of the impact energy as function of the parameter gap/ l_p for different values of k_i N/m with $\Delta = 10^{-3}$ m.	52
4.5	Mean and 90% confidence interval of Λ/λ^{ref} as function of gap/ l_p with $\delta = 0.15$ for (a) $E\{K_i\} = 10^4$ N/m and (b) $E\{K_i\} = 10^5$ N/m .	53
4.6	(a) Mean and 90% confidence interval of Λ as function of gap/ l_p with $\delta = 0.15$ and $E\{K_i\} = 10^6$ N/m and (b) normalized histogram of Λ/λ^{ref} for gap/ $l_p = 0.63$ m, $E\{K_i\} = 10^6$ N/m and $\delta = 0.15$.	54
4.7	Mean and 90% confidence interval of Λ/λ^{ref} as function of gap/ l_p with $E\{K_i\} = 10^4$ N/m for (a) $\delta = 0.25$ and (b) $\delta = 0.35$.	54
4.8	Mean and 90% confidence interval of Λ/λ^{ref} as function of gap/ l_p with $E\{K_i\} = 10^5$ N/m for (a) $\delta = 0.25$ and (b) $\delta = 0.35$.	55
4.9	Mean and 90% confidence interval of Λ/λ^{ref} as function of gap/ l_p with $E\{K_i\} = 10^6$ N/m for (a) $\delta = 0.25$ and (b) $\delta = 0.35$.	55
5.1	Motor-cart-hammer coupled system. The nonlinear component spring is drawn as a linear spring with constant k_{h1} and a nonlinear cubic spring with constant k_{h3} .	58
5.2	Parallelization of the simulations in the sensitivity analysis.	65

- 5.3 For the optimal values $(m_c/m_h)^*$ and Δ^* : (a) graph of π_{imp} as a function of g and k_{h1}/m_h (varying in all its range of values), (b) graph of π_{imp} as a function of g and k_{h1}/m_h (varying in $[0.06, 0.02]$ and $[1\ 250, 1\ 953]$ respectively). 66
- 5.4 (a) Graph of π_{imp} as a function of m_c/m_h with $(k_{h1}/m_h)^*$, g^* , and Δ^* . (b) Graph of π_{imp} as a function of k_{h1}/m_h with $(m_c/m_h)^*$, g^* , and Δ^* . 66
- 5.5 (a) Graph of π_{imp} as a function of g with $(m_c/m_h)^*$, $(k_{h1}/m_h)^*$, and Δ^* . (b) Graph of π_{imp} as a function of Δ with $(m_c/m_h)^*$, $(k_{h1}/m_h)^*$, and g^* . 66
- 5.6 (a) Graph of π_{elec} as a function of m_c/m_h with $(k_{h1}/m_h)^*$, g^* , and Δ^* . (b) Graph of π_{elec} as a function of k_{h1}/m_h with $(m_c/m_h)^*$, g^* , and Δ^* . 67
- 5.7 (a) Graph of π_{elec} as a function of g with fix $(m_c/m_h)^*$, $(k_{h1}/m_h)^*$, and Δ^* . (b) Graph of π_{elec} as a function of Δ with fix $(m_c/m_h)^*$, $(k_{h1}/m_h)^*$, and g^* . 67
- 5.8 Parallelization of the simulations performed to solve the robust optimization problem. 70
- 5.9 (a) Cost function as function of the design parameters for the deterministic case. (b) Cost function as function of the design parameters for the case in which $\delta_{K_i} = \delta_{C_i} = 0.1$ and $\delta_{K_{h1}} = 0$. 71
- 5.10 (a) Cost function as function of the design parameters for the case in which $\delta_{K_i} = \delta_{C_i} = \delta_{K_{h1}} = 0.1$. (b) Cost function as function of the design parameters for the case in which $\delta_{K_i} = \delta_{C_i} = 0.1$ and $\delta_{K_{h1}} = 0.4$. 72
- 5.11 (a) Cost function as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$. (b) Cost function as function of \underline{K}_{h1}/m_h with g^{opt} . In both graphs, the $E\{\Pi_{\text{imp}}(\mathbf{p}_{\text{des}}^{\text{opt}})\}$ is highlighted for each level of uncertainties with markers. 72
- 5.12 (a) Mean value of the time average of electric power as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$. (b) Mean value of the time average of electric power as function of \underline{K}_{h1}/m_h with g^{opt} . In both graphs, the $E\{\Pi_{\text{elec}}(\mathbf{p}_{\text{des}}^{\text{opt}})\}$ is highlighted for each level of uncertainties with markers. 73
- 5.13 (a) Coefficient variation of Π_{imp} as function of g with $(\underline{K}_{h1}/m_h)^{\text{opt}}$. (b) Coefficient variation of Π_{imp} as function of \underline{K}_{h1}/m_h with g^{opt} . In both graphs, the $\delta_{\Pi_{\text{imp}}}(\mathbf{p}_{\text{des}}^{\text{opt}})$ is highlighted for each level of uncertainties with markers. 73

List of Tables

2.1	Values of the motor parameters used in simulations.	25
5.1	Values of the system parameters used in simulations.	64