Referências Bibliográficas

[1] BURNSIDE, W.. **Theory of Groups of Finite Order**. Dover Publications, New York, 2nd edition, 1955.

[2] COLBOURN, C. J.; MAGLIVERAS, S. S. ; STINSON, D. R.. **Steiner Triple Systems of Order 19 with Nontrivial Automorphism Group**. Mathematics of Computation, 59:283–295, 1992.

[3] CONWAY, J. H.; CURTIS, R. T.; NORTON, S. P.; PARKER, R. A. ; WILSON, R. A.. **Atlas of finite groups**. Oxford University Press, New York, 1985.

[4] CONWAY, J. H.; SLOANE, N. J. A.. **Sphere packings, lattices and groups**. Springer-Verlag, New York, 1999.

[5] GRIESS JR, R. L.. **Twelve sporadic groups**. Springer-Verlag, Berlin, 1998.

[6] KASKI, P.; ÖSTERGÅRD, P. R. J.. **The Steiner Triple Systems of Order 19**. Mathematics of Computation, 73:2075–2092, 2004.

[7] MATHIEU, E.. **Memoire sur l'étude des fonctions de plusier quantités, sur la manière de les former et sur les substitutions qui les laissent invariables**. J. Math Pures Appl., 6:241–323, 1873.

[8] MATHIEU, E.. **Sur la fontion cinq fois transitive de 24 quantités**. J. Math Pures Appl., 18:25–46, 1873.

[9] WILSON, R. A.. **The Finite Simple Groups**. Springer-Verlag, Berlin, 2009.

A
Uma Conta Trabalhosa

Neste apêndice veremos como verificar se uma permutação de 24 pontos pertence à M_{24}. Para tanto, basta verificar se ela preserva o código de Golay. Como exemplo, faremos essa verificação para provar que a permutação α definida em 5.1.4 é um elemento de M_{24}.

Afirmação. A permutação α definida em (5.1.4) preserva o Código de Golay.

Demonstração. Considere a seguinte base do Código de Golay formada por octads:

$$\begin{aligned}\beta = \{&[0,0,0,1,0,1,0,0,0,1,0,0,0,0,0,1,0,1,0,0,1,1,1,0],\\ &[1,1,1,1,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0],\\ &[0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1],\\ &[0,0,0,0,0,0,1,1,0,0,1,1,0,0,0,0,0,1,1,0,0,1,0,1],\\ &[0,0,0,0,0,0,0,0,1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0],\\ &[1,1,0,0,1,1,0,0,1,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0],\\ &[1,1,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1,0,1,0,1,1,1,0],\\ &[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1],\\ &[1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0],\\ &[1,1,1,1,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0],\\ &[0,1,1,1,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0],\\ &[1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]\}\end{aligned}$$

Usando o programa Maple, verifica-se facilmente que β é de fato linearmente independente (crie uma matriz A cujas linhas são os vetores de β e calcule o posto de A com o comando *rank(A)*).

A ação de α em cada octad da base β é ainda uma octad. De fato,

1) 0001 0100 0100 0001 0100 1110

$$\begin{array}{|cc|cc|cc|} \hline & & & & & 1 \\ & 1 & 1 & & 1 & 1 \\ & & & & & 1 \\ 1 & & & 1 & & \\ \hline \overline{w} & 1 & 1 & \overline{w} & 1 & \overline{w} \end{array} \quad \xrightarrow{\alpha} \quad \begin{array}{|cc|cc|cc|} \hline & & & & 1 & 1 \\ & 1 & 1 & & 1 & \\ & & & & & \\ & 1 & 1 & & & 1 \\ \hline 0 & w & w & 0 & 1 & \overline{w} \end{array}$$

2) 1111 0000 1111 0000 0000 0000

$$\begin{array}{|cc|cc|cc|}\hline 1 & & 1 & & & \\ 1 & & 1 & & & \\ 1 & & 1 & & & \\ 1 & & 1 & & & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|}\hline 1 & & 1 & & & \\ 1 & & 1 & & & \\ & 1 & & 1 & & \\ & 1 & & 1 & & \\ \hline 1 & 1 & 1 & 1 & 0 & 0 \end{array}$$

3) 0000 0000 0011 0011 0011 0011

$$\begin{array}{|cc|cc|cc|}\hline & & & & & \\ & & & & & \\ & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|}\hline & & & & & \\ & & & & & \\ & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

4) 0000 0011 0011 0000 0110 0101

$$\begin{array}{|cc|cc|cc|}\hline & & & & & \\ & & & & 1 & 1 \\ & 1 & 1 & & 1 & \\ & 1 & 1 & & & 1 \\ \hline 0 & 1 & 1 & 0 & \overline{w} & w \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|}\hline & & & & 1 & 1 \\ & & & & & \\ 1 & & & 1 & & 1 \\ 1 & & & 1 & 1 & \\ \hline 1 & 0 & 0 & 1 & \overline{w} & w \end{array}$$

5) 0000 0000 1100 1100 1100 1100

$$\begin{array}{|cc|cc|cc|}\hline & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ & & & & & \\ & & & & & \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|}\hline & & 1 & 1 & 1 & 1 \\ & & 1 & 1 & 1 & 1 \\ & & & & & \\ & & & & & \\ \hline 0 & 0 & 1 & 1 & 1 & 1 \end{array}$$

6) 1100 1100 1100 1100 0000 0000

$$\begin{array}{|cc|cc|cc|}\hline 1 & 1 & 1 & 1 & & \\ 1 & 1 & 1 & 1 & & \\ & & & & & \\ & & & & & \\ \hline 1 & 1 & 1 & 1 & 0 & 0 \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|}\hline 1 & 1 & 1 & 1 & & \\ 1 & 1 & 1 & 1 & & \\ & & & & & \\ & & & & & \\ \hline 1 & 1 & 1 & 1 & 0 & 0 \end{array}$$

7) 1100 0000 1100 0000 0101 0110

$$\begin{array}{|cc|cc|cc|} \hline 1 & & 1 & & & \\ 1 & & 1 & & 1 & 1 \\ & & & & & 1 \\ & & & & 1 & \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{w} & \multicolumn{1}{c}{\overline{w}} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & & 1 & & 1 & 1 \\ 1 & & 1 & & & \\ & & & & 1 & \\ & & & & & 1 \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{w} & \multicolumn{1}{c}{\overline{w}} \end{array}$$

8) 1111 0000 0000 0000 0000 1111

$$\begin{array}{|cc|cc|cc|} \hline 1 & & & & & 1 \\ 1 & & & & & 1 \\ 1 & & & & & 1 \\ 1 & & & & & 1 \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & & & & 1 & \\ 1 & & & & 1 & \\ & 1 & & & & 1 \\ & 1 & & & & 1 \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} \end{array}$$

9) 1111 0000 0000 0000 1111 0000

$$\begin{array}{|cc|cc|cc|} \hline 1 & & & & 1 & \\ 1 & & & & 1 & \\ 1 & & & & 1 & \\ 1 & & & & 1 & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & & & & & 1 \\ 1 & & & & & 1 \\ & 1 & & & 1 & \\ & 1 & & & 1 & \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} \end{array}$$

10) 1111 0000 0000 1111 0000 0000

$$\begin{array}{|cc|cc|cc|} \hline 1 & & & 1 & & \\ 1 & & & 1 & & \\ 1 & & & 1 & & \\ 1 & & & 1 & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & & & 1 & & \\ 1 & & & 1 & & \\ & 1 & 1 & & & \\ & 1 & 1 & & & \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array}$$

11) 0111 1000 1000 1000 1000 1000

$$\begin{array}{|cc|cc|cc|} \hline & 1 & 1 & 1 & 1 & 1 \\ 1 & & & & & \\ 1 & & & & & \\ 1 & & & & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline & 1 & 1 & 1 & & \\ 1 & & & & 1 & 1 \\ & 1 & & & & \\ & 1 & & & & \\ \hline \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{1} & \multicolumn{1}{c}{1} \end{array}$$

12) 1111 1111 0000 0000 0000 0000

$$\begin{array}{|cc|cc|cc|} \hline 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array} \xrightarrow{\alpha} \begin{array}{|cc|cc|cc|} \hline 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ 1 & 1 & & & & \\ \hline \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} & \multicolumn{1}{c}{0} \end{array}$$

Donde α preserva o Código de Golay, isto é, $\alpha \in M_{24}$.

$\square$