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Introduction

Natural Deduction is a logical system designed by Gentzen in the early 30's on an attempt to create a deductive system more compatible with mathematical reasoning. Afterwards, Prawitz improved this system by formalizing the idea of normal form derivations to minimal, classical and intuitionistic logic and proved normalization for these systems.

Intuitionistic logic is a sublogic of the classical logic and it is the logical counterpart of constructivism. Its basic operators $(\wedge, \rightarrow, \vee, \exists, \forall)$ and constant $(\perp)$ are not interdefinable and form a complete set for any intuitionistic operator. Intuitionistic logic is also related to computable functions proposed by Curry and Schönfinkel under the name of combinatorial logic.

Category Theory can be found in areas such as Mathematics and Computer Science. A category is composed of objects (that can be sets, theories, programs...) and arrows (or morphisms) that represent relations between these objects. Objects are defined up to isomorphism but between the arrows the relation is of equality. If the equalities were also replaced by natural isomorphisms categorical structure would be much richer. With this in mind, categorists have developed what is known as 2-Category Theory. In a 2-category, besides the objects and the arrows, there are arrows between these arrows. A typical example of a 2-category has categories for objects, functors for arrows and natural transformations for arrows between arrows.

Natural Deduction for intuitionistic logic can be seen as a category whose objects are formulas and whose arrows are derivations. In this relationship, equivalent derivations are viewed as the same derivation, that is, reductions are not explicitly represented. The theory that relates Category to Proof Theory is usually known as Categorical Logic. If we expand such a notion to relate Proof Theory to 2-Category Theory we would use the arrows between arrows to represent reductions, enabling us to use different arrows to represent equivalent derivations.

In this dissertation we compare the 2-categorical with the 1-categorical view of Proof Theory. We also investigate how structural rules behave under these interpretations. As the name indicates it, structural rules work on the global structure of a derivation.

In chapter 2 we introduce the system we relate to Category Theory (whose formulation is strongly based on Prawitz's ((14) and (15))) and some properties needed to relate it with 2-Category Theory. We also present a short historical background on the Curry-Howard Isomorphism. This isomorphism relates Natural Deduction and typed λ-Calculus but the isomorphisms between typed λ-Calculus and Category Theory and between Category Theory and Natural Deduction for intuitionistic logic are sometimes also known by this name.

In chapter 3 we introduce Category Theory and show its relation to the Natural Deduction for intuitionistic logic. In chapter 4 we introduce 2-Category Theory and show its relation to Natural Deduction for intuitionistic logic and in the conclusion we discuss the results presented in the earlier chapters and point out further research.