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An Overview on Network Cost Allocation Methods

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Abstract

This work is devoted to study and discuss the main methods to solve the network cost allocation problem both for generators and demands. From the presented, compared and discussed methods, the first one is based on power injections, the second deals with proportional sharing factors, the third is based upon Equivalent Bilateral Exchanges, the fourth analyzes the power flow sensitivity in relation to the power injected, and the last one is based on Z_{bus} network matrix. All the methods are initially illustrated using a 4–bus system. In addition, the IEEE 24–bus RTS system is presented for further comparisons and analysis. Appropriate conclusions are finally drawn.

Key words: Network cost allocation, electricity market, transmission system.

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1 Introduction

The cost of the transmission network can be interpreted as the cost of maintenance, planning and operation of the infrastructure that involves the transmission system. It's the responsibility of the generators and demands, that are the users of the transmission system, to pay for the network usage of this system. For example, in Brazil, the net basic cost of the transmission network,called TUST-RB, whose elements have a voltage equal or above 230 kV, exceeded 3.1 billion dollars in the period of 2005-2006, according to [1].

One of the main challenges to allocate the cost of transmission is how to establish a criterion to split it between generators and demands. According to [2] the methods of network transmission cost allocation should, beyond ensuring the quality of the transmission service (voltage control, static and dynamic stability, etc.), satisfy a set of restrictions for its correct application:

- no cross-subsidies;
- transparency of the cost allocation procedure;
- simple regulatory method;
- adequate remuneration of present and future transmission investments;
- economic signaling for future dimensioning;
- continuity of existing network charges.

Different proposals for transmission network cost allocation have appeared in recent years. The pro rata method, presented in [3] and [4], allocates costs to generators and demands according to the sum of active power produced/consumed by each generator/demand.

Other methods, a bit more complex, distribute the costs based on the active power flow produced by generators and demands through the transmission lines. These methods use the proportional sharing principle, where the flows attributed to each generator and demand in "upstream" lines determine the power flows through "downstream" lines. These flows are associated with the origins and destinations, i.e., generators and demands. Examples of this method can be found in [2], [5], [6], [7].

The network usage method presented in [8] uses Equivalent Bilateral Exchanges ($EBEs$) to allocate costs to generators and demands. In order to create an EBE, each demand is attributed a generation fraction and, in the same way, a fraction of each demand is attributed to each generator. The attribution of costs by the network usage method occurs considering the impact, in terms of power flow, of each EBE in each transmission line, determined by the DC power flow solution.

The $\mathrm{Z_{bus}}$ method [9] presents a solution based on the Z_{bus} matrix and considers the current injection at each bus. The combination of these two elements (Z_{bus} matrix and current injections) determines a measure of sensitivity that indicates what is the individual contribution of each current injection to produce the power flow through of a transmission line.

The nodal method, used in many countries, allocates the network usage costs and provides a measure to determine this allocation based on the power flow sensitivity in each line due to the power injected at each bus. This method can be found in [1].

In the last years, several studies about network cost allocation considering cross-border exchanges in Europe and Asia have appeared. For example, in Europe, there is a need to find a robust and fair mechanism for Inter-Transmission System Operators (Inter-TSO) compensation in the European internal electricity market to replace the provisional European Transmission System Operators (ETSO) mechanism. One of the main challenges is that the system operator of each country does not have information about the electrical networks of other countries, making the application of any network cost allocation method difficult. This issues can be found in [10],[11],[12] and [13]. However, these studies are out of scope of this paper.

The main objective of this paper is to study and discuss the main methods used to allocate the network usage costs. Note that a less exhaustive work that also studies and analyzes several methods to allocate network usage costs in transmission systems is presented in [14]. This paper is organized as follows: section 2 introduces the main methods present in the literature; section 3 illustrates the methods using a 4–bus system; a more complex IEEE 24–bus RTS case study is presented in section 4, and section 5 presents the conclusions reached with the different methods that are analyzed.

2 Network usage cost allocation methods

Any network usage cost method must be both calculated and defined for a certain period, but there are different proposals to allocate this cost. In this section five ways to allocate this cost are presented: the pro rata method(PR), the proportional sharing method (PS), the Equivalent Bilateral Exchange (EBE) method, the nodal method, and the Z_{bus} method.

2.1 Pro rata (PR) method

The PR method, also known as postage stamp method, revised in [3] and [4], allocates costs proportionally to the power injected by each generator or by each demand. But, to do this, it's necessary to decide how much of the cost should be attributed to generators and demands. For example, a 50% proportion can be used.

Consider a system of n buses and nl lines. Assuming a converged power flow, the power generated (P_{Gk}) and consumed (P_{Dk}) at each bus k of the system can be obtained. Then, the total network usage of each line l is calculated as:

$$U_l = \sum_{k=1}^{n} U_l^{Gk} + \sum_{k=1}^{n} U_l^{Dk} = U_l^G + U_l^D. \tag{1}$$

where:

U_l: Usage of line l (MW).

U_l^{Gk}: Usage of line l allocated to generator located at bus k (MW).

U_l^{Dk}: Usage of line l allocated to demand located at bus k (MW).

U_l^G: Usage of line l allocated to generators (MW).

U_l^D: Usage of line l allocated to demands (MW).

According to the PR method the usage of any line l allocated to any generator (demand) should be given as the proportion of its generator (demand) in relation to the generation (consumption) of the system. Thus:

$$U_l^{Gk} = (\frac{P_{Gk}}{\sum_{k=1}^{n} P_{Gk}})U_l^G \tag{2}$$

$$U_l^{Dk} = (\frac{P_{Dk}}{\sum_{k=1}^{n} P_{Dk}})U_l^D. \tag{3}$$

For the sake of simplicity and for each line, we consider a total annualized line cost in $\$/h$, C_l, which includes operation, maintenance and building costs. Note that how this cost is computed is outside the scope of this paper. This cost represents the total cost allocated to generators (C_l^G) plus the total cost allocated to demands (C_l^D).

$$C_l = C_l^G + C_l^D. \tag{4}$$

The corresponding cost rates of line l respectively allocated to generators and demands are:

$$r_l^G = \frac{C_l^G}{U_l^G} \tag{5}$$

$$r_l^D = \frac{C_l^D}{U_l^D}. \tag{6}$$

In this way, the cost of line l respectively allocated to generators and demands located at bus k is:

$$C_l^{Gk} = r_l^G U_l^{Gk} \tag{7}$$

$$C_l^{Dk} = r_l^D U_l^{Dk}. \tag{8}$$

The total cost of the system, C, is obtained as a sum of the total cost of each line:

$$C = \sum_{l=1}^{nl} C_l = \sum_{l=1}^{nl} \sum_{k=1}^{n} (C_l^{Gk} + C_l^{Dk}). \tag{9}$$

2.2 Proportional sharing (PS) method

The network usage cost calculated by the PS method [6] is obtained observing how the power flow distributes itself through the transmission lines, taking into account the power injection that originates it.The idea is that each extraction of power at a bus has a direct relationship with each injection of power at this bus. To apply this method, it's necessary to split it into two parts: first to allocate the cost to generators (upstream looking algorithm) and, second, to allocate cost to demands (downstream looking algorithm). Obviously, it's also necessary to attribute a proportion of the costs allocated to generators and demands, for example, a 50% share.

For the upstream looking algorithm, consider P_{ji} (the power flow from bus j to bus i) and P_j (the nodal power, calculated as a sum of all power injected or extracted at bus j). Considering that the power flow is much bigger than the losses in each line, the losses term can be ignored, therefore we assume that $|P_{ji}| = |P_{ij}|$.

Considering the expression $|P_{ji}| = c_{ji} P_j$, where $c_{ji} = |P_{ji}| / P_j$, then:

$$P_{Gi} = P_i - \sum_{j \in \alpha_i^u} c_{ji} P_j. \tag{10}$$

where:

α_i^u: Set of buses connected and supplying directly bus i.

Rearranging (10):

$$P_G = A_u P, \tag{11}$$

where:

P_G: Total generation vector.

A_u: Upstream distribution matrix (nxn).

P: Power injection vector.

The element (i, j) from A_u matrix relates the power flow arriving bus i with the power injected at bus j. An element (i, j) of this matrix is given by:

$$[A_u]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -c_{ji} = -\left|P_{ji}\right| / P_j & \text{for } j \in \alpha_i^u \\ 0 & \text{otherwise} \end{cases}$$

α_i^u: Set of buses connected and supplying directly by bus i.

The A_u matrix is non-symmetrical. If A_u^{-1} exists, then $P = A_u^{-1} P_G$ and the element i of vector P, is given by:

$$P_i = \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ik} P_{Gk} \ \text{ for } i = 1, 2, ..., n. \tag{12}$$

Applying the proportional sharing principle:

$$|P_{ij}| = \frac{|P_{ij}|}{P_i} P_i = \frac{|P_{ij}|}{P_i} \sum_{k=1}^{n} \left[A_u^{-1}\right]_{ik} P_{Gk} \ \text{ for } j \in \alpha_i^d, \tag{13}$$

where:

α_i^d: Set of buses connected and supplied directly by bus i.

Considering the power flow through any line l (connected to bus i), the power flow contribution from generator k to this line l is:

$$U_l^{Gk} = \frac{|P_{ij}|}{P_i} \left[A_u^{-1}\right]_{ik} P_{Gk} \quad \text{for } j \in \alpha_i^d. \tag{14}$$

Finally, with the cost rate of line l calculated as in (5), the cost of line l allocated to a generator located at bus k is:

$$C_l^{Gk} = r_l^G U_l^{Gk}. \tag{15}$$

Analogously for demands, the power flow $|P_{ij}|$ can be calculated as:

$$|P_{ij}| = \frac{|P_{ij}|}{P_i} P_i = \frac{|P_{ij}|}{P_i} \sum_{k=1}^{n} \left[A_d^{-1}\right]_{ik} P_{Dk} \quad \text{for all } j \in \alpha_i^u, \tag{16}$$

where $\left[A_d^{-1}\right]_{ik}$ relates the power flow leaving bus i with the power injected at bus j :

$$[A_d]_{ij} = \begin{cases} 1 & \text{for } i = j \\ -c_{ji} = -|P_{ji}|/P_j & \text{for } j \in \alpha_i^d \\ 0 & \text{otherwise} \end{cases}$$

The term $\frac{|P_{ij}|}{P_i} \left[A_d^{-1}\right]_{ik} P_{Dk}$ represents the contribution of demand k to the power flow through line ij, representing the network usage of demand k for this line.

Considering the power flow through any line l (connected at bus i), the power flow contribution from demand k to this line l is:

$$U_l^{Dk} = \frac{|P_{ij}|}{P_i} \left[A_u^{-1}\right]_{ik} P_{Dk} \quad \text{for } j \in \alpha_i^u. \tag{17}$$

Finally, with the cost rate of line l calculated as in (6), the cost of line l allocated to demand located at bus k is:

$$C_l^{Dk} = r_l^D U_l^{Dk}. \tag{18}$$

2.3 Equivalent Bilateral Exchange (EBE) method

The term Equivalent Bilateral Exchange (EBE) represents a power injection and an offtake of the same entity. It suggests the existence of a generator and a demand with the same active power, but in opposite directions. To tackle this problem, it proposes unbundling line flows using the EBE methodology [8], which allows identifying the responsibility of each generator/demand on the unbundled flows through every line. This methodology is based on the EBE principle, in which each generator provides a predefined fraction of power to each demand, and each demand receives a predefined fraction of power from each generator. This hypothesis has been used in [8], [12] and [15].

In this way, assuming a generator k and a demand j, the individual contribution from generator k to demand j is given by:

$$GD_{kj} = \frac{P_{Gk}^{DC} P_{Dj}}{P^{Total}}, \tag{19}$$

where:

P^{Total}: Sum of all power demands.

P_{Dj}: Demand at bus j.

P_{Gk}^{DC}: Power generated in any bus k considering a lossless system.

Then, the contribution of each EBE to the power flow in each line of the system can be determined by the following expression:

$$F = \beta P, \tag{20}$$

where P represents the active power vector at each bus (disregarding the slack bus) and β represents the sensitivity matrix of the system [16], [17]. The P vector of power injections describes a generic EBE with one injection GD_{kj} at bus i and one extraction GD_{kj} at bus j, so that:

$$P = \begin{pmatrix} 0 & \cdots & GD_{kj} & \cdots & -GD_{kj} & \cdots & 0 \end{pmatrix}^T .$$
$$\begin{array}{ccccccc} & & \uparrow & & \uparrow & & \\ & & k & & j & & \end{array}$$

The vector F of (20) expresses the power flow in each transmission line due to the EBE formed by generator k and demand j. In order to determine the network usage allocated to generator k in the line l, half of the sum of all $EBEs$ containing generator k must be considered, for this line. Then:

$$U_l^{Gk} = \frac{1}{2} \sum_{j=1}^{n_G} F_l^{kj}. \tag{21}$$

Analogously, for demand k:

$$U_l^{Dk} = \frac{1}{2}\sum_{j=1}^{n_D} F_l^{jk}, \tag{22}$$

where F_l^{kj} is the power flow through line l due to the EBE composed of generator k and the demand j.

Finally, considering the cost rates as in (5) and (6), the cost of line l allocated to generators and demands located at bus k is:

$$C_l^{Gk} = r_l^G U_l^{Gk} \tag{23}$$

$$C_l^{Dk} = r_l^D U_l^{Dk}. \tag{24}$$

2.4 Nodal method

The nodal method is the actual methodology approved by ANEEL (the Electric Energy National Agency of Brazil) for the network usage cost allocation, based on the concept of nodal tariffs. In this methodology the network users pay for network usage considering their location and their maximum capacity of possible power (generated/consumed) to be dispatched, but not necessarily dispatched. Details of the method used, as well as the software to calculate the cost allocated to each agent, can be found in [1].

In this paper the method is modified to be rightly compared with other methods. Thus, the results obtained by this method depend on the network and also on the power (generated or consumed) dispatched at the buses of the system.

The measure that effectively characterizes the cost of network usage allocated to each agent is given by the sensitivity matrix (β). However this matrix does not permit to allocate costs to the generators and loads located at the slack bus since the sensitivity of the power flow in any line with respect to the slack bus is zero. Therefore, the original nodal method [1] needs an adjustment to allocate costs to the slack bus. The original nodal method assigns the following cost of line l to an agent located at node k so that:

$$C_l^{k(before)} = \beta_{kl} r_l P_k F_{Wl}, \tag{25}$$

where:

$C_l^{k(before)}$: Line l cost allocated to agent k before the adjustment of the method.

β_{kl}: Sensitivity factor of line l with respect to node k.

r_l: Cost rate of line l expressed as

$$r_l = \frac{C_l}{U_l} \tag{26}$$

P_k: Power generated or consumed at node k.

F_{Wl}: Weighting factor of the line l whose value is:

$$F_{Wl} = \begin{cases} 1 & \text{if the level of loading of the line is above a pre-determined value;} \\ 0 & \text{if the level of loading of the line is below a pre-determined value;} \\ \frac{F_l}{Cap_l} & \text{otherwise,} \end{cases}$$

where:

F_l: Power flow through line l.

Cap_l: Transmission capacity of line l.

The factor F_{Wl} is used so that lines with low loading do not contribute to the cost allocated to the agents. For simplicity, we assume $F_{Wl} = 1$.

Therefore, the cost of line l allocated to generators and demands located at node k becomes:

$$C_l^{Gk(before)} = \beta_{kl} r_l^G P_k^G \tag{27}$$

$$C_l^{Dk(before)} = \beta_{kl} r_l^D P_k^D, \tag{28}$$

where:

$C_l^{Gk(before)}$: Cost of line l allocated to generator k before of the adjustment.

$C_l^{Dk(before)}$: Cost of line l allocated to demand k before of the adjustment.

In this way, the network usage cost of the entire transmission system allocated to generators and demands located at bus k is, respectively:

$$C^{Gk(before)} = \sum_{l=1}^{nl} \beta_{kl} r_l^G P_k^G \tag{29}$$

$$C^{Dk(before)} = \sum_{l=1}^{nl} \beta_{kl} r_l^D P_k^D, \tag{30}$$

In order to allocate costs to the slack bus and to recover the costs allocated to generators (50% of the total cost), an adjustment is applied. Then:

$$C^{Gk} = C^{Gk(before)} + \Delta_G, \tag{31}$$

where:

$$\Delta_G = \frac{C^G - \sum_{k=1}^{n_G} C^{Gk(before)}(P_{Gk})}{\sum_{k=1}^{n_G} P_{Gk},} \tag{32}$$

C^{Gk}: Total transmission cost allocated to the generator located at bus k.

C^G: Total transmission cost allocated to all generators.

Analogously for demand k:

$$C^{Dk} = C^{Dk(before)} + \Delta_D, \tag{33}$$

where:

$$\Delta_D = \frac{C^D - \sum_{k=1}^{n_D} C^{Dk(before)}(P_{Dk})}{\sum_{k=1}^{n_D} P_{Dk}} \tag{34}$$

C^{Dk}: Total transmission cost allocated to the demand located at bus k.

C^D: Total transmission cost allocated to all demands.

2.5 Z_{bus} method

The Z_{bus} method can be found in [9] and was revised in [14]. To show the method, assume that the complex power injected through transmission line jk is:

$$S_{jk} = E_j \, I_{jk}^* \,, \tag{35}$$

where:

S_{jk}: Complex power flow through line jk.

E_j: Nodal voltage at bus j.

I_{jk}: Current through line jk.

Using the impedance matrix Z_{bus}, obtained as the inverse of the admittance matrix ($Y_{bus}^{-1} = Z_{bus}$) , the voltage at bus j can be calculated by:

$$E_j = \sum_{i=1}^{n} z_{ji}\; I_i \;, \tag{36}$$

where:

z_{ji}: Element (j, i) of the Z_{bus} matrix.

The current through line jk, I_{jk}, is obtained as:

$$I_{jk} = (E_j - E_k)\; y_{j\to k} + E_j\; y^{\text{sh}}_{j\to k} \;, \tag{37}$$

where:

$y_{j\to k}$: Series admittance of the π equivalent circuit of line jk.

$y^{sh}_{j\to k}$: Shunt admittance of the π equivalent circuit of line jk.

Substituting (36) in (37):

$$I_{jk} = \left(\sum_{i=1}^{n} z_{ji} I_i - \sum_{i=1}^{n} z_{ki} I_i \right)\; y_{j\to k} + \sum_{i=1}^{n} z_{ji}\; I_i\; y^{\text{sh}}_{j\to k} \;. \tag{38}$$

Rearranging (38):

$$I_{jk} = \sum_{i=1}^{n} \left[(z_{ji} - z_{ki})\; y_{j\to k} + z_{ji}\; y^{\text{sh}}_{j\to k} \right] I_i \;. \tag{39}$$

Note that the first term of the product in equation (39) is constant, since it only depends on network parameters. Thus, equation (39) can be written as:

$$I_{jk} = \sum_{i=1}^{n}\; a^i_{jk}\; I_i \;, \tag{40}$$

where:

$$a^i_{jk} = (z_{ji} - z_{ki}) y_{j\to k} + z_{ji}\; y^{\text{sh}}_{j\to k} \;. \tag{41}$$

The magnitude of the a^i_{jk} parameter provides a measure of the *electrical distance* between bus i and one point at the beginning of the line jk. The concept of *electrical distance* is important because it serves as a basis to determine the cost allocated to each bus, i.e., the bigger the *electrical distance* between a bus and a transmission line used by this bus, the bigger the cost allocated to this bus tends to be.

Substituting (40) in (35)

$$S_{jk} = E_j \sum_{i=1}^{n} (a^i_{jk}\ I_i)^* = \sum_{i=1}^{n} E_j\ a^{i*}_{jk}\ I^*_i\ . \tag{42}$$

Then, the active power through line jk is:

$$P_{jk} = \Re\{\sum_{i=1}^{n} E_j\ a^{i*}_{jk}\ I^*_i\}\ , \tag{43}$$

where:

$\Re$ - Real part of the complex number. Or, equivalently:

$$P_{jk} = \sum_{i=1}^{n} \Re\{E_j\ a^{i*}_{jk}\ I^*_i\}. \tag{44}$$

In this sense, the active power flow through any transmission line can be split and associated to the nodal current injection at each bus. Then, the power flow through line jk associated to current i is:

$$P^i_{jk} = \Re\{E_j\ a^{i*}_{jk}\ I^*_i\}\ . \tag{45}$$

Knowing that the term a^i_{jk} represents the *electrical distance* from bus i to a point at the beginning of the line jk (P_{jk} is the power flow through line jk calculated at the beginning of the line jk) , it's necessary to reach a better measure of the usage of line jk to better represent the *electrical distance* between one bus and one line. The idea is to calculate the average of the contribution from bus i to the transmission line jk, with the power flow calculated at the beginning and end of this line. Thus:

$$U^i_{jk} = U^i_l = \frac{\mid P^i_{jk} \mid + \mid P^i_{kj} \mid}{2} \quad \forall\, l\ \in\ \Omega_L, \tag{46}$$

where:

U_{jk}^i: Usage of line jk associated with the current injection at bus i. Also referred to as U_l^i.

Ω_L: Set of lines of the system.

The total usage of any line l is:

$$U_l = \sum_{i=1}^{n} U_l^i \ . \tag{47}$$

Without loss of generality, we consider at least a single generator and a single demand at each bus of the network. The usage of line jk apportioned to the generator or demand located at bus i is stated below.

If bus i contains only generation, the usage allocated to generator i pertaining to line l is:

$$U_l^{Gi} = U_l^i \ . \tag{48}$$

On the other hand, if bus i contains only demand, the usage allocated to demand i pertaining to line l is:

$$U_l^{Di} = U_l^i \ . \tag{49}$$

Else, if bus i contains both generation and demand, the usage allocated to the generation at bus i pertaining to line l is:

$$U_l^{Gi} = [P_{Gi}/(P_{Gi} + P_{Di})]U_l^i \ . \tag{50}$$

And the usage allocated to the demand at bus i pertaining to line l is:

$$U_l^{Di} = [P_{Di}/(P_{Gi} + P_{Di})]U_l^i \ . \tag{51}$$

Expressions (48) and (49) correspond to buses with either generation or demand, respectively, and expressions (50) and (51) correspond to buses including both generation and demand.

Finally, considering the cost rate as expressed in (5) and (6), the cost of line l allocated to generators and demands located at bus k is, respectively:

$$C_l^{Gk} = r_l^G U_l^{Gk} \tag{52}$$

$$C_l^{Dk} = r_l^D U_l^{Dk}. \tag{53}$$

3 Example of application

Some premises presented in section 1 indicate certain subjectivity in the evaluation (for example, easiness to promote regulation and transparency of the cost allocation procedure). However, the last three premises can be further evaluated. To illustrate them, several analyses regarding the locational viewpoint, the remuneration of new investments and the stability of tariffs, are presented. All analyses are done for the 4–bus system depicted in Fig. 1.

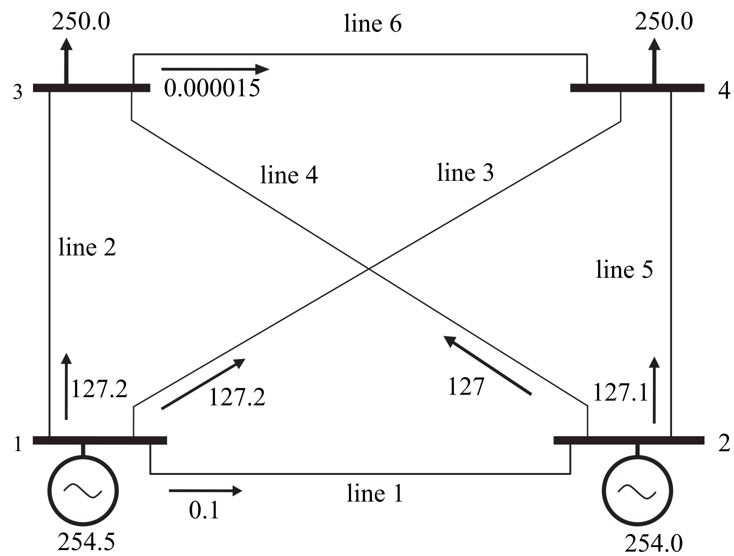

Fig. 1. 4–bus system with the AC power flow solution.

Fig. 1 presents the result of the power flow solution for this system. For simplicity, a symmetrical system is selected, where all the buses are connected to each other. All the lines have the same values of resistance and reactance, $r = 0.01275\,pu$ and $x = 0.097\,pu$, respectively. The transmission system cost is obtained as the sum of the costs of each transmission line, resulting a value of $C = 582.0$ \$/h. The cost of line jk is directly obtained from its reactance value: $C_{jk} = 1000 * x_{j-k}$ \$/h. The power injected and extracted at each bus is practically the same; approximately 254.0 MW for the generators, and 250.0 MW for the demands.

3.1 Locational viewpoint

All the remuneration procedures, considering a locational viewpoint, can be analyzed observing the costs for network usage in the base case (Fig. 1). Since we have a system which is practically symmetrical, and because generators and demands have almost the same values, it's expected that the costs for network usage should be practically the same for all the generators and demands. Table 1 shows the costs allocated to each agent (generator/demand) of the system in the base case. The cost values shown in this table are obtained from the network usage allocated to each agent multiplied by the transmission cost of the system, in this case 582.0 \$/h.

Table 1
Transmission cost allocation.

Bus	Cost (\$/h)				
-	PR	PS	EBE	Nodal	Z_{bus}
1	145.42	121.21	143.56	112.87	145.66
2	145.58	169.79	147.44	178.13	145.81
3	145.50	121.25	145.50	145.50	145.27
4	145.50	169.75	145.50	145.50	145.27
Total	582.0	582.0	582.0	582.0	582.0

The PR, EBE and Z_{bus} methods produce almost the same results, approximately 145 \$/h for each agent (generator/demand). These results are reasonable since the system is relatively symmetrical. On the other hand, the PS and nodal methods present very different results for some agents, indicating a disequilibrium between buses in the system, which, in fact, does not exist.

For the locational analysis, consider a change in the cost of the transmission lines of the base case. For example, an increase in the reactance of line 1, resulting in an increase of the cost of this line. Now, the reactance value and the cost of this line are $x_{1-2} = 0.194\,pu$ and $C_{12} = 194$ \$/h, respectively, twice the original value. Table 2 shows the cost allocation to each agent in this new case.

Table 2
Transmission cost allocation with an increase in the cost of line 1.

Bus	Cost (\$/h)				
-	PR	PS	EBE	Nodal	Z_{bus}
1	169.89	194.03	167.46	45.82	180.22
2	169.61	96.97	172.04	293.68	179.95
3	169.75	145.50	169.75	169.75	159.41
4	169.75	145.50	169.75	169.75	159.41
Total	679.0	679.0	679.0	679.0	679.0

In this new simulation it's expected for the agents electrically close to a certain line to be apportioned most of the usage of this line. In this case, it's desirable that generators 1 and 2 suffer the biggest cost allocation changes due to the increment of the line 1 cost. This aspect is also discussed in [9], and it's called the *proximity effect*. According to Table 2, it can be noted that the Z_{bus} method shows the *proximity effect*, significantly increasing the costs allocated to generators 1 and 2 up to approximately 180 \$/h, and up to 159.41 \$/h for demands 3 and 4. This property cannot be seen in the other methods.

3.2 Remuneration considering new investments

Consider now that two new identical generators are introduced, and they are connected by identical transmission lines to the same bus. For example, assume that two new generators 5 and 6 are added and the power injected by each is 125 MW. These generators are both connected by two transmission lines to bus 3. Generators 1 and 2 modifiy their respective productions to 131 MW and 125 MW, respectively. In this case the demands remain at the same value of 250 MW. Table 3 presents the cost allocation to the generators and demands in the new configuration.

Table 3
Transmission cost allocation with the introduction of two new generators.

-	PR	PS	EBE	Nodal	Z_{bus}
1	100.95	152.12	101.04	13.64	102.27
2	95.68	100.61	101.04	104.58	97.33
3	194.00	167.31	149.54	194.00	136.05
4	194.00	220.69	238.46	194.00	182.78
5	95.68	67.64	92.96	134.89	128.78
6	95.68	67.64	92.96	134.89	128.78
Total	776.0	776.0	776.0	776.0	776.0

Observing Table 3, all the methods produce the same cost allocation for the new generators 5 and 6, as expected. However, in the nodal method, the changes produced in the system indicate a significant variation of the costs allocated to generator 1 (13.64 \$/h) in relation to the other generators. This means that there exists a certain volatility associated to this method.

3.3 Stability of tariffs

Another useful metric to check the advantages of the allocation methods is the analysis of the tariffs' stability.

To understand the advantages of the results presented by this parameter (tariff) we assume that all methods have two factors that determine the cost allocation of the network usage for each agent: first, a factor that depends on the network (also called tariff) and, second, a factor related to the power generated (or consumed) at each bus. This actually happens to all methods presented in this paper, except for the PR method, that does not depend on the network.

For a method to show its network dependence, the value of the tariff, calculated as the cost allocated to each generator (demand) divided by the power generated (consumed) in the bus where it's located, must be different for each agent. However, it is also desirable that these tariffs (for all agents) are within a relatively small range of values. The smaller the range of values, the greater the stability (lesser volatility of tariffs over time).

This idea was, firstly, introduced in [8]. The expressions (54) and (55) describe how to obtain these tariffs for the generators and demands:

$$r_{Gi} = \frac{C_{Gi}}{P_{Gi}} \tag{54}$$

$$r_{Di} = \frac{C_{Di}}{P_{Di}} \ . \tag{55}$$

Table 4 shows the maximum and minimum values of the tariffs as obtained in the base case 4–bus system, as well as the standard deviation, the average results and the volatility for each method, calculated as the ratio between the standard deviation and the average of the rates.

Table 4
Tariffs for generators and demands.

-	PR	PS	EBE	Nodal	Z_{bus}
Maximum ($/MWh)	0.58	0.57	0.57	0.95	0.57
Minimum ($/MWh)	0.57	0.67	0.58	0.58	0.58
Standard deviation	0.01	0.10	0.01	0.07	0.01
Average	0.58	0.58	0.58	0.58	0.58
Volatility (%)	1.72	17.24	0.0	12.06	1.72

According to the results shown in Table 4, it can be seen that the PR, EBE and Z_{bus} methods have lower volatilities than the other methods, nodal and PS. These results show that the latter methods are highly volatile. To confirm this trend we have performed the same analyses in the IEEE 24–bus RTS system. These analyses are presented in the next section.

4 Case study

The IEEE 24–bus RTS system shown in Fig. 2, whose data are depicted in [18], is presented in this case study. The same five methods applied to the 4–bus system are used in this section. The converged power flow corresponds to the IEEE RTS peak load, on the Tuesday of week 51 from 5 p.m to 6 p.m, as in [9]. The aspects referring to location, new investments and statistics rates are also discussed.

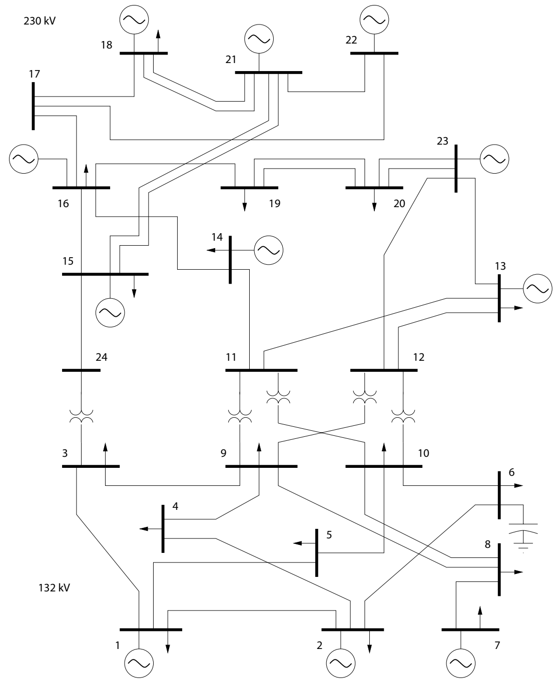

Fig. 2. IEEE 24–bus RTS system.

4.1 Locational viewpoint

Tables 5 and 6 present the base case network usage cost allocations to generators and demands, respectively. Issues related to location can be observed in generators at buses 1 and 16. Both buses have similar power generations, 172 MW and 155 MW, and similar power injections, 64 MW and 55 MW, as compared to the other buses in the system. According to the PR method, the cost allocation does not depend on the network topology, therefore, the cost allocated to both generators is: 81.44 \$/h for generator 1, and 73.99 \$/h for generator 16. The ratio between both is 1.1 (81.44/73.99).

However, we should expect the cost allocation to generator 1 to be much higher in value than the cost allocation to generator 16, because generator 1 uses the transmission network much more, being at the corner of the network. This

Table 5
Transmission cost allocation to the generators.

Bus	Cost ($/h)				
-	PR	PS	EBE	Nodal	Z_{bus}
1	81.44	64.27	120.42	90.39	41.88
2	81.44	138.12	121.94	99.68	51.27
7	113.63	30.70	161.97	37.53	118.78
13	117.12	0.0	76.02	176.93	4.51
15	101.80	0.0	86.08	114.42	34.84
16	73.39	20.36	54.21	86.79	10.48
18	189.39	29.90	171.96	158.95	28.74
21	189.39	309.70	166.79	152.70	370.45
22	142.04	266.98	179.07	83.68	339.17
23	284.08	513.69	235.24	372.63	413.47
Total	1373.72	1373.72	1373.72	1373.72	1413.59

conclusion is ratified by the PS, EBE and Z_{bus} methods, obtaining a cost ratio of the generators of 3.16, 2.22 and 3.99, respectively.

Regarding the demands, the same analysis can be done with the demands located at buses 8 and 14, the former being a heavier user of the network. The power demands are 171 MW and 194 MW, respectively. Although bus 14 has a bigger power demand, a reduction of costs due to its location in the system should take place. Again, this occurs with the PS, EBE, nodal and Z_{bus} methods, that allocate higher costs to bus 8, highlighting the importance of the locational viewpoint.

4.2 Remuneration considering new investments

Consider two new generators at buses 25 and 26, with a power generation of 25 MW each, connected by two identical transmission lines to bus 24. Table 7 presents the cost allocation to the generators in this new configuration.

The cost of the new transmission lines is 135.6 $/h. First, all the methods must have the same cost allocation results for generators 25 and 26, since they are identical generators connected by identical transmission lines to the same bus. Second, most of the costs due to the new investments in transmission should be allocated to the new generators located at buses 25 and 26. This happens because the new transmission lines 24-25 and 24-26 are the only way for buses 25 and 26 to inject or to extract power in the network. This issue is also

Table 6
Transmission cost allocation to the demands.

Bus	Cost ($/h)				
-	PR	PS	EBE	Nodal	Z_{bus}
1	52.06	0.0	82.15	48.36	26.30
2	46.75	0.0	75.10	38.20	28.91
3	86.76	236.99	102.99	66.42	180.24
4	35.67	120.66	56.95	40.03	85.71
5	34.22	62.43	49.26	46.81	71.54
6	65.55	192.68	94.82	72.15	74.74
7	60.25	0.0	96.22	102.12	61.86
8	82.42	99.82	110.35	181.69	179.96
9	84.35	215.62	90.30	70.84	138.29
10	93.99	218.47	104.89	107.37	85.71
13	127.73	7.70	102.16	18.71	4.83
14	93.51	72.77	64.37	78.41	125.48
15	152.79	31.28	101.97	139.83	51.37
16	48.20	0.0	29.32	41.34	6.76
18	160.51	0.0	115.81	191.78	23.93
19	87.24	62.24	55.26	78.56	94.55
20	61.70	53.05	41.77	51.07	93.64
Total	1373.72	1373.72	1373.72	1373.72	1333.82

shown in [9]. In this sense, the PS method is the most adequate, allocating most of the cost difference between the base case and the modified case to the new generators: $59.51 + 59.51 = 119.02$ $/h. The EBE and Z_{bus} methods also allocate higher costs due to the new investments to generators 25 and 26, however with relatively lesser values than the PS method. On the other hand, the PR and nodal methods distribute these costs costs among all the generators and demands, which cannot be considered reasonable.

4.3 Stability of tariffs

Table 8 shows the maximum and minimum values of the generator tariff for the base case, as well as the average, the standard deviation and the volatility. Table 9 presents the same information for the demands.

According to the results presented in Tables 8 and 9, the EBE and nodal

Table 7
Transmission cost allocation to the generators with the new configuration.

Bus	Cost ($/h)				
-	PR	PS	EBE	Nodal	Z_{bus}
1	85.42	62.83	120.05	95.86	42.72
2	85.42	133.51	121.60	105.15	52.45
7	119.20	30.70	161.73	45.16	120.46
13	98.55	0.0	56.77	136.44	18.18
15	106.78	0.0	85.78	121.26	35.80
16	76.98	20.47	54.05	91.72	10.60
18	198.66	29.92	171.43	171.68	28.30
21	198.66	291.46	166.19	165.42	363.63
22	149.00	259.69	178.54	93.23	335.67
23	297.99	493.89	235.14	391.72	407.20
25	12.42	59.51	45.10	11.93	47.47
26	12.42	59.51	45.10	11.93	47.47
Total	1441.5	1441.5	1441.5	1441.5	1509.9

Table 8
Tariffs for the generators.

-	PR	PS	EBE	Nodal	Z_{bus}
Maximum ($/MWh)	0.47	0.89	0.71	0.72	1.13
Minimum ($/MWh)	0.47	0.0	0.31	0.16	0.02
Standard deviation	0.0	0.34	0.14	0.14	0.32
Average	0.47	0.25	0.43	0.53	0.27
Volatility (%)	0.0	137	32.0	26.0	119.0

Table 9
Cost rates for the demands.

-	PR	PS	EBE	Nodal	Z_{bus}
Maximum ($/MWh)	0.48	1.63	0.77	1.06	1.16
Minimum ($/MWh)	0.48	0.0	0.29	0.07	0.02
Standard deviation	0.00	0.33	0.10	0.15	0.30
Average	0.48	0.36	0.34	0.42	0.48
Volatility (%)	0.0	93.0	29.0	37.0	62.0

methods showed the lowest volatility. This means that these methods provide greater stability against changes in the network; however note that the nodal method shows high volatility in the 4–bus system. In general, the nodal method allocates more cost to the buses that induce big power flow changes in the system. In the 4–bus system, which was almost symmetrical, the method allocated the biggest portion of the cost to bus 2, which is also responsible for the high volatility. In bigger systems, such as the IEEE 24–bus RTS system, this trend is lessened, due to the asymmetry of the system and the splitting of the allocations between several buses.

The PS method presents more dispersed values, indicating lower stability. The Z_{bus} method shows intermediate values, indicating relative stability compared to the PS method, but lesser stability compared to the EBE and nodal methods. Finally, the PR method presents the highest level of stability since this method does not depend on the network topology.

In this sense, the results presented using the IEEE 24–bus RTS are more general than the ones of the 4-bus system because they represent a more realistic case study.

5 Conclusions

A comparison of the main methods of network cost allocation present in the literature is presented in this paper. An analysis of the main characteristics of several methods and recommendations for their use in different situations follows.

The PR method is not sensitive to the transmission system, i.e, this method can be considered poor in the locational aspect, and does not make an adequate remuneration method with regard to new investments. However, and for this reason, is a stable method with respect to tariff continuity, which, in some cases, is something desired by the investors.

The PS method shows a good performance considering the locational aspect and an optimal performance with respect to new investments in generation. However it shows the worst performance in terms of tariff stability.

The EBE and nodal methods present good results in terms of stability of tariffs in the IEEE 24–bus RTS system. However, they are less efficient than the Z_{bus} method regarding the locational viewpoint and less efficient than the PS method with regard to the remuneration of new investments.

The Z_{bus} method reveals more efficiency than all the methods with respect to the locational aspect and good performance with respect to new investments. Moreover, it shows relative stability in the tariff continuity, which is worse than the EBE and nodal methods, but better than the PS method, for the IEEE 24–bus RTS system.

The application of one method or another depends on several factors. If looking for a stable method with relation to the tariffs of the system, the PR method can be used. Looking for relative stability, with good performance of the locational aspect, either the EBE or the nodal methods should be used. Searching for a good method to allocate costs due to new investments and a good locational behavior, the PS method can be used, although it's very bad in terms of tariff stability. Finally, the Z_{bus} method is efficient in terms of locational behavior, presents good results with new investments, but not so good in terms of tariffs.

References

[1] Agência Nacional de Energia Elétrica (in portuguese). Online access from October, 2006. Available at http://www.aneel.gov.br.

[2] J. W. L. Marangon, Allocation of transmission fixed charges: an overview, *IEEE* Trans. Power Syst., 11(3):1409-1414, August 1996.

[3] M. Ilic, F. D. Galiana, and L. Fink, *Power Systems Restructuring: Engineering and Economics*, Norwell, MA, 1998.

[4] D. S. Kirschen and G. Strbac, *Fundamentals of Power System Economics*, Chichester, UK, Wiley, 2004.

[5] J. Bialek, Tracing the flow of electricity, *IEE* Proc. Gen. Trans. Distrib., 143(4):313-320, July 1996.

[6] J. Bialek, Topological generation and load distribution factors for supplement charge allocation in transmission open access, *IEEE* Trans. Power Syst., 12(1):52-60, February 1997.

[7] D. S. Kirschen, R. N. Allan, and G. Strbac, G., Contribution of individual generators to loads and flows, *IEEE* Trans. Power Syst., 12(1):52-60, February 1997.

[8] F. D. Galiana, A. J. Conejo, and H. A. Gil, Transmission network cost allocation based on Equivalent Bilateral Exchanges, *IEEE* Trans. Power Syst., 18(4):1425-1431, November 2003.

[9] A. J. Conejo, J. Contreras, D. A. Lima, and A. Padilha-Feltrin, Z_{bus} transmission network cost allocation, *IEEE* Trans. Power Syst., 22(1):342-349, February 2007.

[10] Florence School of Regulation, Study on the Inter-TSO compensation mechanism, European University Institute, October 2005.

[11] I. P. Arriaga, L. O. Camacho and F. J. Odériz, Cost components of cross border exchanges of electricity, Report prepared for the Directorate-General for Energy and Transport/European Commission, Instituto de Investigación Tecnológica Universidad Pontificia Comillas Madrid, Spain November 2002.

[12] H. A. Gil, F. D. Galiana, and A. J. Conejo, Multiarea transmission network cost allocation based on Equivalent Bilateral Exchanges, *IEEE* Trans. Power Syst., 20(3):1293-1301, August 2005.

[13] A. Cattareeya, O. Weerakorn and H. Hans-Jurgen , Allocation of transmission losses and network charges for cross border trades in ASEAN power grid based on tracing methodology, Power Tech IEEE Russia, 27(30):1-6, June 2005.

[14] D. A. Lima, Alocação de perdas e custos pelo uso do sistema transmissão (in portuguese), PhD Dissertation, UNESP-FEIS, Ilha Solteira SP, Brazil, 2007.

[15] D. A. Lima, A. J. Conejo, and J. Contreras, Allocation of the cost of transmission losses in a multi-market framework, *IEE* Proc. Gen. Trans. Distrib., 153(6):670-676, November 2006.

[16] A. R. Bergen and V. Vittal, *Power Systems Analysis*, 2nd edition, Upper Saddle River, New Jersey, Prentice Hall, 1999.

[17] A.G. Expósito, *Análisis y Operación de Sistemas de Energía Eléctrica* (in Spanish), McGraw-Hill, Madrid, 2002.

[18] IEEE Task Force, The IEEE Reliability Test System 1996, *IEEE* Trans. Power Syst., 14(3):1010-1020, August 1999.