Pontifícia Universidade Católica
do Rio de Janeiro

André Rubens França Carneiro

Surface diffeomorphisms with non-trivial invariant measures

MSc Thesis

Thesis presented to the Post–graduate Program in Pure Mathematics of the Mathematics Department, PUC-Rio as partial fulfillment of the requirements for the degree of Master in Pure Mathematics

Adviser: Prof. Julio Cesar de Souza Rebelo

Rio de Janeiro
August 2008

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Prof. Julio Cesar de Souza Rebelo
Adviser
Department of Mathematics — PUC-Rio

Prof. Rafael Osvaldo Ruggiero Rodriguez
Mathematics Department — PUC-Rio

Prof. Marcos Martins Alexandrino da Silva
Mathematics and Statistics Institute — USP

Prof. José Eugênio Leal
Head of the Science and Engineering Center — PUC-Rio

Rio de Janeiro — August 8, 2008

André Rubens França Carneiro

André Rubens França Carneiro graduated from the Pontifícia Universidade Católica do Rio de Janeiro (Brazil) in Pure Mathematics.

Bibliographic data

Carneiro, André Rubens França

Surface diffeomorphisms with non-trivial invariant measures / André Rubens França Carneiro; adviser: Julio Cesar de Souza Rebelo. — Rio de Janeiro : PUC-Rio, Department of Mathematics, 2008.

v., 34 f: il. ; 29,7 cm

1. MsC Thesis - Pontifícia Universidade Católica do Rio de Janeiro, Department of Mathematics.

Bibliography included.

1. Mathematics – Dissertation. 2. Surface diffeomorphisms. 3. Invariant measures. 4. Linear growth. I. Rebelo, Julio Cesar de Souza. II. Pontifícia Universidade Católica do Rio de Janeiro. Department of Mathematics. III. Title.

CDD: 510

Acknowledgments

To my advisor, Professor Julio Rebelo, for his support, availability and for the enormous deal of mathematics I learned from him.

To FAPERJ and PUC-Rio, for their financial support, without which this work would not have been realized.

To my parents, for their unconditional support.

To my professors, guilty of creating the passion for mathematics in me.

To my dear colleagues, who shared advices and accompanied me in this journey, always being a source of motivation.

To the staff of the Mathematic Departament, for their helpfulness and constant aid.

Abstract

Carneiro, André Rubens França; Rebelo, Julio Cesar de Souza. **Surface diffeomorphisms with non-trivial invariant measures**. Rio de Janeiro, 2008. 34p. MScThesis — Department of Mathematics, Pontifícia Universidade Católica do Rio de Janeiro.

Some diffeomorphisms of closed surfaces only have "trivial" invariant probabilities, i.e., those supported on the set of fixed points. Results of this nature make extensive use of the classification of surface homeomorphisms, making them typical of dimension 2. We attack this problem by showing that surface diffeomorphisms admiting "non-trivial" invariant probabilities exhibit some sort of positive linear growth. The techniques used are elementary and a significant part of the results remains valid in higher dimensions.

Keywords

Surface diffeomorphisms. Invariant measures. Linear growth.

Resumo

Carneiro, André Rubens França; Rebelo, Julio Cesar de Souza. **Difeomorfismos de superfície com medidas invariantes não-triviais**. Rio de Janeiro, 2008. 34p. DissertaÃâo de Mestrado — Departamento de Matemática, Pontifícia Universidade Católica do Rio de Janeiro.

Alguns difeomorfismos de superfícies fechadas possuem apenas medidas invariantes "triviais", isto é, medidas cujo suporte está contido no conjunto de pontos fixos. Resultados dessa natureza fazem uso fundamental da classificação dos homeomorfismos de superfície, tornando-os típicos da dimensão 2. Nós atacamos esse problema mostrando que difeomorfismos de superfícies que admitem medidas invariantes "não-triviais" exibem uma forma de crescimento linear positivo. As técnicas utilizadas são elementares e uma parte significativa dos resultados continua válida em dimensões mais altas.

Palavras–chave

Difeomorfismos de superfície. Medidas invariantes. Crescimento linear.

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