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Apêndice A

Apresenta-se neste anexo o código do Maple utilizado para obtenção dos deslocamentos axial e circunferencial em função de um dado deslocamento lateral.

```
restart: with(linalg): with(student): with(PDEtools):

#Equações não-lineares de equilíbrio da casca cilíndrica:
EM1:=diff(T11(x,y),x)+diff(T12(x,y),y);
EM2:=diff(T12(x,y),x)+diff(T22(x,y),y);
EM3:=-diff(M11(x,y),x$2)-2*diff(M12(x,y),x,y)-diff(M22(x,y),y$2)+T22(x,y)*(-
diff(w(x,y),y$2))+T22(x,y)*1/R     +(T11(x,y)-P(t))*(-diff(w(x,y),x$2))     +T12(x,y)*(-
2*diff(w(x,y),x,y));

#Deformações e mudanças de curvatura:
e11:=diff(u(x,y),x)+1/2*diff(w(x,y),x)^2;
e22:=diff(v(x,y),y)+w(x,y)/R+1/2*(-diff(w(x,y),y))^2;
e12:=diff(u(x,y),y)+diff(v(x,y),x)+(diff(w(x,y),x)*diff(w(x,y),y) );
k1:=-diff(w(x,y),x$2);
k2:=-(diff(w(x,y),y$2));
k12:=-(diff(w(x,y),x,y));

#Coeficientes da matriz constitutiva do material linear-elástico
Q11:=expand(E/(1-nu^2)):
Q22:=expand(E/(1-nu^2)):
Q12:=expand(E*nu/(1-nu^2)):
Q66:=expand(E/(2*(1+nu))):
Matriz de tensões ao longo da espessura:
Matrix_sigma:=evalm(matrix(3,3,[Q11,Q12,0,Q12,Q22,0,0,0,Q66])&*
matrix(3,1,[e11+z*k1, e22+z*k2, e12+2*z*k12]));

#Determinação das tensões atuantes
sigma11:=simplify(Matrix_sigma[1,1]):
sigma22:=simplify(Matrix_sigma[2,1]):
sigma12:=simplify(Matrix_sigma[3,1]):

#Resultante dos esforços de membrana e dos momentos atuantes na casca:
T11(x,y):=int(sigma11,z = -1/2*h .. 1/2*h);
T22(x,y):=int(sigma22,z = -1/2*h .. 1/2*h);
T12(x,y):=int(sigma12,z = -1/2*h .. 1/2*h);
M11(x,y):= int(z*sigma11,z=-h/2..h/2);
M22(x,y):= int(z*sigma22,z=-h/2..h/2);
M12(x,y):= int(z*sigma12,z=-h/2..h/2);

#Expansão modal para os deslocamentos transversais:
w(x,y):=W11*h*(sin(m*Pi/L*x))*cos(n*y/R);
```

```
#Substituição dos deslocamentos transversais nas duas primeiras equações de
#movimento
EM1:=expand(powsubs(w(x,y)=w(x,y),EM1)):
EM2:=expand(powsubs(w(x,y)=w(x,y),EM2)):

#Isolando u,xy na primeira equação da casca:
A:=solve(EM2,diff(u(x,y),x,y)):
uxy:=diff(u(x,y),x,y)=A:

#Aplicando identidades trigonométricas para a simplificação das equações
dir:=(rhs(uxy)):
dir:=factor(powsubs(sin(n*y/R)*cos(n*y/R)=1/2*sin(2*n*y/R), sin(m*Pi/L*x)^2=1/2-
1/2*cos(2*m*Pi/L*x), cos(m*Pi/L*x)^2=1/2*cos(2*m*Pi/L*x)+1/2,dir)):
dir:=factor(powsubs(sin(m*Pi/L*x)*cos(2*m*Pi/L*x)=1/2*sin(3*m*Pi/L*x)-
1/2*sin(m*Pi/L*x), cos(n*y/R)*sin(2*n*y/R)=1/2*sin(3*n*y/R)+1/2*sin(n*y/R),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(3*m*Pi/L*x)=1/2*sin(5*m*Pi/L*x)+1/2*sin
(m*Pi/L*x),dir)):
dir:=factor(powsubs(sin(n*y/R)*cos(n*y/R)=1/2*sin(2*n*y/R),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)^3=1/4*cos(6*m*Pi/L*x)+3/4*cos(2*m*Pi/L*x)
,dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)^2=1/2*cos(4*m*Pi/L*x)+1/2,dir)):
dir:=factor(powsubs(1/2*cos(4*m*Pi/L*x)+1/2*cos(8*m*Pi/L*x)=1/2*cos(4*m*Pi/L*
x)+1/2*cos(8*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(3*n*y/R)=1/2*sin(4*n*y/R)+1/2*sin(2*n*y/R),di
r)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*cos(6*m*Pi/L*x)=1/2*cos(4*m*Pi/L*x)+1/2*c
os(8*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(2*n*y/R)=1/2*sin(3*n*y/R)+1/2*sin(n*y/R),dir))
:
dir:=factor(powsubs(cos(n*y/R)*sin(4*n*y/R)=1/2*sin(5*n*y/R)+1/2*sin(3*n*y/R),di
r)):
dir:=factor(powsubs(cos(n*y/R)*sin(5*n*y/R)=1/2*sin(6*n*y/R)+1/2*sin(4*n*y/R),di
r)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(5*m*Pi/L*x)=1/2*sin(7*m*Pi/L*x)+1/2*sin
(3*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(m*Pi/L*x)=1/2*sin(3*m*Pi/L*x)-
1/2*sin(m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(3*n*y/R)=1/2*sin(4*n*y/R)+1/2*sin(2*n*y/R),di
r)):
dir:=factor(powsubs(cos(n*y/R)*sin(n*y/R)=1/2*sin(2*n*y/R),dir)):
uxy:=lhs(uxy)=dir;

#Derivando u,xy duas vezes em x e, em seguida, duas vezes em y:
Axx:=diff(A,x,x):
Ayy:=diff(A,y,y):

#Derivando a segunda equação de equilíbrio uma vez em x e, em seguida, uma
#vez em y:
B:=diff(EM1,x,y):

#Substituindo o valor de u,xxxy e u,xyyy na segunda equação:
BB:=powsubs(diff(u(x,y),y,x$3)=Axx,diff(u(x,y),x,y$3)=Ayy,B):
Eq:=isolate(BB,v(x,y)):
EQ:=collect(Eq,diff(v(x,y),`$`(x,4))):
```

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EQ:=collect(EQ,diff(v(x,y),`$`(x,2),`$`(y,2))):
EQ:=factor(collect(EQ,diff(v(x,y),`$`(y,4)))):
EQ:=expand(simplify(EQ/R^5/L^4/(nu-1))):
esq:=lhs(EQ):

#Aplicando identidades trigonométricas para a simplificação das equações
dir:=(rhs(EQ)):
dir:=factor(powsubs(sin(n*y/R)*cos(n*y/R)=1/2*sin(2*n*y/R),
cos(m*Pi/L*x)^2=1/2*cos(2*m*Pi/L*x)+1/2,dir)):
dir:=factor(powsubs(sin(m*Pi/L*x)*cos(2*m*Pi/L*x)=1/2*sin(3*m*Pi/L*x)-
1/2*sin(m*Pi/L*x),          cos(n*y/R)*sin(2*n*y/R)=1/2*sin(3*n*y/R)+1/2*sin(n*y/R),
cos(n*y/R)*sin(3*n*y/R)=1/2*sin(4*n*y/R)+1/2*sin(2*n*y/R),
cos(n*y/R)*sin(n*y/R)=1/2*sin(2*n*y/R),cos(2*m*Pi/L*x)*sin(3*m*Pi/L*x)=1/2*sin(5
*m*Pi/L*x)+1/2*sin(m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(3*m*Pi/L*x)=1/2*sin(5*m*Pi/L*x)+1/2*sin
(m*Pi/L*x),dir)):
dir:=factor(powsubs(sin(n*y/R)*cos(n*y/R)=1/2*sin(2*n*y/R),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)^3=1/4*cos(6*m*Pi/L*x)+3/4*cos(2*m*Pi/L*x)
,dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)^2=1/2*cos(4*m*Pi/L*x)+1/2,dir)):
dir:=factor(powsubs(1/2*cos(4*m*Pi/L*x)+1/2*cos(8*m*Pi/L*x)=1/2*cos(4*m*Pi/L*
x)+1/2*cos(8*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(3*n*y/R)=1/2*sin(4*n*y/R)+1/2*sin(2*n*y/R),di
r)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*cos(6*m*Pi/L*x)=1/2*cos(4*m*Pi/L*x)+1/2*c
os(8*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(2*n*y/R)=1/2*sin(3*n*y/R)+1/2*sin(n*y/R),dir))
:
dir:=factor(powsubs(cos(n*y/R)*sin(4*n*y/R)=1/2*sin(5*n*y/R)+1/2*sin(3*n*y/R),di
r)):
dir:=factor(powsubs(cos(n*y/R)*sin(5*n*y/R)=1/2*sin(6*n*y/R)+1/2*sin(4*n*y/R),di
r)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(5*m*Pi/L*x)=1/2*sin(7*m*Pi/L*x)+1/2*sin
(3*m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(2*m*Pi/L*x)*sin(m*Pi/L*x)=1/2*sin(3*m*Pi/L*x)-
1/2*sin(m*Pi/L*x),dir)):
dir:=factor(powsubs(cos(n*y/R)*sin(3*n*y/R)=1/2*sin(4*n*y/R)+1/2*sin(2*n*y/R),di
r)):
dir:=factor(powsubs(cos(n*y/R)*sin(n*y/R)=1/2*sin(2*n*y/R),dir)):
EQ:=esq=dir;

#Resolvendo por substituição a EDP de quarta ordem de v(x,y):
fac:=-h*W11*n/L^4/R^5:
term_dir:=simplify(rhs(EQ)/fac):
F:=0:
for i from 1 to nops(term_dir) do
subeq:=op(i,term_dir):
r:=1/(diff(subeq,x$4)+diff(subeq,y$4)+2*diff(diff(subeq,x$2),y$2));
f:=factor(r*subeq^2*fac);
F:=F+f;
od:
v(x,y):=collect(F,    [sin(n*y/R),    sin(2*n*y/R),    sin(3*n*y/R),    sin(4*n*y/R),
sin(5*n*y/R),   sin(6*n*y/R),   sin(m*Pi/L*x),   sin(3*m*Pi/L*x),   sin(5*m*Pi/L*x),
sin(7*m*Pi/L*x),       cos(2*m*Pi/L*x),       cos(4*m*Pi/L*x),       cos(6*m*Pi/L*x),
cos(8*m*Pi/L*x) ], distributed);
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#Verificacao de v.
expand(lhs(EQ)-rhs(EQ)): simplify(%);

#Simplificando a equação de v. Substitui os termos constantes por Aj.
VP:=0:
for i from 1 to nops(v(x,y)) do
   temp:=op(i,v(x,y)):
   a||i:=op(1,temp):
   t||i:=eval(temp/a||i);
   VP:=VP+A||i*t||i;
od:
v(x,y):=VP;

#Determinação de u a partir de u,xxxy
diff(u(x,y),x,x,x,y)=expand(Axx);

#Aplicando identidades trigonométricas para a simplificação das equações
dir:=rhs(%):
dir:=factor(powsubs(cos(n*y/R)*sin(n*y/R)=1/2*sin(2*n*y/R),dir)):
dir:=factor(powsubs(cos(m*Pi/L*x)^2=1/2*cos(2*m*Pi/L*x)+1/2,dir)):
dir:=factor(powsubs(sin(m*Pi/L*x)^2=1/2-1/2*cos(2*m*Pi/L*x),dir)):
diff(u(x,y),x,x,x,y)=dir;

#Resolve a EDP de u(x,y)
pdsolve(%);

#Solução de u(x,y) com a parcela _F4(x) da solução particular. Para isso F1(y),
#F2(y) e F3(y) são assumidos como nulo.
assign(powsubs(_F3(y)=0,_F1(y)=0,_F2(y)=0,%));
Substituicao dos valores de Aj.
for i from 1 to nops(VP) do
  A||i:=eval(a||i);
od:

#Com u, v e w definidos falta apenas determinar as funções desconhecidas e
#suas constantes. Retornando a primeira equação de equilíbrio
simplify(EM1):
em1:=combine(%,trig);

#Obtém-se F4(x)
dsolve(em1,_F4(x));
assign(%);

#Retornando a segunda equação de equilíbrio. O campo de deslocamento
#utilizado atende esta equação
simplify(EM2):
em2:=combine(%,trig);

#Aplicação das condições de contorno. Nesta fase soma-se a u a sua solução
#homogênea com a seguinte forma F2(y)*x + F3(y). Essa forma é a solução
#homogênea de u,xxxy.
u(x,y):=u(x,y)+_F2(y)*x+_F3(y):
Condicao de contorno Nx=0 em x = 0 ou L.
a:=combine(powsubs(_F2(x)=_F2(x),x=0,eval(T11(x,y)))):
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a:=collect(a,[_C1,_F2(y),cos(2*n*y/R)],distributed);

#Por inspeção dos termos tem-se que:
coeff(a,_F2(y))*_F2(y)+coeff(a,cos(2*n*y/R))*cos(2*n*y/R):
_F2(y):=solve(%,_F2(y));
coeff(a,_C1)*_C1+(a-coeff(a,_C1)*_C1):
_C1:=solve(%,_C1);

#Condicao de contorno u(L/2)=0.
a:=collect(combine(powsubs(x=L/2/m,eval(u(x,y))),trig),[cos(2*n*y/R),_F3(y)],
distributed);

#Por inspeção dos termos:
(a-coeff(a,cos(2*n*y/R))*cos(2*n*y/R)-_F3(y)):
_C2:=solve(%,_C2);
collect(combine(powsubs(x=L/2/m,eval(u(x,y))),trig),[cos(2*n*y/R),_F3(y),_F1(y)],
distributed):
_F3(y):=simplify(solve(%,_F3(y)));

#Verificação do campo de deslocamento obtido. (u,v,w).
#Primeira equação de equilíbrio atendida.
simplify(em1,trig);

#Segunda equação de equilíbrio atendida.
simplify(em2,trig);

#Nx=0 em x=0 e x =L.
combine(eval(powsubs(_F2(x)=_F2(x),x=L/m,T11(x,y))));

#u(L/2)=0.
combine(eval(a));

#Simplificação do campo de deslocamento u(x,y) obtido
a:=collect((eval(powsubs(_C1=_C1,_C2=_C2,u(x,y)))),[cos(m*Pi/L*x),
sin(2*m*Pi/L*x), cos(n*y/R), cos(2*n*y/R), x],distributed):
prov:=0:
for i from 1 to nops(a) do
    temp:=factor((op(i,a)));
    prov:=prov+temp;
od:
u(x,y):=prov;
```